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On quasishape and quasihomology

Let X be a topological space, and let

$$\mathbf{SX} := (N\mathcal{U}, \mathcal{U} \in NCOV(X))$$

be a pro-space consisting of Vietoris nerves $N\mathcal{U}$, where $\mathcal{U}$ runs over all **normal** coverings of X. It is well-known (due to Bernd Günther) that X and **SX** are strong shape equivalent. Consider now the following pro-space

$$\mathbf{QX} := (N\mathcal{U}, \mathcal{U} \in COV(X))$$

where $COV(X)$ is the set of **all** coverings. The pro-space **QX** represents a class [**QX**] in the homotopy category

$$pro - CW\left[SSE^{-1}\right]$$

where SSE is the class of strong shape equivalences of pro-spaces.

Definition. The class [**QX**] will be called the *quasishape* of X.

Definition. The *quasishape category* QSh is the full subcategory of $pro - CW\left[SSE^{-1}\right]$ having classes [**QX**] as objects.

Remark. Since the strong homology $\overline{H}_*$ is well-defined on the category $pro - CW\left[SSE^{-1}\right]$, it is equally well-defined on the category QSh. The corresponding homology will be called *quasihomology* and denoted by QH_*.

Remark. It is clear that the quasishape and quasihomology of X is equivalent to the strong shape and strong homology of X, when X is paracompact.

Examples.

1. If X is a finite topological space, then its quasishape is that of a compact polyhedron.
2. If X is a locally finite topological space (i.e. every point has a finite open neighborhood), then its quasishape is that of a polyhedron.
3. Let X be the 4-point circle (the space with 4 points, weakly equivalent to a circle), and let $Y = \bigvee X$ be the wedge of countably many copies of X. Then Y has the quasishape of the Hawaiian earring.

3'.
$$QH_*(Y) \approx \prod_{i=1}^{\infty} H_*(S^1).$$

Theorem. QH_* satisfies the Eilenberg-Steenrod axioms and the wedge axiom, i.e.

$$QH_*\left(\bigvee X_\alpha\right) \approx \prod QH_*(X_\alpha).$$