Scaling up a water-gas flow model with mass exchange

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Outline

Two-phase immiscible, incompressible flow

Upscaling procedure

Application to a natural oil reservoir

Two-phase compressible flow with mass exchange

Upscaling

Two-phase flow model, with no mass exchange

We consider two-phases incompressible, immiscible flow through heterogeneous porous medium made of different rock-types:

- Two incompressible fluid phases, w, and o: wetting and non-wetting;
- No mass exchange, between phases;
- Temperature is constant;
- Heterogeneous porous medium, with different rock-types.

Two-phase incompressible, immiscible flow equations

Conservation of mass for each fluid + Muskat's(generalized Darcy's) law + capillary pressure law:

$$\phi(\mathbf{x})\frac{\partial S}{\partial t} + \operatorname{div}\mathbf{q}_w = 0$$

$$-\phi(\mathbf{x})\frac{\partial S}{\partial t} + \operatorname{div}\mathbf{q}_o = 0$$

$$\mathbf{q}_w = -\mathbb{K}(\mathbf{x})\lambda_w(S,\mathbf{x})(\nabla p_w - \rho_w \mathbf{g})$$

$$\mathbf{q}_o = -\mathbb{K}(\mathbf{x})\lambda_o(S,\mathbf{x})(\nabla p_o - \rho_o \mathbf{g})$$

$$p_c(S,\mathbf{x}) = p_o - p_w$$

- $S = S_w, \quad S_w + S_o = 1;$
- $\lambda_w(S,\mathbf{x}) = kr_w(S,\mathbf{x})/\mu_w =$ mobility of water;
- $\lambda_o(S,\mathbf{x}) = kr_o(S,\mathbf{x})/\mu_o =$ mobility of oil;

Scaling up problem

The goal of scaling up is to find an effective representation for a heterogeneous medium, with rapidly oscillating properties, such that at large scale the flow can be correctly represented.
Starting from rapidly oscillating data:

$$\mathbf{x} \to \phi(\mathbf{x}),\ \mathbb{K}(\mathbf{x}),\ \lambda_w(\mathbf{x}, S), \lambda_o(\mathbf{x}, S),\ p_c(\mathbf{x}, S);$$

find, if possible(*), effective values:

$$\phi^*,\ \mathbb{K}^*,\ \lambda_w^*(S), \lambda_o^*(S),\ p_c^*(S);$$

Remark: (*)*it is not always possible, see for instance [A.B., M.Panfilov;in C.G. 2, 1998]*

Effective or macroscopic model

Effective equations [A.B., A. Hidani; in A.A. 1995]have the same form as the microscopic ones:

$$\phi^* \frac{\partial S^*}{\partial t} + \operatorname{div} \mathbf{q}_w^* = 0$$

$$-\phi^* \frac{\partial S^*}{\partial t} + \operatorname{div} \mathbf{q}_o^* = 0$$

$$\mathbf{q}_w^* = -\mathbb{K}^* \boldsymbol{\lambda}_w^*(S^*) \left(\nabla p_w^0 - \rho_w \mathbf{g} \right)$$

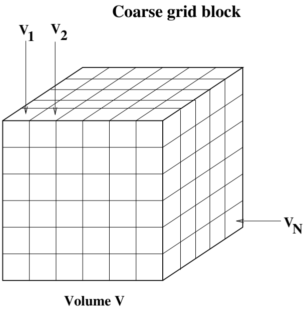
$$\mathbf{q}_o^* = -\mathbb{K}^* \boldsymbol{\lambda}_o^*(S^*) \left(\nabla p_o^0 - \rho_o \mathbf{g} \right)$$

$$p_c^*(S^*) = p_o^0 - p_w^0.$$

How to compute effective properties in a real case?[A.B.,M.Jurak; ELF 1998 and A.B.,A. Piatnitski; in A.I.H.P. 2004]

Scaling up Technique

Homogenization of the coarse grid-block



$$\phi(\mathbf{x}) = \sum_{i=1}^{N} \chi_{V_i}(\mathbf{x}) \phi^i,$$

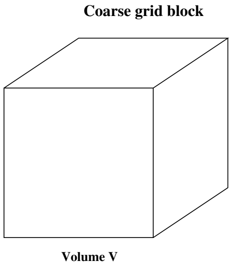
$$\mathbb{K}(\mathbf{x}) = \sum_{i=1}^{N} \chi_{V_i}(\mathbf{x}) \mathbb{K}^i$$

$$\lambda_\xi(\mathbf{x}, S) = \sum_{i=1}^{N} \chi_{V_i}(\mathbf{x}) \lambda_\xi^i(S), \quad (\xi \in \{o, w\}),$$

$$p_c(\mathbf{x}, S) = \sum_{i=1}^{N} \chi_{V_i}(\mathbf{x}) Pc^i(S).$$

Calculation of p_c^* in a coarse grid-block

Mean porosity: ϕ^*.
Effective capillary pressure: From any Capillary Pressure value u (one value for one coarse grid block $V = \bigcup_{i=1,N}$), find the saturation distribution in each small V_i:



$$u = p_c^1(S^1) = p_c^2(S^2) = \cdots = p_c^N(S^N).$$

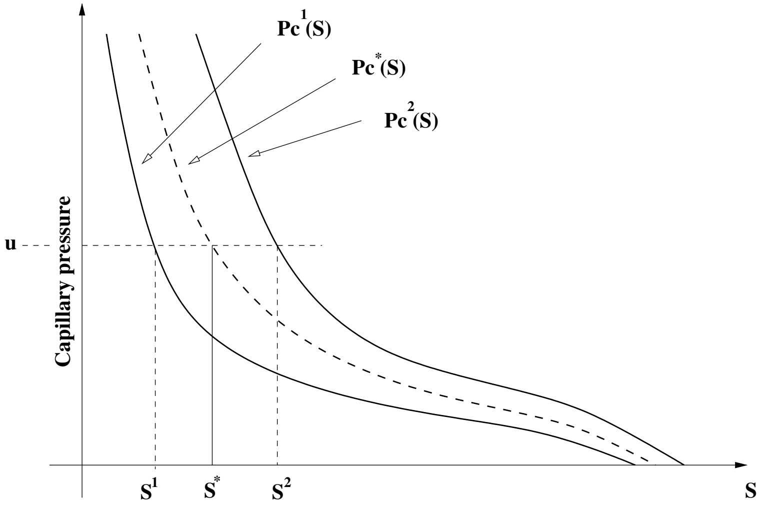
Then set S^*:

$$\phi^* S^* = \sum_{i=1}^{N} \text{vol}(V_i)\phi_i S^i,$$

and p_c^*:

$$p_c^*(S^*) = u.$$

Calculation of p_c^* in a coarse grid-block. Example, N=2.



Effective mobility tensors

For any $S^0 == \sum_{i=1}^{N} \chi_{V_i} S^i$, compute the following local problems ($\xi \in \{w, o\}$ and $k = 1, \dots, d$),

$$\begin{aligned} \operatorname{div}\left(\mathbb{K}(\mathbf{x})\lambda_\xi(\mathbf{x}, S^0)\nabla N_k^\xi\right) = 0, \quad \text{in } V, \\ N_k^\xi = x_k \quad \text{on } \partial V. \end{aligned}$$

Then the Effective Mobilities are : ($\xi \in \{w, o\}$ and $k = 1, \dots, d$),

$$\boldsymbol{\lambda}_\xi^*(S^*)\mathbf{e}_k = \frac{1}{\operatorname{vol}(V)} \int_V \mathbb{K}(\mathbf{x})\lambda_\xi(\mathbf{x}, S^0)(\nabla N_k^\xi + \mathbf{e}_k)\, d\mathbf{x}. \qquad (1)$$

Conclusion

- Constant capillary pressure in the coarse grid block has decoupled local and global computations (local problems are then linear).
- This comes from the dominance of the capillary forces at global level, i.e. a small Peclet number (Capillary number).

Literature: A. Bourgeat, A. Hidani: Effective model of two-phase flow in a porous medium made of different rock types, *Applicable Analysis*, (1995), no. 56, pp. 381-399. A. Bourgeat, A. Piatnitski: Approximations of effective coefficients in stochastic homogenization, *Ann. I. H. Poincaré*, PR 40 (2004) 153-165. A. Bourgeat, M. Panfilov: Effective two-phase flow through highly heterogeneous porous media: Capillary non-equilibrium effects, *Computational Geosciences*, (1998), 2, pp. 191-215. A. Bourgeat, M. Jurak: Scalingup two-phase flow in porous media:comparison of methods, *Rapport Interne -ELF*, (1998).

How to apply to natural oil reservoirs?

Problems:

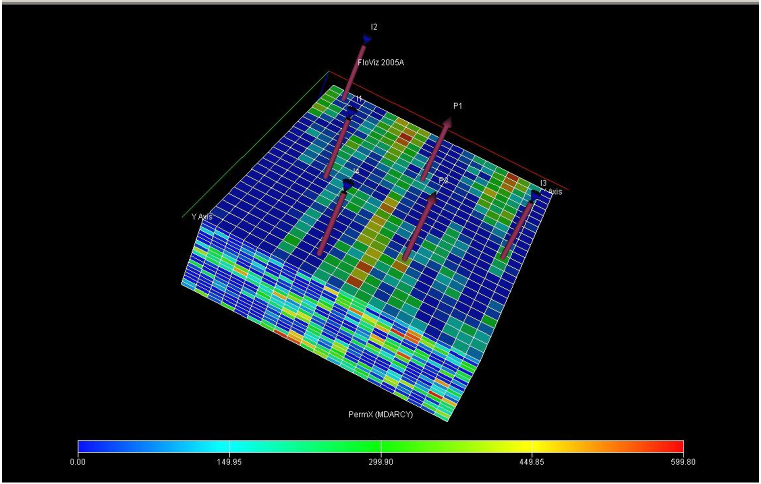
- Dominance of capillary force is usually not satisfied;
- Separation of scales may be weak.

Answer: Yes, if it is done in a clever way:

- The upscaling of absolute permeabilities and relative permeabilities (mobilities) should be done separately, on different coarse grid blocks.
- The upscaling of absolute permeability has to be done on a coarse grid that is well adapted to the heterogeneity of the reservoir.
- The upscaling of mobilities (relative permeabilities) should be done in larger volumes, preferably in several layers.

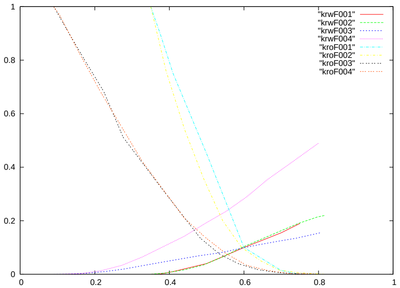
An example

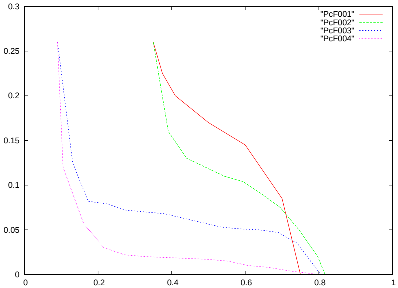
Fine grid of $20 \times 25 \times 20$ blocks of dimensions $50 \times 50 \times 2$ meters.
Horizontal permeability.



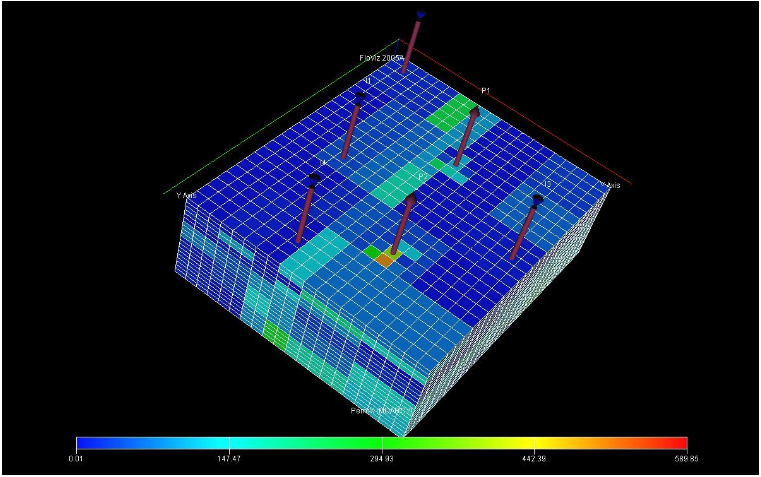
Fine grid relative permeabilities

Fine grid relative permeabilities and capillary pressures for 4 different rock types:



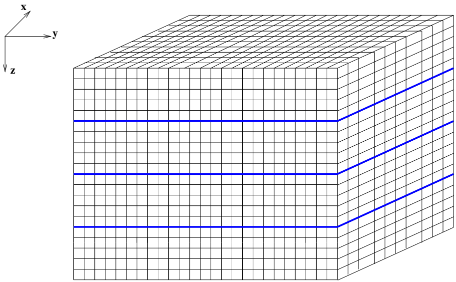


Upscaled absolute permeability



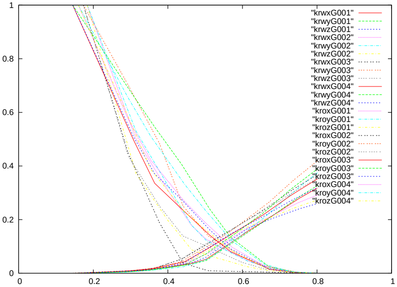
Coarse grid for upscaling mobilities

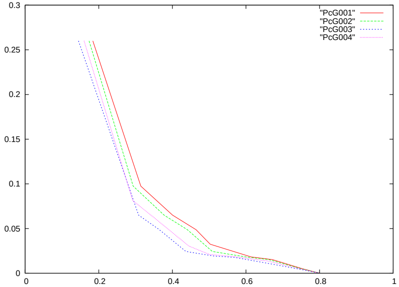
Upscaling of relative permeabilities should be done in large volumes, preferably horizontal layers.



Effective mobilities and effective capillary pressures

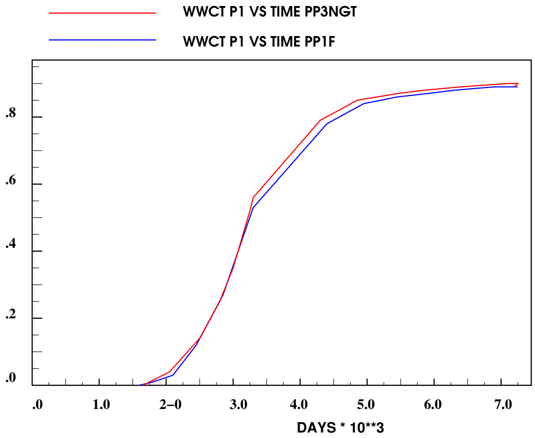
Three directional Effective Relative Permeability curves and one Effective Capillary Pressure curve in 4 horizontal layers :





Comparison: heterogeneous and upscaled simulation

Well P1 water-cut. Coarse grid: $6 \times 5 \times 6$, non-uniform aggregation for absolute permeability, and $1 \times 1 \times 4$ uniform grid for upscaling relative permeabilities. Well P1 water-cut:



Goal

- To extend this upscaling method to the case of two phase partially miscible flow with diffusive fluxes (water and gas).

Two-phase compressible flow with mass exchange

We consider a multiphase, multicomponent flow through an heterogeneous porous medium:

- Two fluid phases: liquid and gas;
- Two components: water (incompressible) and hydrogen (compressible);
- Mass exchange between the phases given by thermodynamic equilibrium:
  - Dissolution of hydrogen in water; Water does not evaporate;
- Diffusivity of dissolved hydrogen;
- Temperature is constant;
- Porous medium is rigid.

Fluid components (w=water, h=hydrogen)

Partial densities:

- ρ_w^l = mass density of water in the liquid phase;
- ρ_h^l = mass density of hydrogen in the liquid phase;
- ρ_g = mass density of the gas phase (hydrogen);

Ideal gas law:

$$\rho_g(p_g) = C_g p_g, \quad C_g = \frac{M_h}{RT},$$

Henry's law (saturated case):

$$\rho_l = \rho_w^l + \rho_h^l, \quad \rho_h^l = C_h p_g, \quad C_h = M_h H(T)$$

Unsaturated case (the gas phase may disappear) : ρ_h^l is an independent variable.

Mass conservation for components

Mass conservation for water and hydrogen components:

$$\rho_w^l \Phi \frac{\partial S_l}{\partial t} + \text{div}\left(\rho_w^l \mathbf{q}_l + \rho_g \mathbf{q}_g + \mathbf{j}_w^l\right) = Q_w$$

$$\Phi \frac{\partial}{\partial t}\left(S_l \rho_h^l + S_g \rho_g\right) + \text{div}\left(\rho_h^l \mathbf{q}_l + \rho_g \mathbf{q}_g + \mathbf{j}_h^l\right) = Q_h.$$

Diffusive fluxes ($D_h^l = D_w^l$ =const):

$$\mathbf{j}_h^l = -\rho_l S_l \Phi D_h^l \nabla\left(\frac{\rho_h^l}{\rho_l}\right), \quad \mathbf{j}_w^l = -\rho_l S_l \Phi D_w^l \nabla\left(\frac{\rho_w^l}{\rho_l}\right)$$

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}\left(S_l \rho_l + S_g \rho_g\right) + \text{div}\left(\rho_l \mathbf{q}_l + \rho_g \mathbf{q}_g\right) = Q_w + Q_h.$$

Saturated case ($S_g > 0$). Variables $p = p_g$ and $S = S_l$

$$\Phi \frac{\partial S}{\partial t} + \mathrm{div}\left(\mathbf{q}_l - \mathbf{J}\right) = Q_w/\rho_w^l$$

$$\Phi \frac{\partial}{\partial t}\left(S\rho_l(p) + (1-S)\rho_g(p)\right) + \mathrm{div}\left(\rho_l(p)\mathbf{q}_l + \rho_g(p)\mathbf{q}_g\right) = Q_h + Q_w.$$

$$\mathbf{q}_l = -\mathbb{K}(\mathbf{x})\frac{kr_l(\mathbf{x},S)}{\mu_l}\left(\nabla(p - p_c(\mathbf{x},S)) - \rho_l(p)\mathbf{g}\right),$$

$$\mathbf{q}_g = -\mathbb{K}(\mathbf{x})\frac{kr_g(\mathbf{x},S)}{\mu_g}\nabla p$$

$$\mathbf{J} = \mathbf{j}_h^l/\rho_w^l = -S\frac{\Phi(\mathbf{x})}{\rho_l(p)}D_h^l\nabla\rho_h^l(p).$$

$$\rho_g(p) = C_g p, \quad \rho_h^l(p) = C_h p, \quad \rho_w^l = \text{const.}, \quad \rho_l(p) = \rho_h^l(p) + \rho_w^l.$$

Unsaturated case ($S_g = 0$). Variables $p = p_l$ and ρ_h^l

$$\operatorname{div}(\mathbf{q}_l - \mathbf{J}) = Q_w / \rho_w^l$$

$$\Phi \frac{\partial \rho_h^l}{\partial t} + \operatorname{div}\left((\rho_w^l + \rho_h^l) \mathbf{q}_l \right) = Q_h + Q_w.$$

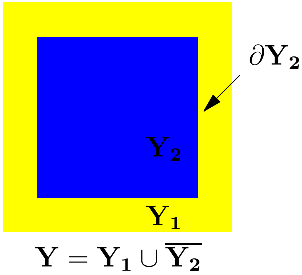
$$\mathbf{q}_l = -\mathbb{K}(\mathbf{x}) \frac{kr_l(1)}{\mu_l} \left(\nabla p - (\rho_h^l + \rho_w^l) \mathbf{g} \right),$$

$$\mathbf{J} = -\frac{\Phi(\mathbf{x})}{\rho_h^l + \rho_w^l} D_h^l \nabla \rho_h^l,$$

$$0 \leq \rho_h^l \leq C_h p.$$

Application of periodic homogenization technique

- Periodic blocks of porous medium composed of two (or more) rock types;
- Porous media is obtained by repeating a scaled unit cell εY.



We seek asymptotic expansions of the following form:

$$S = S^0(\mathbf{x},\mathbf{y},t) + \varepsilon S^1(\mathbf{x},\mathbf{y},t) + \varepsilon^2 S^2(\mathbf{x},\mathbf{y},t) + \cdots \qquad \mathbf{y} = \frac{\mathbf{x}}{\varepsilon}$$

$$p = p^0(\mathbf{x},t) + \varepsilon p^1(\mathbf{x},\mathbf{y},t) + \varepsilon^2 p^2(\mathbf{x},\mathbf{y},t) + \cdots$$

all functions periodic in $\mathbf{y}$.

Local cell problems

- Capillary pressure is constant and the local distribution of the saturation $\mathbf{y} \mapsto S^0(\mathbf{x}, \mathbf{y}, t)$ is given by this constant capillary pressure;
- Effective capillary pressure relating capillary pressure and mean saturation $S^* = \langle S^0 \rangle$ is constructed as before.

For given p^0 and S^* solve the following local problems:

$$\mathrm{div}_y \left(\mathbb{K}\lambda_l(S^0)(\nabla_y \phi_i + \mathbf{e}_i) \right) = 0$$

$$\mathrm{div}_y \left([C_g p^0 \mathbb{K}\lambda_g(S^0) + \Phi S^0 D^l_w C_h](\nabla_y \chi_i + \mathbf{e}_i) \right) = 0$$

$$\mathrm{div}_y \left(\mathbb{K}\lambda_l(S^0)\,(\nabla_y u_i + \mathbf{e}_i)\, \right) = \frac{C_h D^l_w}{C_h p^0 + \rho^l_w} \mathrm{div}_y \left(\Phi S^0(\nabla_y \chi_i + \mathbf{e}_i) \right)$$

Effective tensors

For each $i = 1, \dots, d$, there are four different effective tensors:
Liquid mobilities:

$$\Lambda_l^1(S^*, p^0)\mathbf{e}_i = \langle \mathbb{K}\lambda_l(S^0)\,(\nabla_y u_i + \mathbf{e}_i)\rangle$$

$$\Lambda_l^2(S^*)\mathbf{e}_i = \langle \mathbb{K}\lambda_l(S^0)(\nabla_y \phi_i + \mathbf{e}_i)\rangle$$

Gas mobility:

$$\Lambda_g(S^*, p^0)\mathbf{e}_i = \langle \mathbb{K}\lambda_g(S^0)(\nabla_y \chi_i + \mathbf{e}_i)\rangle$$

Diffusivity:

$$\mathcal{D}(S^*, p^0)\mathbf{e}_i = D_w^l \langle \Phi S^0(\nabla_y \chi_i + \mathbf{e}_i)\rangle.$$

Effective fluxes

Diffusive:

$$\langle \mathbf{J}^0 \rangle = -\frac{1}{\rho_h^{l,0} + \rho_w^l} \mathcal{D}(S^*, p^0) \nabla_x \rho_h^{l,0},$$

Liquid:

$$\langle \mathbf{q}_l^0 \rangle = -\Lambda_l^1(S^*, p^0) \nabla_x p^0 - \Lambda_l^2(S^*)(\nabla_x p_c^*(S^*) + \rho_l^0 \mathbf{g})$$

Gas:

$$\langle \mathbf{q}_g^0 \rangle = -\Lambda_g(S^*, p^0) \nabla_x p^0$$

where $\rho_h^{l,0} = C_h p^0$ in the saturated case $(S^* < 1)$. $\rho_l^0 = \rho_w^l + \rho_h^{l,0}$.

Effective equations

Saturated case:

$$\Phi^* \frac{\partial S^*}{\partial t} + \mathrm{div}_x \left(\langle \mathbf{q}_l^0 \rangle - \langle \mathbf{J}^0 \rangle \right) = Q_w / \rho_w^l$$

$$\Phi^* \frac{\partial}{\partial t} (\rho_l^0 S^* + C_g (1 - S^*) p^0) + \mathrm{div}_x \left(\rho_l^0 \langle \mathbf{q}_l^0 \rangle + C_g p^0 \langle \mathbf{q}_g^0 \rangle \right) = Q_h + Q_w$$

where $\rho_h^{l,0} = C_h p^0$, $\rho_l^0 = \rho_w^l + \rho_h^{l,0}$. Unsaturated case:

$$\mathrm{div}_x \left(\langle \mathbf{q}_l^0 \rangle - \langle \mathbf{J}^0 \rangle \right) = Q_w / \rho_w^l$$

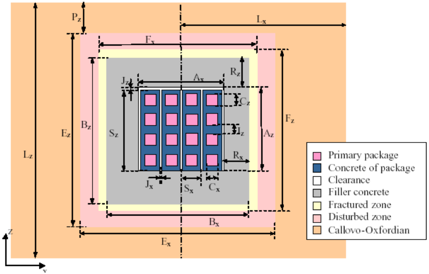
$$\Phi^* \frac{\partial \rho_l^0}{\partial t} + \mathrm{div}_x \left(\rho_l^0 \langle \mathbf{q}_l^0 \rangle \right) = Q_h + Q_w$$

where $\rho_l^0 = \rho_w^l + \rho_h^{l,0}$.

Conclusion

- The local problems are linear due to dominance of capillary forces;
- Coupling between local and global problems is stronger due to diffusive fluxes;
- Application to non periodic media is straightforward.
- Some theoretical work left?
- Efficient implementation has to be tested (CouplexGaz 1)
- Scaling up the Source terms? (CouplexGaz 2); F. Smai.

NEXT STEP :



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