Formulation of Compressible Immiscible Two-Phase Flow Model by Means of Global Pressure

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Outline

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Flow equations

Mass conservation: for $\alpha \in \{w, g\}$,

$$\Phi \frac{\partial}{\partial t}(\rho_\alpha S_\alpha) + \mathrm{div}(\rho_\alpha \mathbf{q}_\alpha) = \mathcal{F}_\alpha,$$

The Darcy-Muscat law: for $\alpha \in \{w, g\}$ (gravity neglected),

$$\mathbf{q}_\alpha = -\lambda_\alpha(S_\alpha)\mathbb{K}\nabla p_\alpha,$$

Capillary law: $p_c(S_w) = p_g - p_w,$

No void space: $S_w + S_g = 1.$

Water is incompressible $\rho_w = cte$, gas is compressible $\rho_g(p_g) = c_g p_g$.

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Global pressure formulation

Goal: Reformulate flow equations in order to

- Make coupling between the two differential equations less strong,
- Give to the system well defined mathematical structure.

We consider:

1. Compressible flow.
   1. Fully equivalent formulation
   2. Simplified formulation

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Compressible flow: equations

Rewrite two-phase flow equations as (keep conservative form):

Total flow: $(\mathbf{Q}_t = \rho_w \mathbf{q}_w + \rho_g \mathbf{q}_g)$

$$\mathbf{Q}_t = -\lambda(S_w, p_g)\mathbb{K}\left(\nabla p_g - f_w(S_w, p_g)\nabla p_c(S_w)\right),$$

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w + (1 - S_w)\rho_g(p_g)) + \operatorname{div}\left(\mathbf{Q}_t\right) = \mathcal{F}_w + \mathcal{F}_g,$$

Water mass conservation:

$$\Phi \rho_w \frac{\partial S_w}{\partial t} + \operatorname{div}(f_w(S_w, p_g)\mathbf{Q}_t) = \operatorname{div}(\mathbb{K}a(S_w, p_g)\nabla S_w) + \mathcal{F}_w.$$

Compressible flow: decoupling

In total flow eliminate saturation gradient:

$$\mathbf{Q}_t = -\lambda(S_w, p_g)\mathbb{K}\left(\nabla p_g - f_w(S_w, p_g)p_c'(S_w)\nabla S_w\right),$$

- Idea: introduce a new pressure-like variable that will eliminate ∇S_w term (*Chavent (1976)*, *Antontsev-Monakhov (1978)*)
- Introduce a new pressure variable p, called global pressure, such that $p_g = \pi(S_w, p)$. Find functions $\pi(S_w, p)$ and $\omega(S_w, p)$ that satisfy:

$$\nabla p_g - f_w(S_w, \pi(S_w, p))p_c'(S_w)\nabla S_w = \omega(S_w, p)\nabla p \qquad (1)$$

Compressible flow: global pressure

1. $$\begin{cases} \dfrac{d\pi(S,p)}{dS} = \dfrac{\rho_w \lambda_w(S) p_c'(S)}{\rho_w \lambda_w(S) + \rho_g(\pi(S,p))\lambda_g(S)}, & 0 < S < 1 \\ \pi(1,p) = p. \end{cases}$$

2. $$\omega(S_w, p) = \exp\left(\int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) \pi(s,p))^2} p_c'(s)\, ds \right),$$

3. $$p_w \le p \le p_g.$$

Compressible flow: transformed equations

Variables: p (global pressure) and S_w (water saturation).

Total flow:

$$\mathbf{Q}_t = -\lambda^n(S_w, p)\omega(S_w, p)\mathbb{K}\nabla p.$$

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w + c_g(1 - S_w)\pi(S_w, p)) + \mathrm{div}\mathbf{Q}_t = \mathcal{F}_w + \mathcal{F}_g.$$

Water mass conservation:

$$\Phi \rho_w \frac{\partial S_w}{\partial t} + \mathrm{div}(f_w^n(S_w, p)\mathbf{Q}_t) = \mathrm{div}(\mathbb{K}a^n(S_w, p)\nabla S_w) + \mathcal{F}_w.$$

These equations are fully equivalent to original equations.

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Simplified global formulation

- Coefficients in global formulations are “difficult” to calculate since they depend on $\pi(S_w, p)$: $\lambda^n(S_w, p) = \lambda(S_w, \pi(S_w, p))$ etc.
- Can we make reasonable approximation?

Yes, but we need a hypothesis:

- In all coefficients of differential equations gas density $\rho_g(\pi(S_w, p))$ can be replaced by $\rho_g(p)$ without introducing a significant error.
- Replace $\lambda^n(S_w, p) = \lambda(S_w, \pi(S_w, p))$ by $\lambda(S_w, p)$ etc.

Approximation

Replace gas pressure function

$$\pi(S_w, p) = p - \int_{S_w}^{1} f_w(s, \pi(s,p)) p_c'(s)\, ds,$$

by

$$\pi(S_w, p) = p - \int_{S_w}^{1} f_w(s, p) p_c'(s)\, ds,$$

(*Chavent-Jaffré (1986)*)

Approximation

Replace

$$\omega(S_w, p) = \exp\left(\int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) \pi(s,p))^2} p_c'(s)\, ds\right),$$

by

$$\omega(S_w, p) = \exp\left(\int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) p)^2} p_c'(s)\, ds\right),$$

Simplified global formulation: equations

The system written in unknowns p and S_w.

Total flow:

$$\mathbf{Q}_t = -\lambda(S_w, p)\omega(S_w, p)\mathbb{K}\nabla p,$$

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}(S_w \rho_w + c_g(1 - S_w)p) + \mathrm{div}\mathbf{Q}_t = \mathcal{F}_w + \mathcal{F}_g,$$

Water mass conservation:

$$\Phi \rho_w \frac{\partial S_w}{\partial t} + \mathrm{div}(f_w(S_w, p)\mathbf{Q}_t) = \mathrm{div}(\mathbb{K}a(S_w, p)\nabla S_w) + \mathcal{F}_w.$$

Note that the equations are the same, only the coefficients are different.

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1D Flow simulations

Van Genuchten's functions with parameters (Couplex test case):

$$n = 2,\ Pr = 2 \text{ MPa},\ \mu_w = 0.86 \cdot 10^{-3} \text{ Pas},\ \mu_g = 9 \cdot 10^{-6} \text{ Pas},$$
$$\rho_w = 996.53 \text{ kg/m3},\ c_g = 0.808 \text{ and } T = 300K.$$

- Domain $\Omega = (0, L)$, $L = 100$ m $\mathbb{K} = 1$ mD, $\Phi = 0.1$;
- Discretization by vertex centered finite volume method;
- Simulation 1 - gas injection:

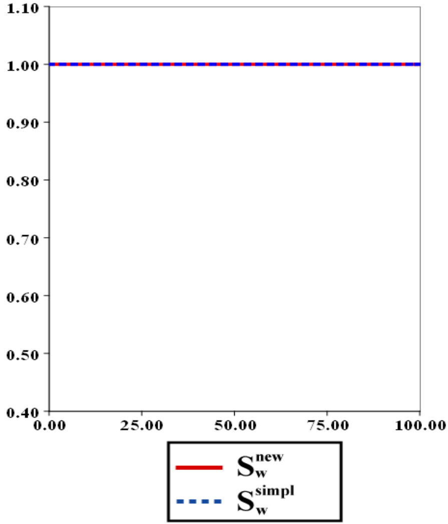
$$p(0,t) = 1.0,\ p(L,t) = 0.1,\ S_w(0,t) = 0.4,\ \frac{\partial}{\partial x} S_w(L,t) = 0,$$
$$S_w(x,0) = 1.0, \quad p(x,0) = 0.1.$$

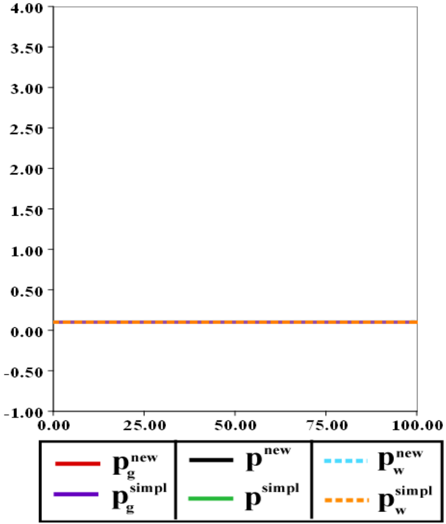
- Simulation 2 - water injection:

$$p(0,t) = 4.0,\ p(L,t) = 0.5,\ S_w(0,t) = 1.0,\ \frac{\partial}{\partial x} S_w(L,t) = 0,$$
$$S_w(x,0) = 0.7, \quad p(x,0) = 0.5.$$

Simulation 1

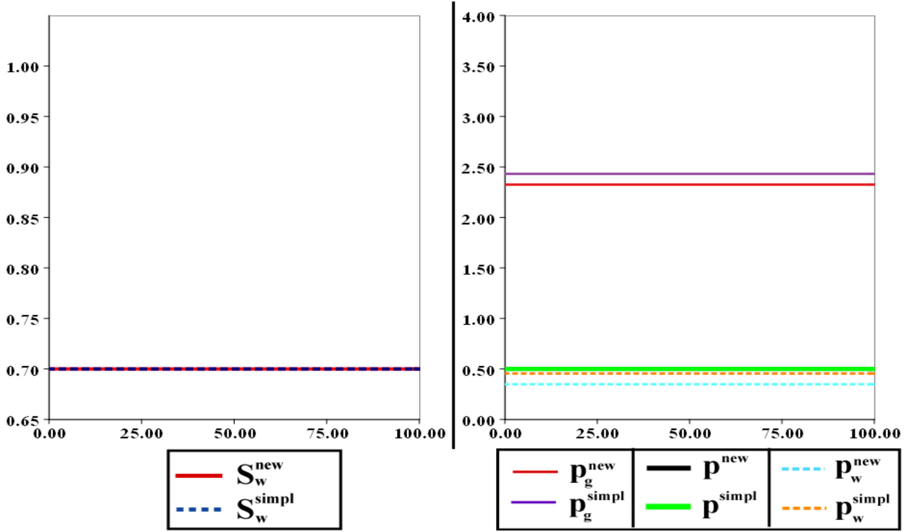
Gas injection





Simulation 2

Water injection



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Conclusion

- Simplification of the system can safely be used:
  - in applications with high mean field pressure.
  - in applications with relatively small capillary pressure.
- The difference between simplified and fully equivalent model becomes significant:
  - for small global pressure.
  - for relatively large capillary pressure.
- The difference in computed saturations is always small, but the differences in gas pressure (global pressure) may be significant.

Work in progress

- An extension to multiphase, multicomponent models is straightforward and it is at present in the course of study.
- New and simplified formulation are well adapted for the mathematical analysis of the model. At present we study existence etc.
- Implementation of FV method in two and three dimensions.
- Treatment of multiple rock types is currently studied.

References: B. Amaziane, M. Jurak: *A new formulation of immiscible compressible two-phase flow in porous media*, C. R. A. S. Mécanique 336 (2008) 600-605.