Ljiljana Arambašić: **On three concepts of orthogonality in Hilbert C^*-modules**

In this talk we consider three concepts of orthogonality in a Hilbert C^*-module V over a C^*-algebra $\mathscr{A}$: the Birkhoff–James orthogonality $\perp_B$, the strong Birkhoff–James orthogonality $\perp_B^s$, and the orthogonality with respect to the $\mathcal{A}$-valued inner product on V. If x and y are elements of a normed linear space X, then *x is orthogonal to y in the Birkhoff–James sense* if

$$\|x\| \leq \|x + \lambda y\|, \quad \lambda \in \mathbb{C}.$$

It is easy to see that in an inner product space the Birkhoff–James orthogonality becomes the usual one.

Hilbert C^*-modules generalize Hilbert spaces by allowing inner products to take values in an arbitrary C^*-algebra instead of the C^*-algebra of complex numbers. Therefore, a concept of orthogonality in a Hilbert C^*-module can be defined with respect to the C^*-valued inner product in a natural way, that is, two elements x and y of a Hilbert C^*-module V over a C^*-algebra $\mathscr{A}$ are *orthogonal* if $\langle x, y\rangle = 0$, where $\langle \cdot, \cdot\rangle$ denotes the $\mathscr{A}$-valued inner product on V.

When x and y are elements of a Hilbert $\mathscr{A}$-module V, we say that *x is orthogonal to y in the strong Birkhoff–James sense* if

$$\|x\| \leq \|x + ya\|, \quad a \in \mathscr{A}, \tag{0.1}$$

i.e., if the distance from x to $\overline{y\mathscr{A}}$, the $\mathscr{A}$-submodule of V generated by y, is exactly $\|x\|$.

We characterize the classes of Hilbert C^*-modules in which any two of these three types of orthogonalities coincide.

References

[1] Lj. Arambašić, R. Rajić, *On three concepts of orthogonality in Hilbert C^*-modules*, Linear and Multilinear Algebra 63 (7) (2015), 1485–1500.