Erratum to: Quadratic approximation in $\mathbb{Q}_p$, Int. J. Number Theory 11 (2015), 193-209.

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In order to obtain inequality (2.12) we used that inequality (2.10) holds for every matrix in $\mathbb{R}^{2\times 2}$. This is not correct, since the matrix norm from the Lemma that was employed [R. A. Horn and C. R. Johnson, Matrix analysis, Cambridge Univ. Press, Cambridge, 1990, Lemma 5.6.10] depends on the matrix.

Therefore, inequality (2.10) should be deleted and the following modifications made.

Let $\|\cdot\|$ be the following (submultiplicative) matrix norm on $\mathbb{R}^{2\times 2}$

$$\left\| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\| = 2\max\{|a|,|b|,|c|,|d|\}.$$

It can be shown using diagonalization that for any $\ell \in \mathbb{Z}_{\geq 1}$ and $r \geq 2$, putting

$$A = \begin{pmatrix} 1 & 1 \\ r & 0 \end{pmatrix},$$

we have

$$\|A^\ell\| < 2^4 \rho(A)^{\ell+1},$$

where $\rho(A)$ denotes the spectral radius of the matrix A. Now the corrected inequality (2.11) says

$$\begin{aligned}
&\left\| \begin{pmatrix} p_n & p_{n-1} \\ q_n & q_{n-1} \end{pmatrix} \right\| = \left\| \prod_{i=0}^{n} \begin{pmatrix} 1 & 1 \\ p^{a_{i,w}} & 0 \end{pmatrix} \right\| \\
&\leq \left\| \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\| \cdot \prod_{j=1}^{k-1} \left\| \begin{pmatrix} 1 & 1 \\ p^{b+3j+\varepsilon_j} & 0 \end{pmatrix}^{\lfloor w^{j+1} \rfloor - \lfloor w^j \rfloor - 1} \right\| \\
&\qquad \cdot \left\| \begin{pmatrix} 1 & 1 \\ p^{b+3k+\varepsilon_k} & 0 \end{pmatrix}^{n-\lfloor w^k \rfloor} \right\| \cdot \prod_{j=0}^{k} \left\| \begin{pmatrix} 1 & 1 \\ p^{b+3j+2} & 0 \end{pmatrix} \right\| \\
&< 2^{8k+8} \prod_{j=0}^{k-1} \beta_j^{\lfloor w^{j+1} \rfloor - \lfloor w^j \rfloor} \cdot \beta_k^{n-\lfloor w^k \rfloor + 1} \prod_{j=1}^{k+1} \beta_j^2 \\
&\leq n^{M \log n} p^{S_n},
\end{aligned}$$

where M is a positive number independent of n.

Omitting the lines between (2.11) and (2.12) in the article, we now have

$$p^{S_n} \leq p_n \leq n^{M \log n} p^{S_n}, \quad (n \geq 1).$$

The inequality below (2.13) becomes

$$w^{-M'j^2} p^{2\sum_{i=1}^{\lfloor w^j \rfloor} a_{i,w} - 2S_{\lfloor w^j \rfloor}} \ll \mathrm{H}(\xi_{w,j}) \ll p^{a_{\lfloor w^j \rfloor, w}} w^{M'j^2} p^{2S_{\lfloor w^j \rfloor}},$$

where again M' is a positive number independent of n.

Fortunately, the rest of the proof is correct since we only used the above inequality to prove (2.15) and it certainly remains valid even with the new bounds.

In the proof of Theorem 2, bounds (2.17) and (2.19) should be changed in an analogous manner, but everything else remains the same.