7.2. HIPERBOLIČKE JEDNADŽBE 2. REDA

- POOPĆENJE JEDNADŽBE

$$u_{tt} - \Delta u = 0$$

7.2.1. DEFINICIJE

$U \subseteq \mathbb{R}^n$ OTVOREN, OGRANIČEN

$U_T = U \times \langle 0, T]$, $T > 0$

ZADAĆA:

$$(*) \begin{cases} u_{tt} + Lu = f & \text{u } U_T \\ u = 0 & \text{NA } \partial U \times [0,T] \quad \text{R.U.} \\ u = g & \text{NA } U \times \{0\} \quad \text{P.U.} \\ u_t = h & \end{cases}$$

ZADANO: $f : U_T \to \mathbb{R}$

$g, h : U \to \mathbb{R}$

$$Lu = -\operatorname{div}(A \nabla u) + b \cdot \nabla u + cu$$

$$\text{ILI}$$

$$Lu = -A \cdot H_u + b \cdot \nabla u + cu$$

$$A : U_T \to M_n(\mathbb{R})$$

$$b : U_T \to \mathbb{R}^n$$

$$c : U_T \to \mathbb{R}$$

TRAŽIMO: $u : U_T \to \mathbb{R}$, $u(x,t)$

DEF: OPERATOR $\frac{\partial^2}{\partial t^2} + L$ JE (UNIFORMNO) HIPERBOLIČKI AKO $\exists \theta > 0$ T.D.

$$A(x,t)\xi \cdot \xi \geq \theta |\xi|^2, \quad (x,t) \in U_T, \ \xi \in \mathbb{R}^n.$$

NAP. $\forall t \in \langle 0, T\rangle$ $A(\cdot, t)$ JE UNIFORMNO ELIPTIČKI

PRIMJER: $A = I$, $b = 0$, $c = 0$, $f = 0$

$L = -\Delta$ ... $u_{tt} - \Delta u = 0$

DEFINICIJA SLABOG RJEŠENJA

L u DIVERGENTNOM OBLIKU

$A \in C^1(\overline{U}_T; M_n(\mathbb{R}))$, $A = A^T$

$b \in C^1(\overline{U}_T; \mathbb{R}^n)$

$c \in C^1(\overline{U}_T; \mathbb{R})$

$f \in L^2(U_T; \mathbb{R})$

$g \in H^1_0(U, \mathbb{R})$

$h \in L^2(U, \mathbb{R})$

$$B[u,v;t] = \int_U A(\cdot,t)\nabla u \cdot \nabla v + b(\cdot,t)\nabla u\, v + c(\cdot,t) u v$$

BILINEARNA FORMA $\qquad u,v \in H^1_0(U)$, $t \in [0,T]$

MOTIVACIJA

$u(x,t)$ SHVATIMO KAO $t \longmapsto u(t) \in H^1_0(U)$

u DOVOLJNO GLATKO

$$u_{tt} + Lu = f \qquad \Big/ \cdot v \in H^1_0(U)$$

$$(u''(t), v) + B[u(t), v; t] = (f(t), v) \,, \qquad v \in H^1_0(U),\ t \in [0,T]$$

KAO I PRIJE

$$(u''(t), v) = \int_U \underbrace{\left(-A(\cdot,t)\nabla u(t)\right)}_{h_x(t)}\nabla v + \underbrace{\left(f(t) - b(\cdot,t)\cdot\nabla u(t) - c(\cdot,t)u(t)\right)}_{h^0(t)} v$$

$\in L^2(U)$ s.s. t

$$\Rightarrow \quad u''(t) \in H^{-1}(U) \quad \text{s.s. } t$$

U TOM KONTEKSTU $\quad {}_{H^{-1}}\langle u''(t), v\rangle_{H^1_0}$

DEF: FUNKCIJU

$$u \in L^2(0,T; H_0^1(U)) \text{ t.d. } u' \in L^2(0,T; L^2(U)), \quad u'' \in L^2(0,T; H^{-1}(U))$$

NAZIVAMO <u>SLABIM RJEŠENJEM</u> ZADAĆE (*) AKO

(i) $\langle u''(t), v\rangle + B[u(t), v; t] = (f(t), v), \quad v \in H_0^1(U)$, $\text{s.s. } t \in [0,T]$

(ii) $u(0) = g, \quad u'(0) = h$

NAP: ULAGANJA $u, u' \Rightarrow u \in C([0,T]; L^2(U)) \Rightarrow u(0)$ OK

$u', u'' \Rightarrow u' \in C([0,T]; H^{-1}(U)) \Rightarrow u'(0)$ OK

7.2.2 EGZISTENCIJA SLABOG RJEŠENJA

GALERKINOVA METODA

- KONAČNO-DIMENZIONALNE APROKSIMACIJE
- KONVERGENCIJA

$(w_k)_k$ OB u $H_0^1(U)$

ONB u $L^2(U)$

$m \in \mathbb{N}$

$$u_m(t) := \sum_{k=1}^{m} d_m^k(t)\, w_k \quad \text{- APROKSIMACIJA}$$

$(d_m^k)_{k=1,\dots,m}$ ZADOVOLJAVAJU

$$(\text{ODJ}) \begin{cases} (u_m''(t), w_k) + B[u_m(t), w_k; t] = (f(t), w_k) & k=1,\dots,m \\ d_m^k(0) = (g, w_k) & \\ d_m^{k\,\prime}(0) = (h, w_k) & k=1,\dots,m \end{cases}$$

NAP: $u_m(0) = \sum_{k=1}^{m} d_m^k(0)\, w_k = \sum_{k=1}^{m} (g, w_k)\, w_k \to g \quad H_0^1(U)$

$u_m'(0) = \sum_{k=1}^{m} (d_m^k)'(0)\, w_k = \sum_{k=1}^{m} (h, w_k)\, w_k \longrightarrow h \quad L^2(U)$

TEOREM 1 $\forall m \in \mathbb{N}$ $\exists!$ u_m ZADANOG OBLIKA KOJI ZADOVOLJAVA (ODJ)

DOK: OZNAČIMO:

$$D_m(t) := \begin{bmatrix} d_m^1(t) \\ \vdots \\ d_m^m(t) \end{bmatrix}, \quad F_m(t) := \begin{bmatrix} \int_U f(t) w_1 dx \\ \vdots \\ \int_U f(t) w_m dx \end{bmatrix}$$

$$G_m := \begin{bmatrix} (g, w_1) \\ \vdots \\ (g, w_m) \end{bmatrix}, \quad H_m := \begin{bmatrix} (h, w_1) \\ \vdots \\ (h, w_m) \end{bmatrix} \qquad \in L^2(0,T)$$

$$M_m(t) := \left[\int_U A(\cdot, t) \nabla w_j \cdot \nabla w_k \, dx \right]_{k,j=1,\dots,m} \qquad C^1$$

(ODJ) $\Rightarrow$

$$\begin{cases} D_m''(t) + M_m(t) D_m(t) = F_m(t) \\ D_m(0) = G_m \\ D_m'(0) = H_m \end{cases}$$

C.Z. SUSTAV ODJ 2. REDA, LINEARNI

$$\Rightarrow \quad D_m \in H^2(0,T)^m \equiv H^2(0,T; \mathbb{R}^m)$$

ENERGETSKE OCJENE

$m \to \infty$ u jednadžbi (ODJ)

TREBAMO OCJENE, UNIFORMNE PO m.

TEOREM 2 (ENERGETSKE OCJENE)

$\exists C(U,T,L) > 0$ T.D.

$$\max_{t\in[0,T]} \left(\|u_m(t)\|_{H_0^1(U)} + \|u_m'(t)\|_{L^2(U)} \right) + \|u_m''\|_{L^2(0,T;H^{-1}(U))}$$

$$\leq C \left(\|f\|_{L^2(0,T;L^2(U))} + \|g\|_{H_0^1(U)} + \|h\|_{L^2(U)} \right), \quad m \in \mathbb{N}.$$

DOK: 1. KORAK

$$(u_m''(t), w_k) + B[u_m(t), w_k, t] = (f(t), w_k), \quad k=1,\dots,m \quad \text{s.s. } t$$

MNOŽIMO s $d_m^{k\,\prime}(t)$, $\sum_{k=1}^{m}$

$$(u_m''(t), u_m'(t)) + B[u_m(t), u_m'(t); t] = (f(t), u_m'(t)) \quad \text{s.s. } t$$

$$\frac{d}{dt}\left(\frac{1}{2}\|u_m(t)\|^2_{L^2(U)}\right) \quad , B, f \text{ OCJENJUJEMO}$$

1) $$B[u_m(t), u_m'(t); t] = \int_U A(\cdot,t)\nabla u_m(t) \cdot \nabla u_m'(t)$$

$$+ \int_U b(\cdot,t)\cdot\nabla u_m(t)\, u_m'(t) + C(\cdot,t)u_m(t)\, u_m'(t)$$

$$= B_1 + B_2$$

$\uparrow$ ODOZDO $\quad$ $\uparrow$ ODOZGO

$$B_1 = \frac{d}{dt}\left(\frac{1}{2}A[u_m(t), u_m(t); t]\right) - \frac{1}{2}\int_U A_t(\cdot,t)\nabla u_m(t)\cdot\nabla u_m(t)$$

ZAPRAVO OVO MOŽEMO U B_2 PREBACITI

$$A[u,v;t] = \int_U A(\cdot,t)\nabla u \cdot \nabla v \, dx$$

TU NAM TREBA DERIVACIJA PO t

$$\left| \int_U A_t(\cdot,t)\, \nabla u_m(t) \cdot \nabla u_m(t) \right| \leq C \, \| u_m(t) \|^2_{H^1_0(U)}$$

$$\Rightarrow \quad B_1 \geq \frac{d}{dt}\left(\frac{1}{2} A[u_m(t), u_m(t); t] \right) - C \, \| u_m(t) \|^2_{H^1_0(U)}$$

$$|B_2| \leq C \left(\| u_m(t) \|^2_{H^1_0(U)} + \| u_m'(t) \|^2_{L^2(U)} \right) \leftarrow \text{KAO PRIJE}$$

2) $$|(f(t), u_m'(t))| \leq C \left(\| f(t) \|^2_{L^2(U)} + \| u_m'(t) \|^2_{L^2(U)} \right)$$

SVE SKUPA:

$$\frac{d}{dt}\left(\frac{1}{2} \| u_m'(t) \|^2_{L^2(U)} + \frac{1}{2} A[u_m(t), u_m(t); t] \right)$$

$$\leq C \left(\| u_m'(t) \|^2_{L^2(U)} + \| u_m(t) \|^2_{H^1_0(U)} + \| f(t) \|^2_{L^2(U)} \right)$$

VRIJEDI:

$$A[u_m(t), u_m(t); t] = \int_U A(\cdot,t)\, \nabla u_m(t) \cdot \nabla u_m(t) \, dx$$

$$\geq \theta \int_U \nabla u_m(t) \cdot \nabla u_m(t) = \theta \, \| \nabla u_m(t) \|^2_{L^2(U)}$$

DOBIVAMO:

$$\frac{d}{dt}\Big(\overbrace{\| u_m'(t) \|^2_{L^2(U)} + A[u_m(t), u_m(t); t]}^{\eta(t)} \Big)$$

$$\leq C \Big(\| u_m'(t) \|^2_{L^2(U)} + A[u_m(t), u_m(t); t] + \underbrace{\| f(t) \|^2_{L^2(U)}}_{\xi(t)} \Big)$$

$$\eta'(t) \leq C_1 \eta(t) + C_2 \xi(t)$$

GRONWALL:

$$\eta(t) \leq e^{C_1 t} \left(\eta(0) + C_2 \int_0^t \xi(s)\, ds \right)$$

$$\eta(t) \le C\left(\eta(0) + \|f\|^2_{L^2(0,T;L^2(U))}\right)$$

$$\eta(0) = \|u_m'(0)\|^2_{L^2(U)} + A[u_m(0), \underbrace{u_m(0)}_{\sim g}; 0]$$
$$\le \|h\|^2_{L^2(U)} + \|g\|^2_{H^1_0(U)}$$

$$\Rightarrow \quad \|u_m'(t)\|^2_{L^2(U)} + A[u_m(t), u_m(t); t] \le C\left(\|g\|^2_{H^1_0(U)} + \|h\|^2_{L^2(U)} + \|f\|^2_{L^2(0,T;L^2(U))}\right)$$
$$\text{s.s. } t \in [0,T]$$

$$A[u_m(t), u_m(t); t] \ge \theta \|\nabla u_m(t)\|^2_{L^2(U)} \overset{\text{POINCARE}}{\ge} C\|u_m(t)\|^2_{H^1_0(U)}$$

$$\Rightarrow \quad \max_{t\in[0,T]} \left(\|u_m'(t)\|^2_{L^2(U)} + \|u_m(t)\|^2_{H^1_0(U)}\right) \le C\left(\|g\|^2_{H^1_0(U)} + \|h\|^2_{L^2(U)} + \|f\|^2_{L^2(0,T;L^2(U))}\right)$$

3. KORAK

OSTAJE OCIJENITI u_m''

$v \in H^1_0(U)$, $\|v\|_{H^1_0(U)} \le 1$, $v = v^1 + v^2$, $v^1 \in L\{w_1, \dots, w_m\}$
$v^2 \perp L\{w_1,\dots,w_m\}$ u $L^2(U)$

$${}_{H^{-1}}\langle u_m''(t), v\rangle_{H^1_0} = (u_m''(t), v) = (u_m''(t), v^1) = (f(t), v^1) - B[u_m(t), v^1; t]$$

$$\Rightarrow \quad |\langle u_m''(t), v\rangle| \le C\left(\|f(t)\|_{L^2(U)} + \|u_m(t)\|_{H^1_0(U)}\right) \quad \text{s.s. } t$$

$$\Rightarrow \quad \|u_m''(t)\|_{H^{-1}(U)} \le C\left(\qquad\qquad\qquad\right) \quad \text{s.s. } t \quad |^2 \quad \left(\int_0^T\right.$$

$$\Rightarrow \quad \|u_m''\|^2_{L^2(0,T;H^{-1}(U))} \le C\left(\|f\|^2_{H^1_0(U)} + \|u_m\|^2_{L^2(0,T;H^1_0(U))}\right)$$
$$\le f, g, h$$

IZ DOKAZANOG

EGZISTENCIJA I JEDINSTVENOST

TEOREM 3 $\exists$ SLABO RJEŠENJE $(*)$.

DOK: 1.K Iz TM 2 : u_m OGR u $L^2(0,T;H_0^1(U))$ (ZAPRAVO L^∞ ...)
u_m' OGR u $L^2(0,T;L^2(U))$ (ZAPRAVO L^∞ ...)
u_m'' OGR u $L^2(0,T;H^{-1}(U))$

$\Rightarrow$ $\exists$ PODNIZ u_{m_ℓ} & $u \in L^2(0,T;H_0^1(U))$
$u' \in L^2(0,T;L^2(U))$
$u'' \in L^2(0,T;H^{-1}(U))$ T.D.

$$u_{m_\ell} \rightharpoonup u \quad L^2(0,T;H_0^1(U))$$
$$u_{m_\ell}' \rightharpoonup u' \quad L^2(0,T;L^2(U))$$
$$u_{m_\ell}'' \rightharpoonup u'' \quad L^2(0,T;H^{-1}(U))$$

2. KORAK $N \in \mathbb{N}$, $v \in C^1([0,T];H_0^1(U))$ OBLIKA $v(t) = \sum_{k=1}^{N} d^k(t) w_k$.
d^k, $k=1,\dots,N$ GLATKE

UZMEM $m \geq N$ I JOŠ

$$\langle u_{m_\ell}''(t), w_k \rangle + B[u_{m_\ell}(t), w_k; t] = (f(t), w_k) \quad \Big| d^k \Big| \sum_k \int_0$$

$$(**_{m}) \Rightarrow \int_0^T \langle u_{m_\ell}''(t), v(t)\rangle dt + \int_0^T B[u_{m_\ell}(t), v(t); t] dt = \int_0^T (f(t), v(t))\, dt$$

$$\downarrow$$

$$(**) \quad \int_0^T \langle u''(t), v(t)\rangle dt + \int_0^T B[u(t), v(t); t]\, dt = \int_0^T (f(t), v(t))\, dt$$

PO GUSTOĆI PROŠIRIM NA $v \in L^2(0,T;H_0^1(U))$!

ZA TEST FUNKCIJE OBLIKA: $v(t) = \varphi(t)\, w$, $\varphi \in L^2(0,T)$, $w \in H^1_0(U)$

$$\langle u''(t), w\rangle + B[u(t), w; t] = (f(t), w), \quad w \in H^1_0(U).$$

s.s. $t \in [0,T]$

DOBILI SMO JEDNADŽBU.

3. KORAK

OSTAJE DOBITI POČETNE UVJETE. $u(0)=g$, $u'(0)=h$

$$\left.\begin{array}{l} u \in L^2(0,T; H^1_0(U)) \\ u' \in L^2(0,T; L^2(U)) \\ u'' \in L^2(0,T; H^{-1}(U)) \end{array}\right\} \begin{array}{l} u \in C([0,T]; L^2(U)) \\ u' \in C([0,T]; H^{-1}(U)) \end{array}$$

$v \in C^2([0,T]; H^1_0(U))$, $v(T) = v'(T) = 0$ u (**)

P.I. PO t DVA PUTA

$$\int_0^T (v'', u)\, dt + \int_0^T B[u, v; t]\, dt = \int_0^T (f, v)\, dt - (u(0), v'(0)) + \langle u'(0), v(0)\rangle$$

IZ (**$_m$) P.I. PO t DVA PUTA

$$\int_0^T (v'', u_m)\, dt + \int_0^T B[u_m, v; t]\, dt = \int_0^T (f, v)\, dt - (u_m(0), v'(0)) + (u_m'(0), v(0))$$

ZA $m = m_\ell$ PUSTIMO LIMES

$$\int_0^T (v'', u)\, dt + \int_0^T B[u, v; t]\, dt = \int_0^T (f, v)\, dt - (g, v'(0)) + (h, v(0))$$

$$(u(0) - g, v'(0)) + (u'(0) - h, v(0)) = 0, \quad v \in C^2([0,T]; H^1_0(U))$$

NEOVISNO MOGU IZABRATI $v(0) = 0$ & $v'(0) = 0$ $\Rightarrow$ P.U.

TEOREM 4 (JEDINSTVENOST) SLABO RJEŠENJE (*) JE JEDINSTVENO.

NAP: DOKAZ BI BIO BITNO LAKŠI DA JE $u' \in L^2(0,T; H^1_0(U))$
PA PA GA MOŽEMO UVRSTITI U (**).

DOK: ZBOG LINEARNOSTI ZADAĆE MOŽEMO UZETI $f=0, g=h=0$.

DEF:
$$v(t) := \begin{cases} \int_t^s u(\tau)\,d\tau, & 0 \le t \le s, \\ 0, & s \le t \le T. \end{cases}$$
ZA S FIKSAN

$\Rightarrow$ $v(t) \in H^1_0(U)$, $t \in [0,T]$ DOBRA TEST FUNKCIJA

$$\int_0^T \langle u''(t), v(t)\rangle\,dt + \int_0^T B[u(t), v(t); t]\,dt = 0$$

$$\underbrace{\int_0^s \langle u''(t), v(t)\rangle\,dt}_{P.I.} + \int_0^s B[u(t), v(t); t]\,dt = 0$$

$$-\int_0^s (u'(t), v'(t))\,dt + \underbrace{\cancel{(u'(t), v(t))\Big|_0^s}}_{u'(0)=0,\ v(s)=0} + \int_0^s B[u(t), v(t); t]\,dt = 0$$

$$v'(t) = -u(t), \quad 0 \le t \le s$$

$$\Rightarrow \int_0^s (u'(t), u(t))\,dt - \int_0^s B[v'(t), v(t); t]\,dt = 0$$

$$\| \quad \frac{d}{dt}\left(\frac{1}{2}\|u(t)\|^2_{L^2(U)}\right)$$

↑ OCJENJUJEMO

$$-\int_0^s B[v'(t), v(t); t]\,dt = -\int_0^s \int_U A(\cdot,t)\nabla v'(t)\cdot\nabla v(t) + b(\cdot,t)\cdot\nabla v'(t)\,v(t) + C(\cdot,t)\,v'(t)\,v(t)$$

$$= -\int_0^s \frac{d}{dt}\left(\frac{1}{2}\int_U A(\cdot,t)\nabla v(t)\cdot\nabla v(t) + b(\cdot,t)\cdot\nabla v(t)\,v(t) + c(\cdot,t)\,v(t)^2\right)$$

$$+ \int_0^s \frac{1}{2}\int_U A_t(\cdot,t)\nabla v(t)\cdot\nabla v(t) + b_t(\cdot,t)\cdot\nabla v(t)\,v(t) + b(\cdot,t)\nabla v(t)\,v'(t)$$

$$- b(\cdot,t)\cdot\nabla v'(t)\,v(t) + C_t(\cdot,t)\,v(t)^2$$

P.I. po x!

$$+\operatorname{div}(b(\cdot,t)\,v(t))\,v'(t) = (\operatorname{div} b(\cdot,t))\,v(t)\,v'(t) + b(\cdot,t)\cdot\nabla v(t)\,v'(t)$$

$$= -\int_0^s \frac{d}{dt}\left(\frac{1}{2}B[v(t), v(t); t]\right)dt + \text{OST}$$

$$|\text{OST}| \le \int_0^s \frac{1}{2}\int_U A_t(\cdot,t)\nabla v(t)\cdot\nabla v(t) + b_t(\cdot,t)\cdot\nabla v(t)\,v(t) + C_t(\cdot,t)\,v(t)^2$$

$$+ 2\,b(\cdot,t)\cdot\nabla v(t)\,\underbrace{v'(t)}_{=-u(t)} + (\operatorname{div} b(\cdot,t))\,v(t)\,\underbrace{v'(t)}_{=-u(t)}$$

$$\le C\int_0^s \left(\|v(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}\right)dt$$

$$\Rightarrow \int_0^s \frac{d}{dt}\left(\frac{1}{2}\|u(t)\|^2_{L^2(U)} - \frac{1}{2}B[v(t), v(t); t]\right) \le C\int_0^s \|v(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}$$

$$\Rightarrow \|u(s)\|^2_{L^2(U)} - \underbrace{\|u(0)\|^2_{L^2(U)}}_{=0} + B[v(0), v(0); t] \le \quad -\,\text{"}\,-$$

$$B[v(0), v(0); 0] = \int_U A(\cdot,0)\nabla v(0)\cdot\nabla v(0) + b(\cdot,0)\cdot\nabla v(0)\,v(0) + C(\cdot,0)\,v(0)^2$$

$$\int_U b(\cdot,0)\cdot\nabla v(0)\,v(0) = -\int_U \operatorname{div}(b(\cdot,0)\,v(0))\,v(0)$$

$$= -\int_U (\operatorname{div} b(\cdot,0))\,v(0)^2 + b(\cdot,0)\cdot\nabla v(0)\,v(0)$$

$$\Rightarrow \int_U b(\cdot,0)\cdot\nabla w'(0)\, v(0) = -\frac{1}{2}\int_U \operatorname{div} b(\cdot,0)\, v(0)^2$$

$$\Rightarrow B[v(0),v(0);0] = \int_U A(\cdot,0)\nabla v(0)\cdot\nabla v(0) - \frac{1}{2}\operatorname{div} b(\cdot,0)\, v(0)^2 + c(\cdot,0)\,v(0)^2$$

OCJENA

$$\Rightarrow \|u(s)\|^2_{L^2(U)} + \theta\|\nabla v(0)\|^2_{L^2(U)} \le C\left(\int_0^s\left(\|v(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}\right)dt + \|v(0)\|^2_{L^2}\right)$$

$$\|u(s)\|^2_{L^2(U)} + \|v(0)\|^2_{H^1_0(U)} \le C\left(\int_0^s\left(\|v(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}\right)dt + \|v(0)\|^2_{L^2(U)}\right)$$

<u>2. KORAK</u> DEF:

$$w(t) := \int_0^t u(\tau)\,d\tau, \qquad t\in[0,T]$$

$$\|u(s)\|^2_{L^2(U)} + \|w(s)\|^2_{H^1_0(U)} \le C\left(\int_0^s\left(\underbrace{\|w(s)-w(t)\|^2_{H^1_0(U)}}_{\le\, 2\|w(s)\|^2_{H^1_0(U)} + 2\|w(t)\|^2_{H^1_0(U)}} + \|u(t)\|^2_{L^2(U)}\right)dt + \underbrace{\|w(s)\|^2_{L^2(U)}}_{\le \int_0^s \|u(t)\|^2_{L^2(U)}}\right)$$

$$\le C_1\int_0^s\left(\|w(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}\right)dt + (2sC)\|w(s)\|^2_{L^2(U)}$$

$$\Rightarrow \|u(s)\|_{L^2(U)} + (1-2sC)\|w(s)\|^2_{L^2(U)} \le C_1\int_0^s\left(\|w(t)\|^2_{H^1_0(U)} + \|u(t)\|^2_{L^2(U)}\right)dt$$

NEKA JE $T_1>0$ T.D. $1-2T_1C \geq \frac{1}{2}$

$\Rightarrow$ ZA $s\in[0,T_1]$

$$\|u(s)\|^2_{L^2(U)} + \|w(s)\|^2_{H^1_0(U)} \le C\int_0^s \|u(t)\|^2_{L^2(U)} + \|w(t)\|^2_{H^1_0(U)}$$

GRONWALL $\Rightarrow u\equiv 0$ NA $[0,T_1]$

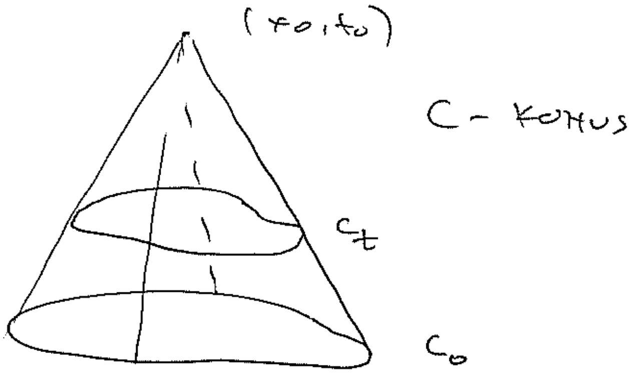
<u>3. KORAK</u> NA $[T_1,2T_1]$ PRIMIJENIM ISTI ARGUMENT. I.T.D.

7.2.3. REGULARNOST

- TEHNIKE ISTE KAO KOD PARABOLIČKIH JEDNADŽBI
- KLJUČNA GLATKOĆA PODATAKA & UVJET KOMPATIBILNOSTI

7.2.4. KONAČNA BRZINA ŠIRENJA POREMEĆAJA

- KAO KOD $u_{tt} - \Delta u = 0$
- SVOJSTVO SUPROTNO ∞ BRZINI ŠIRENJA KOD PARABOLIČKIH
- NEMA J. PRINCIPA MAKSIMUMA



TM

u - GLATKO RJ. $u_{tt} - \mathrm{div}(A \nabla u) = 0$

AKO JE $u \equiv u_t \equiv 0$ NA $C_0 \implies u \equiv 0$ u C

NAP: TOČKE IZVAN C_0 NE UTJEČU NA RJ u (x_0, t_0).

POTREBNO JE KONAČNO VRIJEME DA SE POREMEĆAJ IZVAN C_0 PROŠIRI DO (x_0, t_0).