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Modeling and Numerical Simulations of Water-Gas Flow in Porous Media using the Concept of Global Pressure

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Abstract. *We present a global pressure formulation of immiscible, compressible two-phase flow in porous media that is completely equivalent to the original formulation. The new model is discretized by a finite volume method in order to demonstrate advantages of the global pressure formulation for the simulation of water-gas flow in the context of nuclear waste underground disposal.*

1 Introduction

Historically, there have been two main approaches to modeling multiphase flow in porous media. The first is based on individual balance equations for each of the fluids, while the second involves manipulation and combination of those balance equations into modified forms, with concomitant introduction of ancillary functions that we will refer to as the fractional flow or global pressure-saturation formulation. The notion of global pressure was first introduced by [4], [5] and was then revisited by other authors, see for instance [6]. It has been since used in a wide range of engineering specialties related to numerical simulation in hydrology and petroleum reservoir engineering, see for instance [7] and references therein. It has been proven that this fractional flow approach is far more efficient than the original two-pressure approach from the computational point of view [7]. Numerical methods are very sensitive to the choice of form of the governing equation. In the light of the new and continuing developments in numerical methods for the solution of the multiphase flow equations, it is worthwhile revisiting

the question of the form of the governing equations and exploring the implications of this equation form for a numerical method based on it. In this talk, we will present a new formulation [2] to describe immiscible compressible two-phase flow in porous media. The main feature of this formulation is the introduction of a global pressure. The resulting equations are written in a fractional flow formulation and lead to a coupled system which consists of a nonlinear parabolic (the global pressure equation) and a nonlinear diffusion-convection one (the water saturation equation) which can be efficiently solved numerically. Finally, we will present some numerical results obtained by using a finite volume scheme to illustrate the performance of the new formulation for water-gas flow in the context of nuclear waste underground disposal.

2 Governing equations

The usual equations describing immiscible compressible two-phase flow in a porous medium are given by the mass balance equation and Darcy's law for each of the fluid phases (see, e.g., [5], [7]):

$$\Phi\frac{\partial}{\partial t}(\rho_\alpha S_\alpha) + \mathrm{div}(\rho_\alpha \vec{V}_\alpha) = 0 \quad \text{and} \quad \vec{V}_\alpha = -\mathbb{K}\frac{kr_\alpha(S_\alpha)}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha \vec{g}), \tag{1}$$

where Φ and $\mathbb{K}$ are the porosity and the absolute permeability of the porous medium; $\alpha = w$ denotes the wetting phase (e.g. water), $\alpha = n$ indicates the nonwetting phase (e.g. gas), ρ_α, S_α, p_α, $\vec{V}_\alpha$, μ_α and kr_α are, respectively, the density, (reduced) saturation, pressure, volumetric velocity, viscosity and relative permeability of the α-phase, and $\vec{g}$ is the gravitational, downward-pointing, constant vector. In addition to (1), we also have the customary property for saturations and the capillary pressure function:

$$S_w + S_n = 1 \quad \text{and} \quad p_c(S_w) = p_n - p_w. \tag{2}$$

The primary variables are S_α, p_α, and $\vec{V}_\alpha$. Here we assume that the porosity Φ and the absolute permeability $\mathbb{K}$ are functions of space and viscosities μ_w, μ_n are constant. Finally, we assume that the capillary pressure and relative permeabilities depend upon the saturation solely. For notational simplicity, we neglect their dependence on space variable. With respect to the mass densities we do not need any particular restrictions, but for simplicity we will consider water-gas system where ρ_w is a constant and ρ_n is given by the ideal gas law: $\rho_n(p_n) = c_g p_n$, with constant c_g.

3 Global pressure formulation

For expository convenience we neglect gravity terms and we introduce the phase mobilities $\lambda_\alpha(S_w) = kr_\alpha(S_w)/\mu_\alpha$, $\alpha = w, n$, total mobility $\lambda(S_w, p_n) = \rho_w\lambda_w(S_w) + \rho_n(p_n)\lambda_n(S_w)$, water fractional flow functions $f_w(S_w, p_n) = \rho_w\lambda_w(S_w)/\lambda(S_w, p_n)$, diffusion coefficient $a(S_w, p_n) = -\rho_w\rho_n(p_n)\lambda_w(S_w)\lambda_n(S_w)p_c'(S_w)/\lambda(S_w, p_n)$, and total flux $\vec{Q}_t = \rho_w\vec{V}_w + \rho_n(p_n)\vec{V}_g$.

The idea of global pressure is to eliminate capillary pressure gradient term from the total flux $\vec{Q}_t$, or, in other words, to find some *mean pressure* p such that

$$\nabla p_n - f_w(S_w, p_n)p_c'(S_w)\nabla S_w = \omega(S_w, p)\nabla p,$$

where a function $\omega(S_w, p)$ is to be determined. To that aim we introduce the unknown function π such that $p_n = \pi(S_w, p)$ which relates gas pressure p_n and a new variable p, that is called

global pressure. By an easy calculation (see [2]) it follows that π is defined as the solution of the following Cauchy's problem for ordinary differential equation:

$$\frac{d\pi(S,p)}{dS} = f_w(S,\pi(S,p))p_c'(S), \quad S<1, \quad \pi(1,p)=p. \tag{3}$$

This definition implies $p_w \le p \le p_n$. Having found the function π we can express ω as follows:

$$\omega(S_w,p) = \exp\left(\int_{S_w}^1 \frac{\rho_w c_g \lambda_w(s)\lambda_n(s)p_c'(s)}{(\rho_w\lambda_w(s)+c_g\pi(s,p)\lambda_n(s))^2}\,ds\right), \tag{4}$$

which shows that ω is strictly positive. Finally, using the change of variables $p_n = \pi(S_w,p)$, we can transform the system (1)–(2) to the following system of equations (see [2] for details):

$$\Phi\frac{\partial}{\partial t}(S_w\rho_w + c_g\pi(S_w,p)(1-S_w)) - \mathrm{div}\Big(\lambda^n(S_w,p)\omega(S_w,p)\mathbb{K}\nabla p\Big) = 0, \tag{5}$$

$$\vec{Q}_t = -\lambda^n(S_w,p)\omega(S_w,p)\mathbb{K}\nabla p, \tag{6}$$

$$\Phi\frac{\partial}{\partial t}(S_w\rho_w) + \mathrm{div}(f_w^n(S_w,p)\vec{Q}_t) = \mathrm{div}(a^n(S_w,p)\mathbb{K}\nabla S_w), \tag{7}$$

where the coefficients denoted by superscript n are obtained from corresponding coefficients without superscript by replacing p_n by $\pi(S_w,p)$: $\lambda^n(S_w,p) = \lambda(S_w,\pi(S_w,p))$ etc. System (5)–(7) is completely equivalent to system (1)–(2) and it has sound mathematical structure: equation (7) is convection-diffusion equation for saturation S_w, while equation (5) can be interpreted as a parabolic equation for the global pressure p, with a source term coming from the time derivative of water saturation S_w.

In numerical simulation based on system (5)–(7) one needs to calculate the coefficients by integrating equation (3) for different initial values of global pressure p. From a practical point of view one can solve (3) approximately for certain values of initial data and then use an interpolation procedure to extend these values to the whole range of interest. Necessary calculations can be done in a prepocessing phase, without penalizing the flow simulation.

4 Numerical simulations

Spatial discretization of system (5)–(7) is performed by a vertex centered finite volume method (see [1]), which is applied to the both equations. Implicit time stepping is used and the whole discrete nonlinear system is solved by the Newton method.

We chose here to present only one 1D simulation in which gas is injected into a porous domain initially saturated by water. The porous domain is 100 meters long with homogeneous permeability $\mathbb{K}$ of 1 mD and porosity $\Phi = 0.1$. Fluid properties are taken from Couplex test case [3], with incompressible water and hydrogen as gaseous phase. We have, $\mu_w = 7.98\cdot 10^{-3}$ Pas, $\mu_n = 9\cdot 10^{-6}$ Pas, $\rho_w = 10^3$ kg/m^3 and $c_g = 0.808$ kg/(m^{3}MPa). Two-phase saturation functions are of van Genuchten's type with parameters $n=2$ and $P_r = 2$ MPa.

Boundary conditions are taken as follows $S_w(0,t) = 0.4, \quad p(0,t) = 2.0, \quad p(100,t) = 0.1, \quad \frac{\partial}{\partial x}S_w(100,t) = 0$, and the initial conditions are: $S_w(x,0) = 1.0, \quad p(x,0) = 0.1$ MPa. The results of this simulation are shown on Figure 1 for the time instance of 40 days. On the left part of the figure we plot the three pressures, global pressure p, and gas and water pressures p_n and p_w; on the right figure we show water saturation.

From the plot of pressures we see that in the region of small water saturation the water pressure has very steep gradient, which is not the case for the global and gas pressure. Similarly,

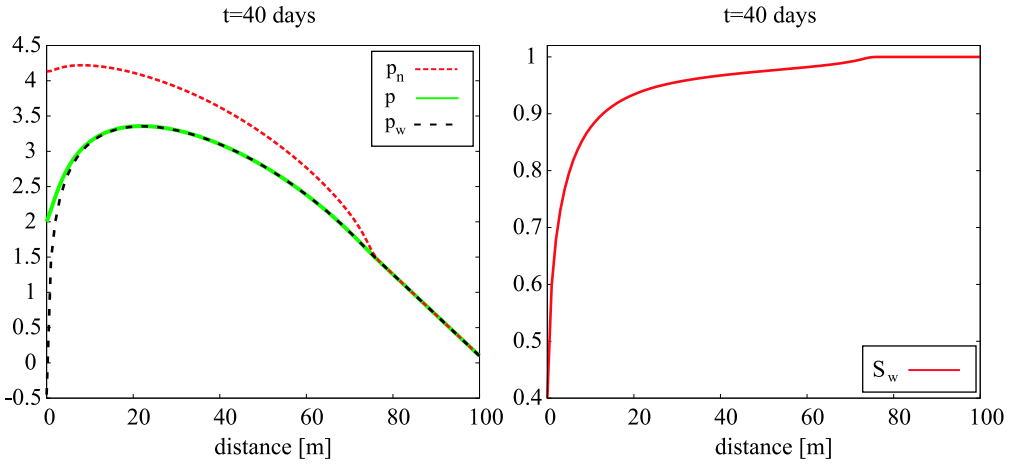

Figure 1: Global and phase pressures (left figure) and water saturation (right figure) at 40 days.

in the point where water saturation reaches value $S_w = 1$ the gas pressure has a steep change of gradient while global pressure remains smooth.

These simple observations lead us to conclude that introduction of the global pressure helps not only to make the coupling between equations (5) and (7) less strong, but also leads to a new pressure variable with more smoothness, which is therefore more convenient for a numerical treatment.

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