Ivan Ivanšić*, *University of Zagreb, Croatia*

Extension Dimension of Stratifiable Spaces

Let X be a topological space, and K be a CW-complex. The notation $\dim X \leq K$, i.e., K is an absolute extensor of X, means that for every closed subspace A of X and map $f\colon A \to K$, there exists a map $F\colon X \to K$ which is an extension of f. Two other notations, $K \in \mathrm{AE}(X)$ and $X \tau K$ are used in the literature. For covering dimension dim, one of the results from the beginning is the Alexandroff Theorem: If X is a compact Hausdorff space, then $\dim X \leq n$ if and only if $\dim X \leq S^n$ (S^n denotes the n-sphere). Such and the analogous characterizations of cohomological dimension in terms of extensions of maps into Eilenberg-MacLane CW-complexes $K(G,n)$ have led A. Dranishnikov to define a notion of extension dimension.

Let $\mathcal{C}$ be a class of spaces, $\mathcal{T}$ a class of CW-complexes, and K, $K' \in \mathcal{T}$. If it is true that for all $X \in \mathcal{C}$, $\dim X \leq K$ implies that $\dim X \leq K'$, then we write $K \leq_{(\mathcal{C},\mathcal{T})} K'$. This defines a preorder on $\mathcal{T}$. One specifies $K \sim_{(\mathcal{C},\mathcal{T})} K'$ if and only if $K \leq_{(\mathcal{C},\mathcal{T})} K'$ and $K' \leq_{(\mathcal{C},\mathcal{T})} K$; then $\sim_{(\mathcal{C},\mathcal{T})}$ is an equivalence relation on $\mathcal{T}$, and $[K]_{(\mathcal{C},\mathcal{T})}$ denotes the equivalence class under this relation. We then write $\dim X \leq [K]_{(\mathcal{C},\mathcal{T})}$ to mean that $\dim X \leq K'$ for some (and hence for all) $K' \in [K]_{(\mathcal{C},\mathcal{T})}$.

For a given space $X \in \mathcal{C}$ and for a given class $\mathcal{T}$ of CW-complexes, we may ask if there is a minimal element in the following class of extension types:

$$\{[L]_{(\mathcal{C},\mathcal{T})} \mid \dim X \leq L\}\,.$$

If there is a minimal element $[K]_{(\mathcal{C},\mathcal{T})}$, then it is called the *extension dimension* of X relative to $(\mathcal{C},\mathcal{T})$ and is denoted by $\text{ext-dim}_{(\mathcal{C},\mathcal{T})} X = [K]_{(\mathcal{C},\mathcal{T})}$.

We prove a theorem on the existence of extension dimension for stratifiable spaces relative to the class of all CW-complexes. Since the class of stratifiable spaces contains that of metrizable spaces, then our result applies to metrizable spaces. In order to achieve this we introduce the class of so called dd-spaces. We define X to be a dd-space if it has the property that $\dim X \leq |K|$ for every simplicial complex K whose polyhedron $|K|$ is contractible. A wedge theorem for dd-spaces is then shown to be true. This is used in proving that the extension dimension of a given stratifiable space is always represented by a certain wedge of polyhedra. We also show that our techniques would apply to determine the existence of extension dimension for any class of compact Hausdorff spaces relative to the class of all CW-complexes.

*This is a joint work with Leonard R. Rubin