Sören Illman, *University of Helsinki, Finland*

Three Basic Results for Real Analytic Proper G-Manifolds

In this talk we will try to cover the main results of the paper

Sören Illman and Marja Kankaanrinta, Three basic results for real analytic proper G-manifolds, *Math. Ann.* 316 (2000), 169–183,

and also say something about the paper

Sören Illman and Marja Kankaanrinta, A new topology for the set $\mathrm{C}^{\infty,G}(M,N)$ of G-equivariant smooth maps.

By a real analytic proper G-manifold M we mean a real analytic manifold M on which a Lie group G acts by a real analytic and proper action. We wish to address the following three basic questions.

(*i*) Given a real analytic proper G-manifold M, does there exist a G-invariant real analytic Riemannian metric on M?

(*ii*) Let $f\colon M \to N$ be a G-equivariant C^r smooth map, $1 \le r \le \infty$, between two real analytic proper G-manifolds. Can one then always approximate f by a G-equivariant real analytic map $h\colon M \to N$?

(*iii*) Suppose G is a linear Lie group and let M be a real analytic proper G-manifold with only finitely many isotropy types. Does there then exist a G-equivariant real analytic imbedding of M into some finite-dimensional linear representation space for G?

We say that a Lie group is *good* if it is isomorphic to a closed subgroup of a Lie group with only finitely many connected components. Note in particular that every linear Lie group is good. Our main results are as follows.

Theorem I. *The answer to question* (*i*) *is affirmative when* G *is a good Lie group.*

Concerning question (*ii*) we prove that if G is a good Lie then every G-invariant C^r smooth map $f\colon M \to N$, $1 \le r \le \infty$, can be approximated arbitrarily well in the *strong-weak topology* by a G-equivariant real analytic map $h\colon M \to N$. More precisely we prove the following.

Theorem II. *Let* M *and* N *be real analytic proper* G*-manifolds, where* G *is a good Lie group. Then* $\mathrm{C}^{\omega,G}_{\mathrm{SW}}(M,N)$ *is dense in* $\mathrm{C}^{r,G}_{\mathrm{SW}}(M,N)$, $1 \le r \le \infty$.

As a corollary of Theorem II we obtain:

Corollary. *Let* M, N *and* G *be as in Theorem II. If* M *and* N *are* G*-equivariantly* C^1 *diffeomorphic they are also* G*-equivariantly real analytically isomorphic.*

Concerning question (*iii*) we prove:

Theorem III. *The answer to question* (*iii*) *is affirmative.*