5. Jednadžba orbite

Definirajmo $s := \frac{1}{r}$, tj. $r = \frac{1}{s}$ pa

$K = \frac{1}{2} r^2 \dot{\theta} \Rightarrow \dot{\theta} = 2K s^2$

diferencijalni operator

Označimo s ' derivaciju po θ, tj. $\frac{d}{d\theta} := \frac{\frac{d}{dt}}{\dot{\theta}}$

fizičari:

$s, \dot{s}, s'$

$r(t) \ldots \dot{r}$

$r(\theta) \ldots r'$

$r(t) = \tilde{r}(\theta(t))$

$\Downarrow \frac{d}{dt}$

$\dot{r}(t) = \tilde{r}'(\theta(t)) \cdot \dot{\theta}$

$\frac{d}{d\theta}$ se ponaša kao "obična" derivacija

$r = \frac{1}{s} \Rightarrow r' = -\frac{1}{s^2} \cdot s'$ i $\dot{r} = r' \cdot \dot{\theta} \Rightarrow \boxed{\frac{d}{dt} = \frac{d}{d\theta} \cdot \frac{d\theta}{dt}}$

$\Rightarrow \ddot{r} = (r'\dot{\theta})' \cdot \dot{\theta}$

$\ddot{r} = \left(-\frac{1}{\cancel{s^2}} \cdot s' \cdot 2K\cancel{s^2}\right)' \cdot 2Ks^2 = -4K^2 s^2 s''$

$$\ddot{r} - r\cdot\dot{\theta}^2 = -\frac{GM}{r^2}$$

$$-4K^2 s^2 s'' - r(2Ks^2)^2 = \frac{-GM}{r^2} = -GM\cdot s^2 \quad /: s^2$$

$$\Rightarrow \boxed{s'' + s = \frac{GM}{4K^2}}$$

e i φ su konst.

možemo pretp. da je $\varphi = 0$ ako zarotiramo koordinat. sustav oko za $-\varphi$

$$s = \frac{GM}{4K^2}\left(1 + e\cos(\theta + \cancel{\varphi})\right)$$

pretp. $e > 0$ (možemo zarotir. koord. sustav za π
$\cos(x+\pi) = -\cos x$)

$$\Rightarrow \frac{1}{r} = \frac{GM}{4K^2}(1 + e\cos\theta).$$

Označimo s $p = \frac{4K^2}{GM(1+e)}$, tada

6. Keplerov prvi zakon

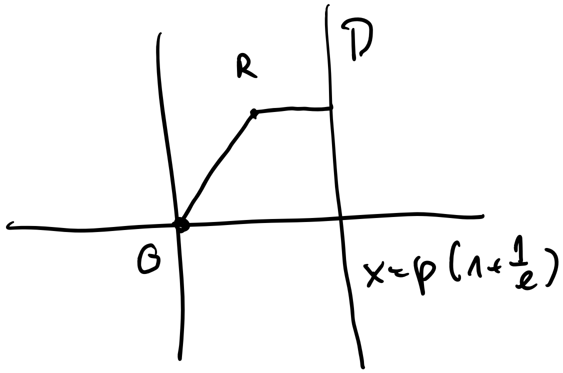
$$\frac{1}{r} = \frac{1+e\cos\theta}{p(1+e)}$$

$$\Rightarrow p(1+e) = r(1+e\cos\theta)$$

$$= r + er\cos\theta$$

$$= r + ex$$

$$\Rightarrow r = e\left[p\left(1+\frac{1}{e}\right) - x\right]$$



Papo–Boškovićev teorem

Neka je D pravac $x = p(1+\frac{1}{e})$. Tada je udaljenost točke R do ishodišta O jednaka e puta udaljenost od R do pravca D. Kažemo, R leži na konici

slijedi

su fokusom O ekscentricitetom e

i direktrisom D.

- za $e>1$ konika je hiperbola
- za $e=1$ konika je parabola
- za $0<e<1$ konika je elipsa.

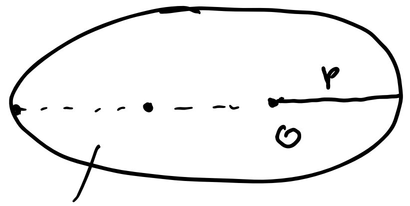
7. Keplerov zakon

$$\frac{1}{r}=\frac{1+e\cos\theta}{p(1+e)}$$

za $\theta=0 \Rightarrow r=p$

za $\theta=\pi \Rightarrow r=\frac{p(1+e)}{1-e}$

za $e<1$ (elipsa)



$$\frac{p(1+e)}{1-e}$$

$$2\cdot a=p+\frac{p(1+e)}{1-e}$$

$$\Rightarrow a=\frac{p}{1-e},\quad p=a(1-e)$$

$$\Rightarrow K^2=\frac{1}{4}GMa(1-e^2)$$

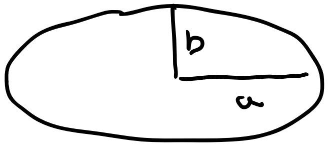
i $\underbrace{r}_{\sqrt{x^2+y^2}}=a(1-e^2)-ex \quad /^2$

$$x^2+y^2=a^2(1-e^2)^2+e^2x^2-2a(1-e^2)ex$$

$$(1-e^2)x^2+2a(1-e^2)ex+y^2=a^2(1-e^2)^2 \quad /:a^2(1-e^2)^2$$

$$\left(\frac{x}{a}\right)^2+2\frac{x}{a}e+\left(\frac{y}{a\sqrt{1-e^2}}\right)^2=1-e^2$$

$$\Rightarrow \left(\frac{x+ae}{a}\right)^2+\left(\frac{y}{a\sqrt{1-e^2}}\right)^2=1$$



$$A = \pi \cdot a \cdot b$$

$$\Rightarrow A = \pi a^2 \sqrt{1-e^2}$$

$$A = T \cdot K$$

$$\Rightarrow \boxed{T^2 = \frac{4\pi^2}{GM} \cdot a^3}$$

treći Keplerov zakon.