

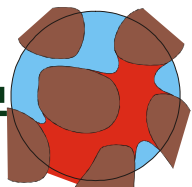
# Upscaling “dissolution” mechanisms in porous media

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Financial support: IFP, CNRS/INSU/PNRH, SNECMA, DGA



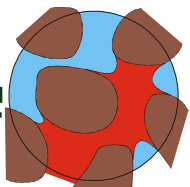
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**M. Quintard**

**dissolution**

# Outline

- **Background**
- **Pore-scale model and effective surface**
- **Darcy-scale models**
- **Stability**
- **Large-scale models?**
- **Conclusions**



# Introduction

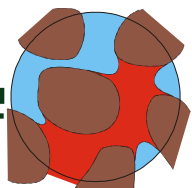
- **Dissolution:**  
geochemistry, karsts, salt mines, NAPL, petrol. Engng, aerospace industry...
- **Problems:**
  - Multiple-scale analysis
  - History effects
  - Instabilities

*Daccord et al., 1993*

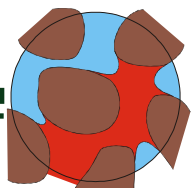
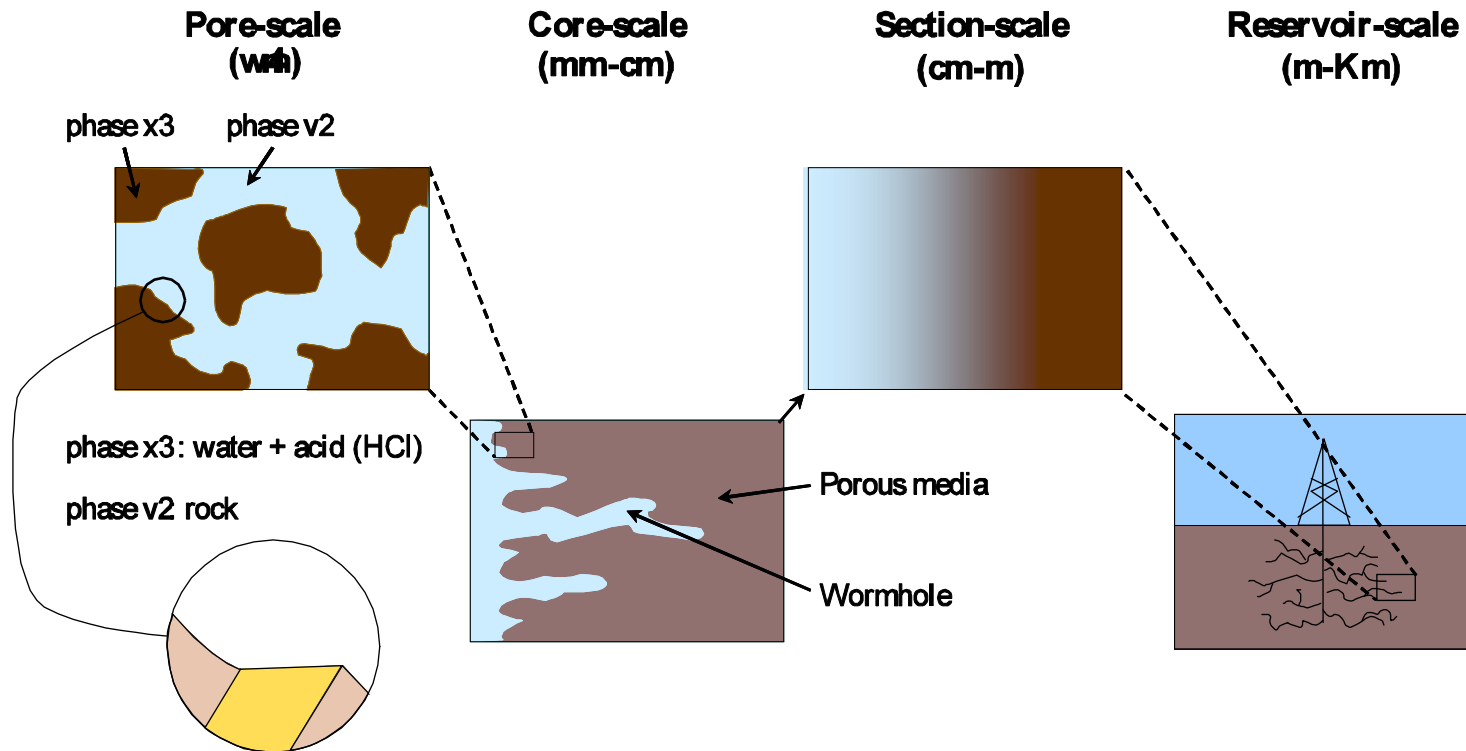


Ha Long Bay

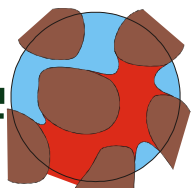
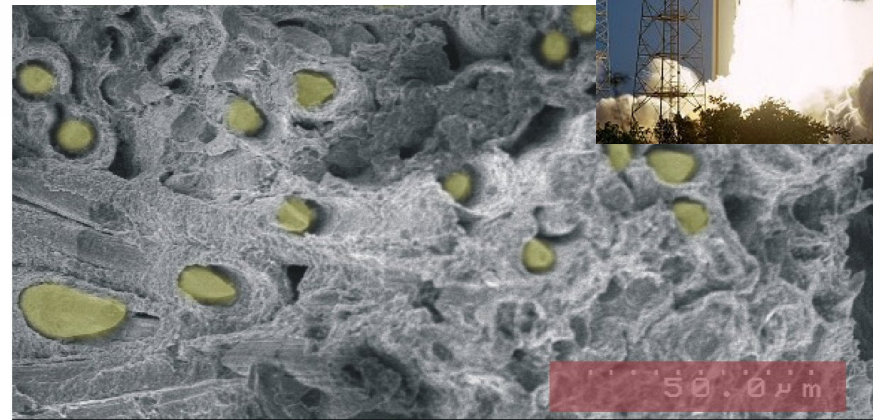
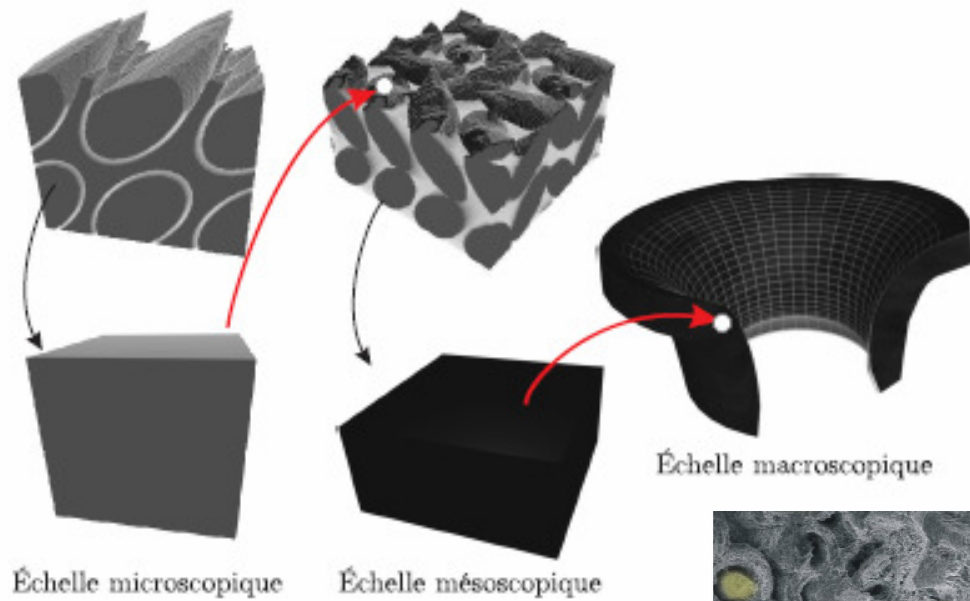
Igue de Planagrèze



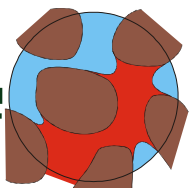
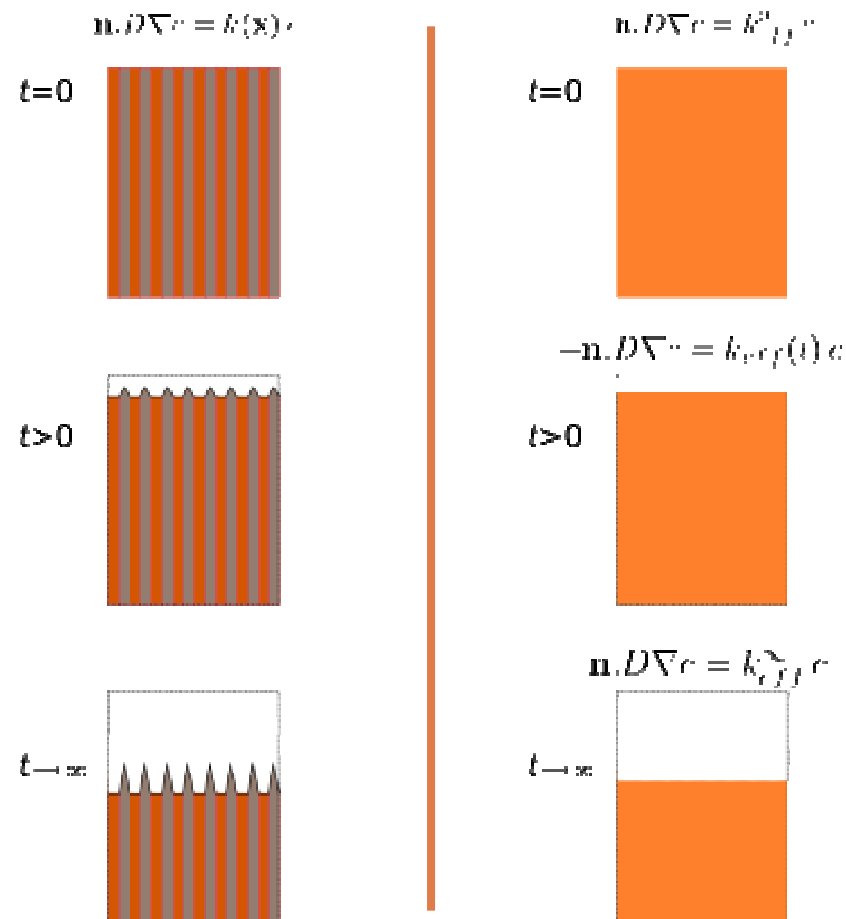
# Example 1: acidizing treatment



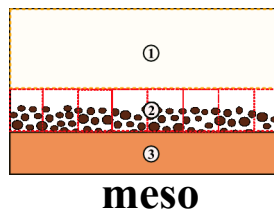
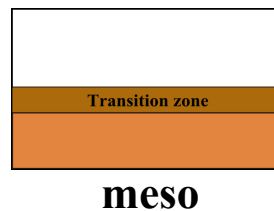
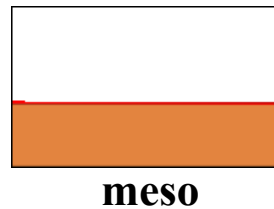
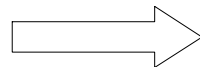
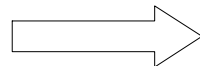
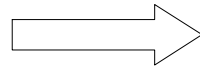
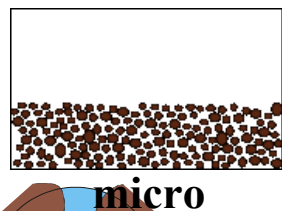
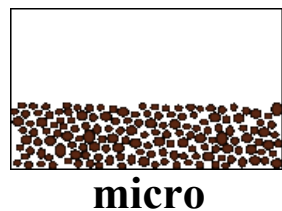
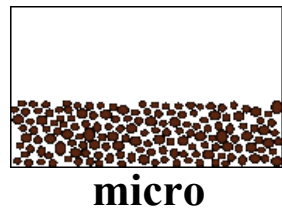
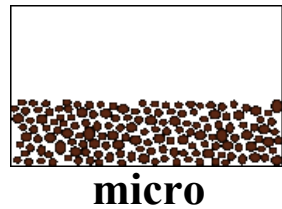
# Example 2: ablation of composite structures



# Upscaling Surface Heterogeneities: The concept of Effective Surface



# Non-ablative case: various approaches



## Direct Simulation

$\langle c \rangle + \tilde{c} \rightarrow$  Effective surface,  
effective BC (jump conditions)

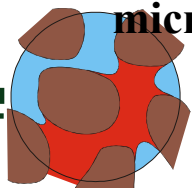
*Eg : Ochoa et al., Wood et al.,  
Valdès-Parada et al.*

Meso-scale modelling, GTE  $\rightarrow$   
Effective surface, effective BC

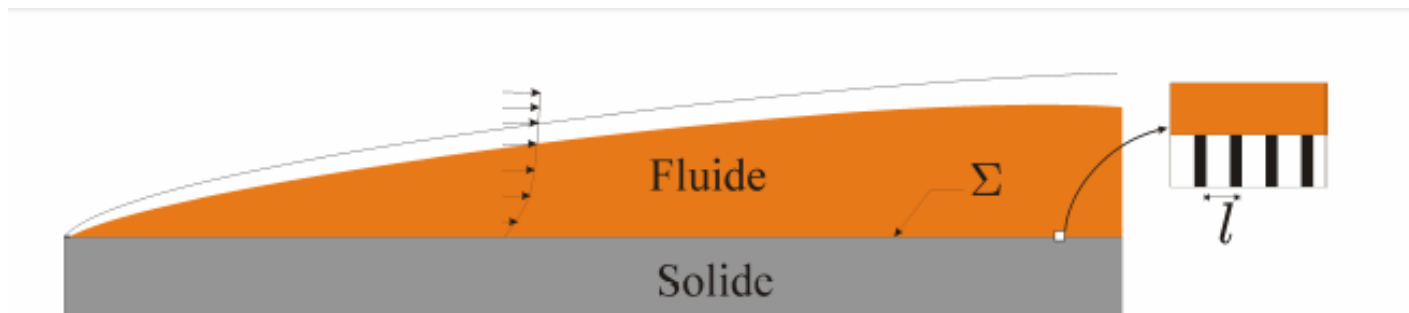
*Eg : Chandesris et al., Goyeau et al.*

## Domain decomposition

*Achdou et al., Jäger and Mikelić, ...*



# Extension of Wood et al. (2000)



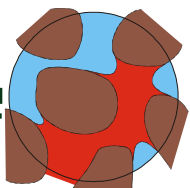
Flow: Blasius

$$\mathbf{u} \cdot \nabla c = \nabla \cdot D \nabla c \quad \text{in the fluid domain}$$

$$-\mathbf{n}_{\gamma\kappa} \cdot D \nabla c = k(\mathbf{x}) c \quad \text{at } \Sigma$$

$$c(y = h) = C_0 \quad \text{at top of B.L.}$$

$$c(x = x_0) = c(x = x_0 + l)$$





# Extension of Wood et al. (2000)

$$c = \langle c \rangle + \tilde{c}$$

with

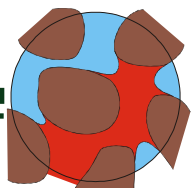
$$\mathbf{u} \cdot \nabla \langle c \rangle = \nabla \cdot D \nabla \langle c \rangle \quad \text{in the fluid domain}$$

$$-\mathbf{n}_{\gamma\kappa} \cdot D \nabla \langle c \rangle = \langle k(\mathbf{x}) c \rangle_{\Sigma} \quad \text{at } \Sigma$$

and

$$\mathbf{u} \cdot \nabla \tilde{c} = \nabla \cdot D \nabla \tilde{c} \quad \text{in the fluid domain}$$

$$-\mathbf{n}_{\gamma\kappa} \cdot D \nabla \tilde{c} = k \tilde{c} + \tilde{k} c - \langle k \tilde{c} \rangle_{\Sigma} \quad \text{at } \Sigma$$



# Extension of Wood et al. (2000)

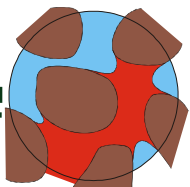
$$\tilde{c} = s(\mathbf{x}) \langle c \rangle + \dots$$

with

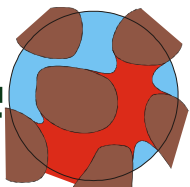
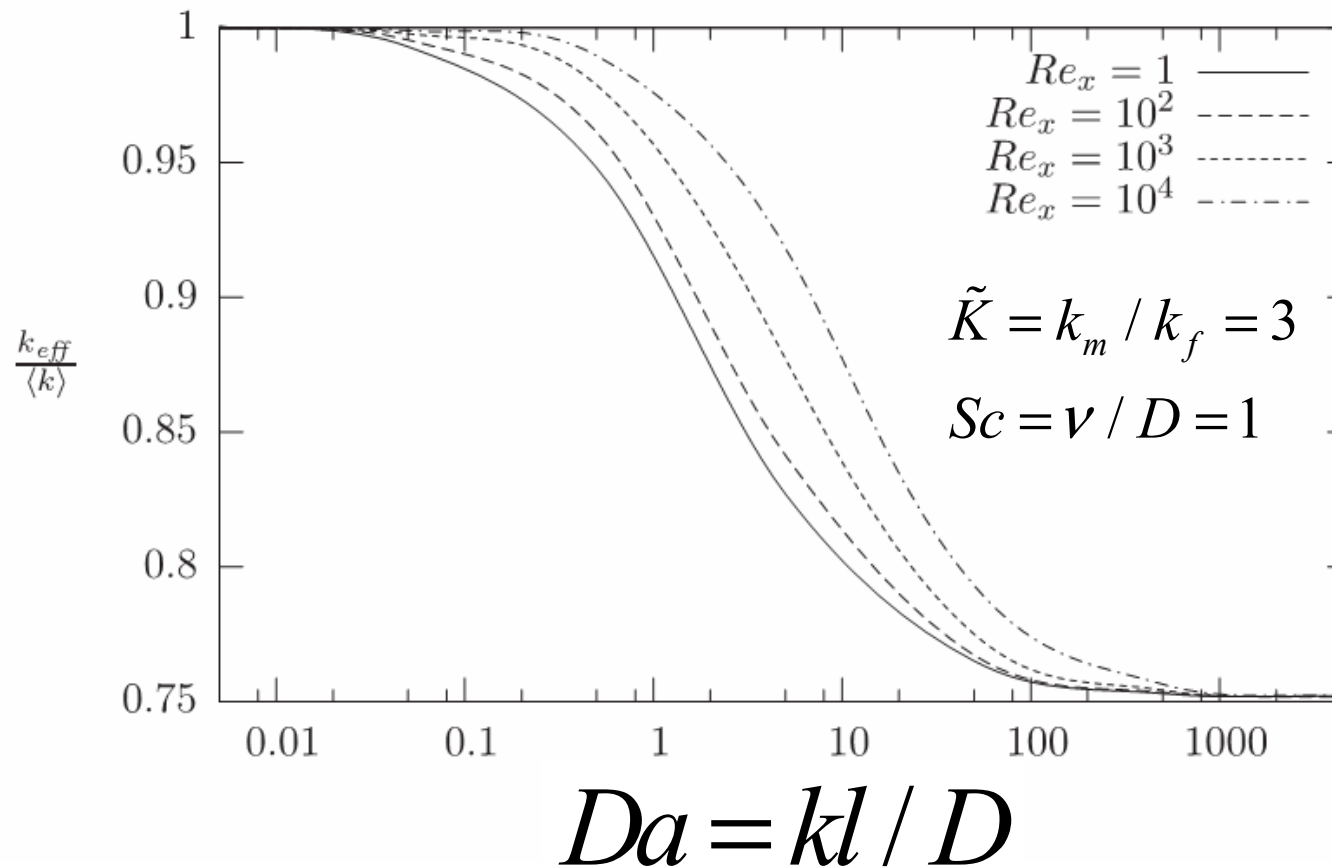
$$\mathbf{u} \cdot \nabla s = \nabla \cdot D \nabla s \quad \text{in the fluid domain}$$

$$-\mathbf{n}_{\gamma\kappa} \cdot D \nabla s = k s + \tilde{k} - \langle \tilde{k} s \rangle_{\Sigma} \quad \text{at } \Sigma$$

$$k_{eff} = \langle k \rangle_{\Sigma} + \langle \tilde{k} s \rangle_{\Sigma}$$



# Results for circular patches (far from entrance region)



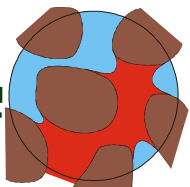
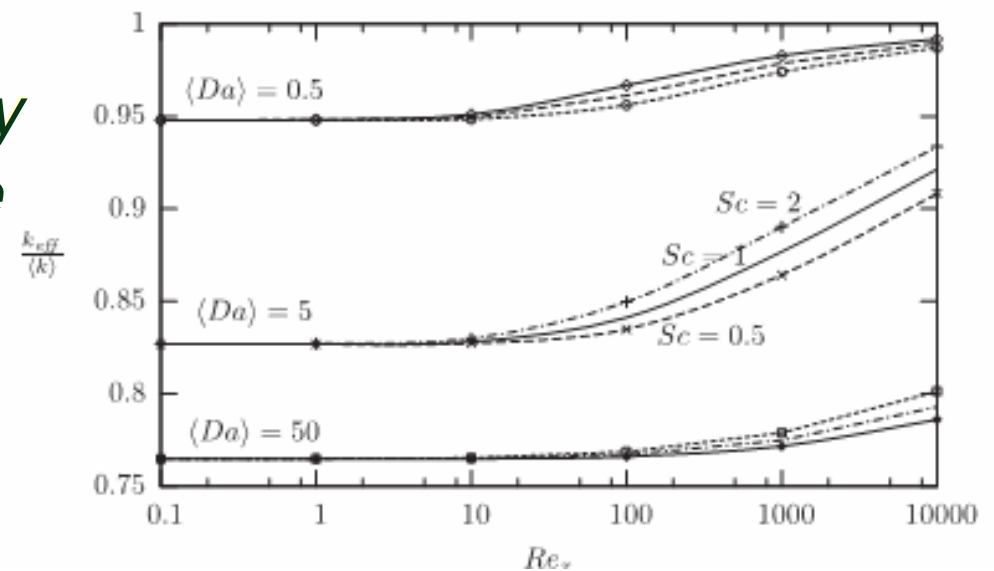
# Results

- **two limit cases**

- $Da \ll 1$ ,  $c(x) = C_0 \rightarrow k_{eff} = \langle k \rangle$
- $Da \gg 1$ ,  $k_{eff} = k^*$  (harmonic mean of the reactivities)

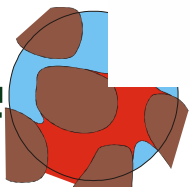
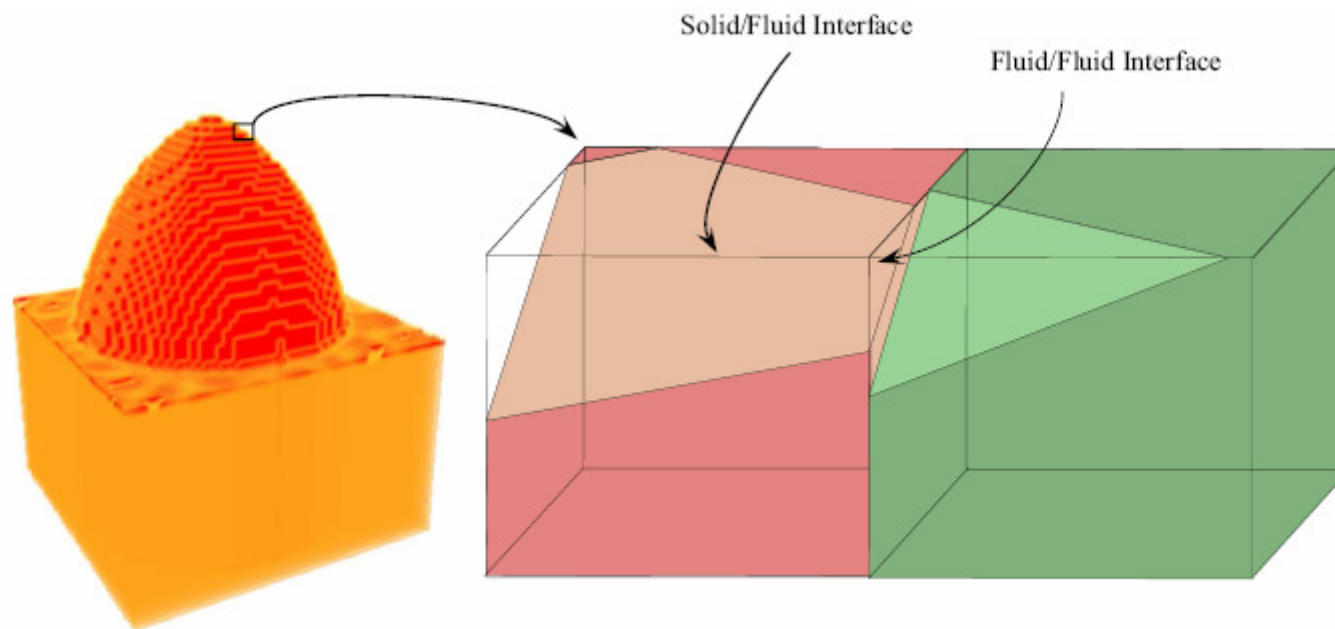
- **general case**

- *influence of geometry*
- *slight influence of  $Re$  (for low  $Re...$ )*

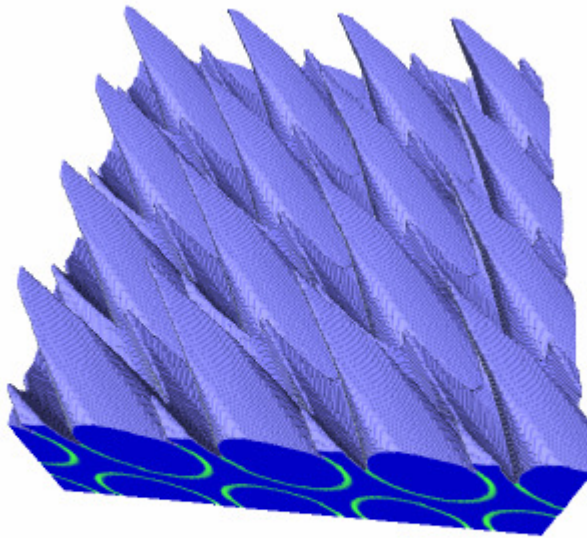


# Ablative case: transient $\rightarrow$ DNS

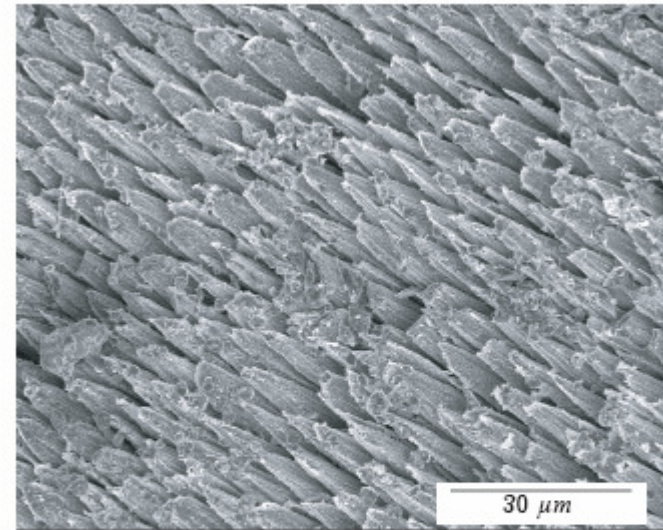
- Ablation leads to non-differentiable surfaces
- Limits of ALE and phase field methods  $\rightarrow$  adapted VOF method (Aspa, 2006)



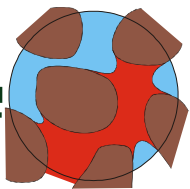
# Example 1: steady-state surface



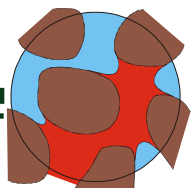
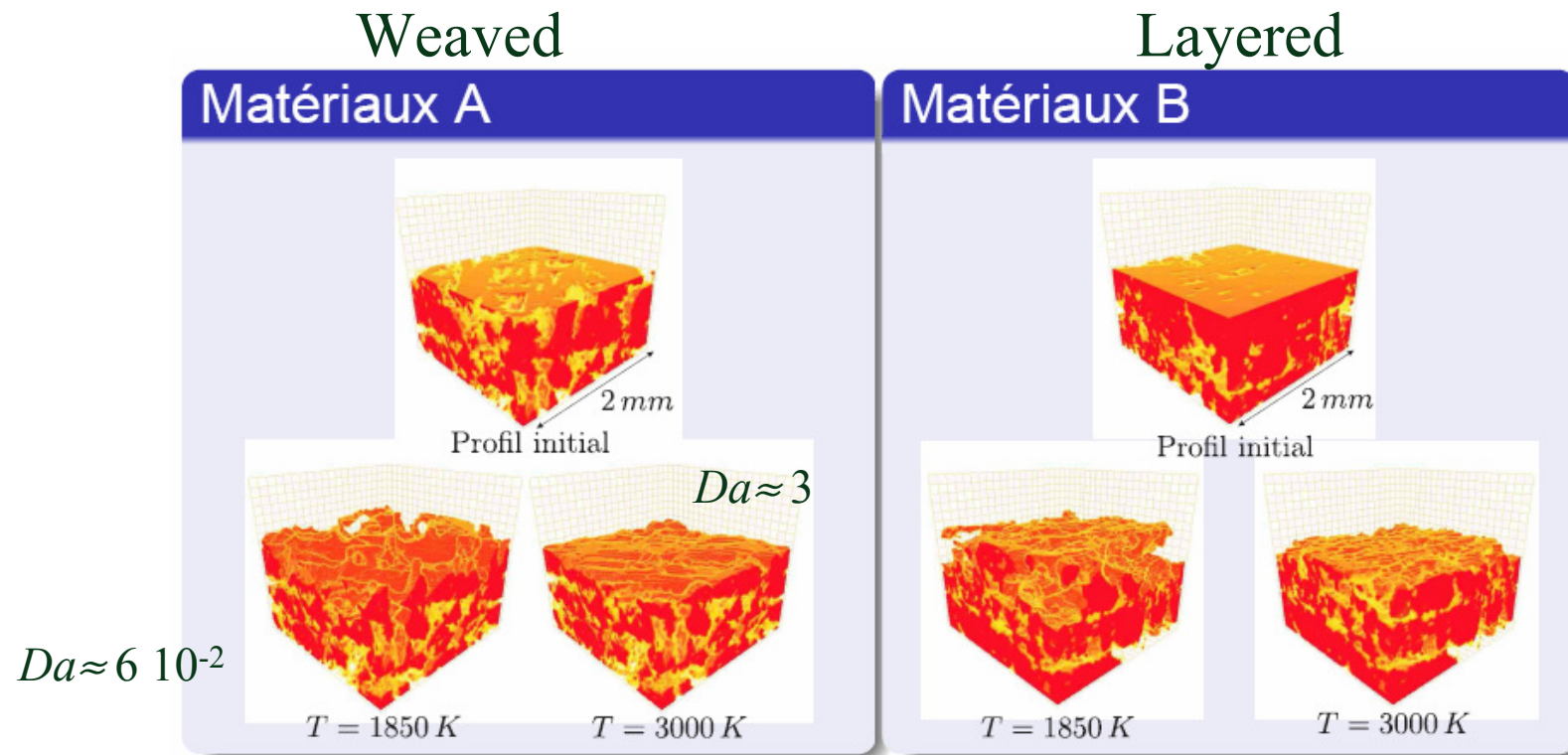
(a) Simulation avec  $k_f = k_m = 0.4 \text{ m/s}$  et  $k_i = 3.2 \text{ m/s}$



(b) Observation



# Example 2: porous composites



Note:  $T$  changes the “diffusion” coefficient

# $K_{eff}$ ? case $Da \ll 1$

- Projected Areas:  $A_{m,p}$  and  $A_{f,p}$
- Steady-state ablation: uniform velocity implies

$$\xi k_f \cos(\theta_f) = \xi k_m \cos(\theta_m) \Rightarrow$$

$$k_f \frac{A_f}{A_{f,p}} = k_m \frac{A_m}{A_{m,p}} \Rightarrow \frac{A_f}{A_{f,p}} = \tilde{k}$$

$$k_{eff} = \langle k \rangle \approx k_m$$

$$\neq \langle k \rangle_{t=0}$$

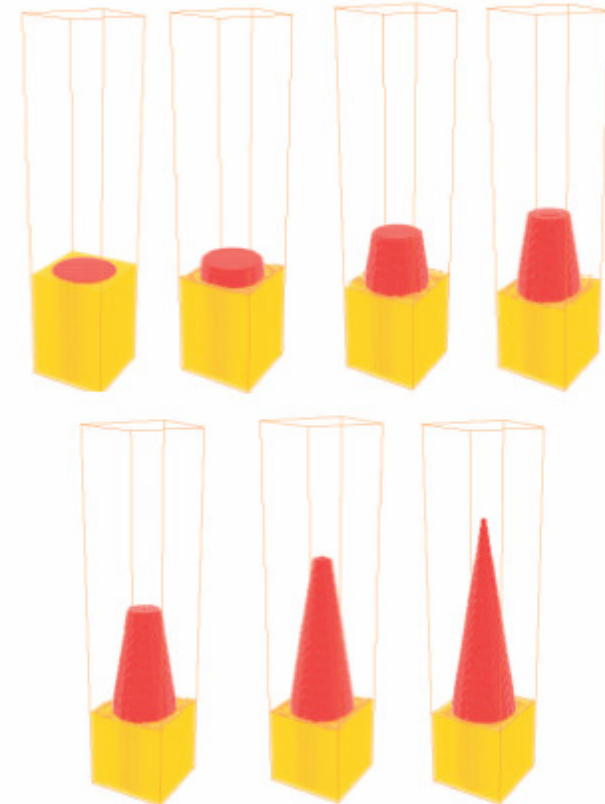
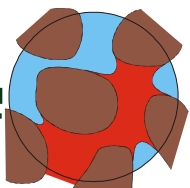


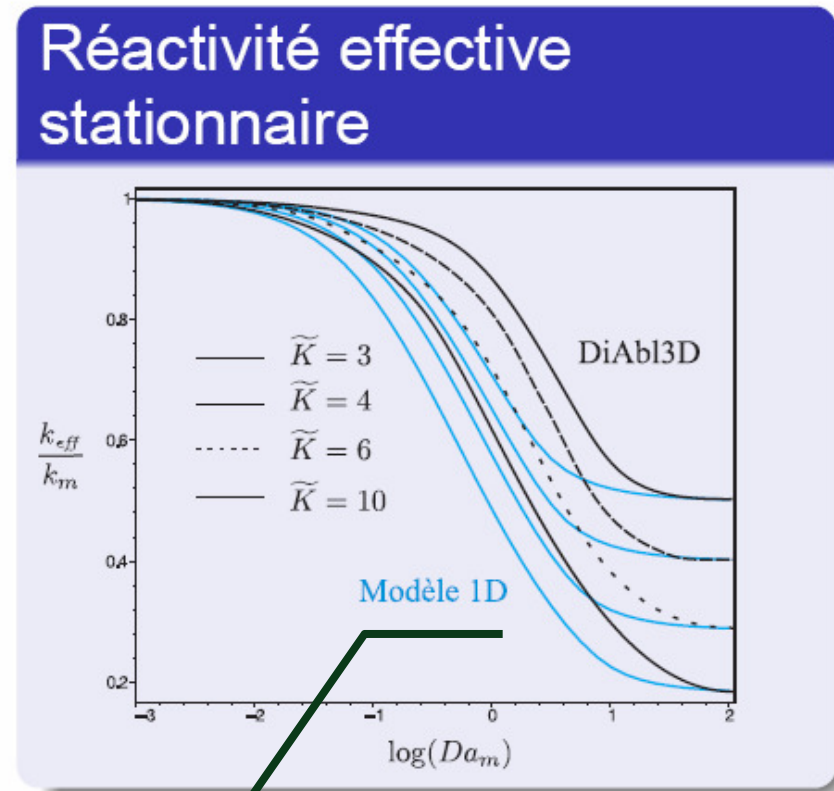
Fig. 1 - Évolution Morphologique d'une cellule avec  $k = 7$



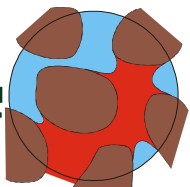


# $K_{eff}$ ?

- **Limit Cases: simple models**
  - $Da \ll 1$ ,  $\max(k)$
  - $Da \gg 1$ ,  $k$ -harmonic mean
- **Intermediate  $Da \rightarrow$  complex simulations**



Simplified model

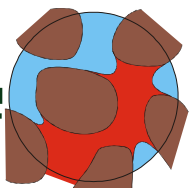
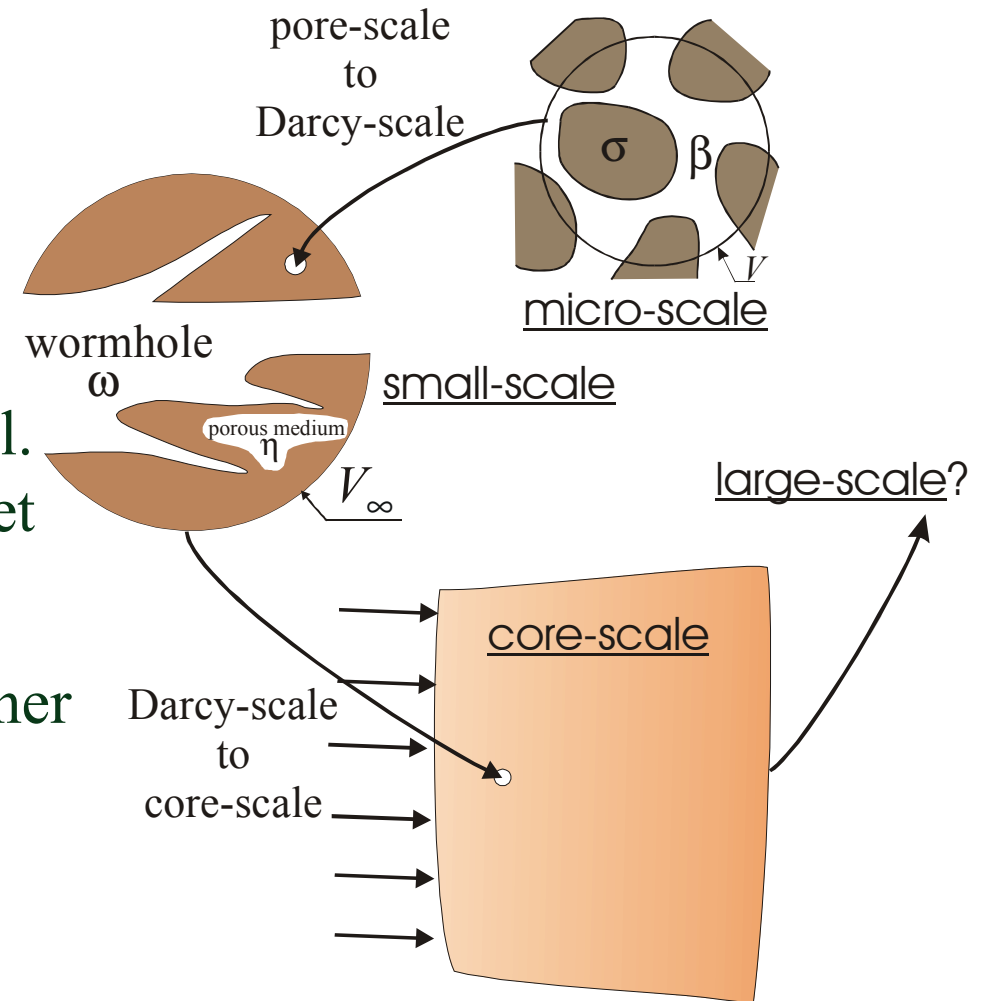


# Dissolution: Darcy-scale (core-scale) models?

- **Pore-scale: non-local effects (space and time)**

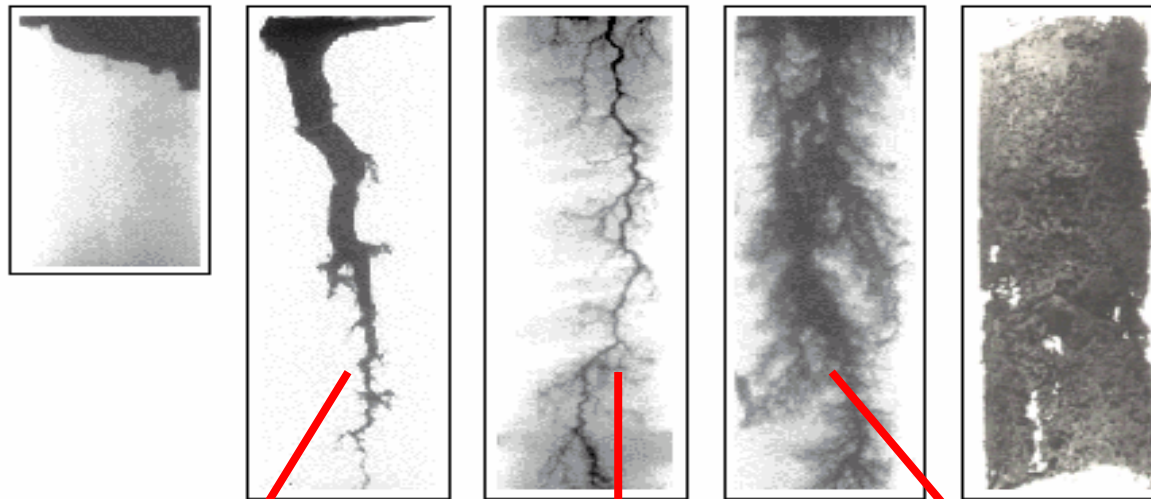
- *Direct Simulation* : Bekri et al. (1995), Mercet (2000), Zhang et Smith (2001), ...

- *Network models* (Fredd, Hoefner et al.)



# Target 1: Dissolution Instabilities (Wormholing)

■ ■ ■  FLOW RATE



HCl-calcite  
system (*Fredd*  
*et al., SPE*)

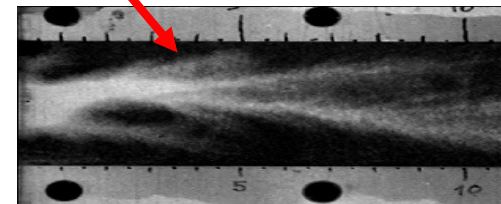
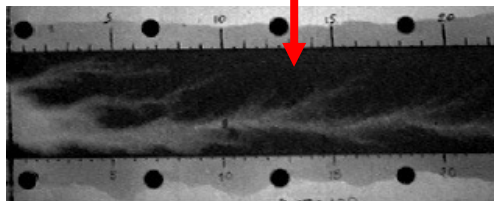
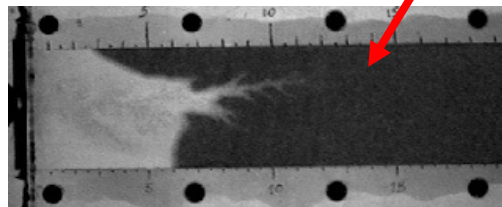
Face  
Dissolution<sup>1</sup>

Conical  
Wormhole<sup>14</sup>

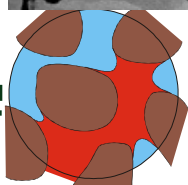
Dominant  
Wormhole<sup>1</sup>

Ramified  
Wormhole<sup>1</sup>

Uniform  
Dissolution<sup>2</sup>



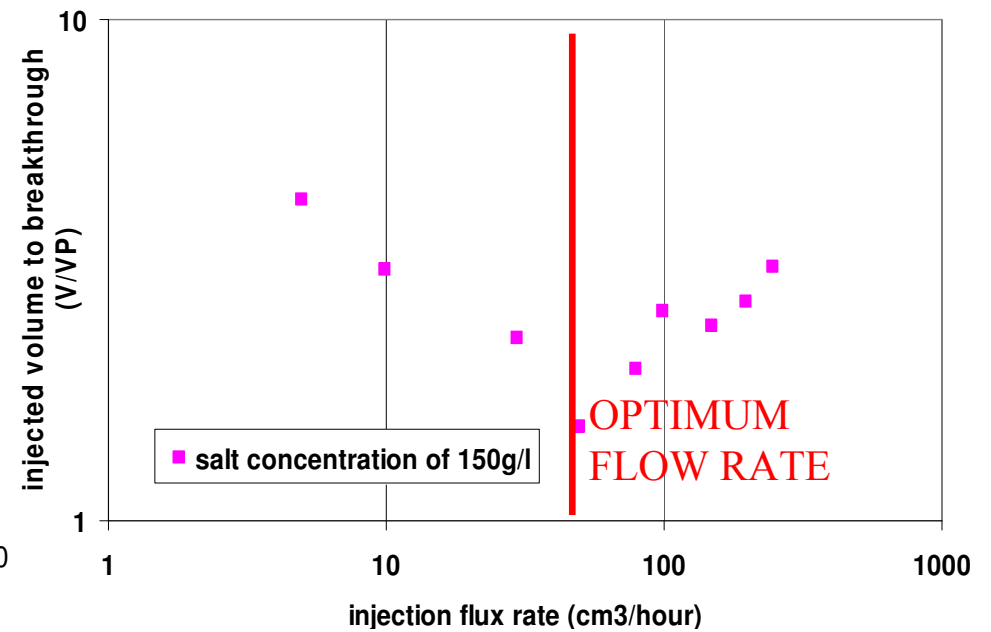
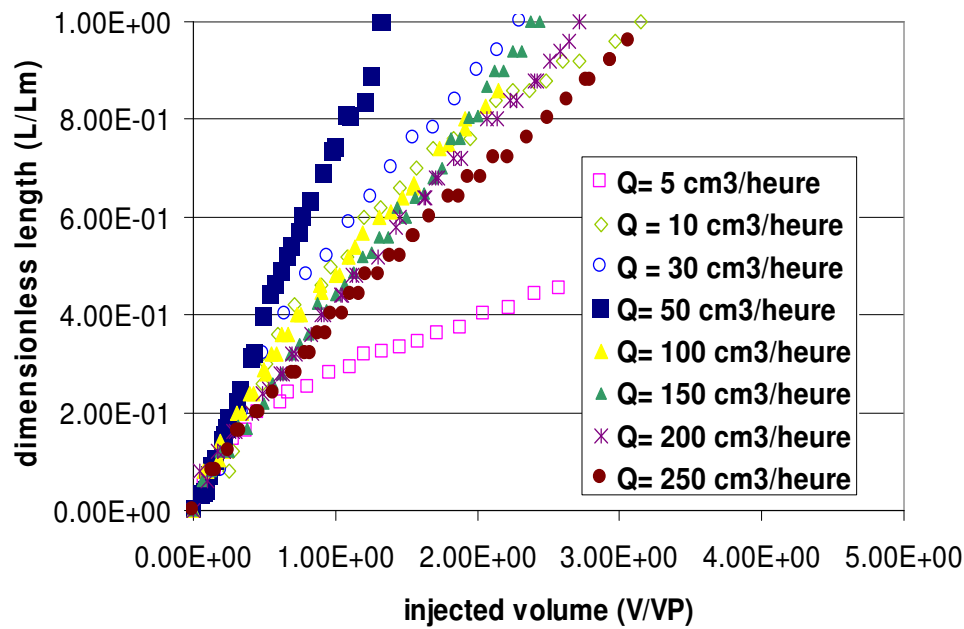
water-NaCl system (*Zarcone et al.*)



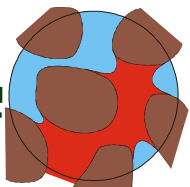
# Target 2: “Optimum flow rate”

*Optimum injection rate: minimum injected acid volume to breakthrough*

$$Q_{opt} = f(\text{length core, } C_{NaCl} \dots)$$



## NaCl Concentration of 150 g/l



# Simplified Pore-scale problem

$$N.-S.$$

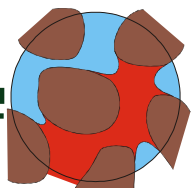
$$+$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (\mathbf{v}c) = \nabla \cdot (D\nabla c) \quad \text{in the fluid domain}$$

$$c = C_{eq} \quad \text{at } A_{\beta\sigma}$$

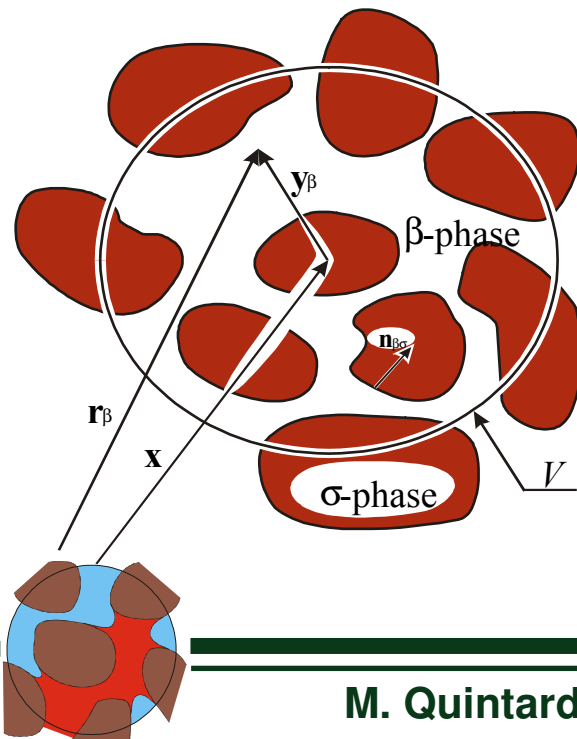
**Note (binary case):**

$$-\mathbf{n} \cdot D\nabla c = kc \quad \Leftrightarrow \quad c = C_{eq} \approx 0 \quad \text{at } A_{\beta\sigma} \quad \text{if } Da \gg 1$$



# Darcy-scale: Local Non-Equilibrium Models

- **Local equilibrium dissolution:**  $C_{A\beta} = \langle c_{A\beta} \rangle^\beta = C_{eq}$  produces sharp fronts
- **LNE:** Heuristic model classically used in Chemical Engineering (discussion in Quintard & Whitaker, 1994, 1999)



$$C_{A\beta} = \langle c_{A\beta} \rangle_{\mathbf{x}}^\beta = \frac{1}{V_\beta} \int_{V_\beta} c_{A\beta}(\mathbf{x}+\mathbf{y}) dV$$

$$\varepsilon \frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \nabla \cdot (\mathbf{D}^* \cdot \nabla C) - \alpha C$$

+ additional terms

+ other macro-scale equations

# Upscaling (framework)

(Quintard and Whitaker, 1999; Golfier et al., 2001)

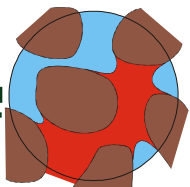
- **Deviations**  $\mathbf{v}_\beta = \boldsymbol{\varepsilon}_\beta^{-1} \mathbf{V}_\beta + \tilde{\mathbf{v}}_\beta$   $c_{A\beta} = C_{A\beta} + \tilde{c}_{A\beta}$
- **Coupled pore-scale and averaged equations**

$$\frac{\partial(\boldsymbol{\varepsilon}_\beta C_{A\beta})}{\partial t} + \nabla \cdot (\mathbf{V}_\beta C_{A\beta}) + \frac{1}{V} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot c_{A\beta} (\mathbf{v}_{A\beta} - \mathbf{w}) dA = \nabla \cdot \left[ D \left( \boldsymbol{\varepsilon}_\beta \nabla C_{A\beta} + \frac{1}{V} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \tilde{c}_{A\beta} dA \right) \right] - \nabla \cdot \langle \tilde{\mathbf{v}}_\beta \tilde{c}_{A\beta} \rangle$$

+

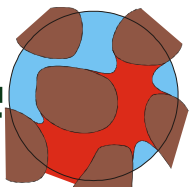
$$\frac{\partial \tilde{c}_{A\beta}}{\partial t} + \tilde{\mathbf{v}}_\beta \cdot \nabla C_{A\beta} + \mathbf{v}_\beta \cdot \nabla \tilde{c}_{A\beta} - \underbrace{\boldsymbol{\varepsilon}_\beta^{-1} \nabla \cdot \langle \tilde{\mathbf{v}}_\beta \tilde{c}_{A\beta} \rangle}_{\ll \mathbf{v}_\beta \cdot \nabla \tilde{c}_{A\beta}} = \nabla \cdot (D \nabla \tilde{c}_{A\beta}) - \underbrace{\boldsymbol{\varepsilon}_\beta^{-1} D \nabla \cdot \left[ \frac{1}{V} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \tilde{c}_{A\beta} dA \right]}_{\ll \nabla \cdot (D \nabla \tilde{c}_{A\beta})} - \frac{\boldsymbol{\varepsilon}_\beta^{-1}}{V} \int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma} \cdot D \nabla \tilde{c}_{A\beta} dA$$

+ .....



# Upscaling : problems

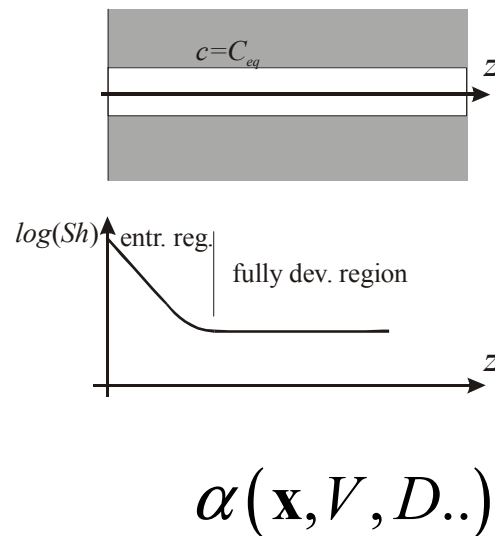
- **Quasi-steady dissolution: terms like  $\mathbf{n}_{\beta\sigma} \cdot (\mathbf{v}_{A\beta} - \mathbf{w})$  may be neglected in the problem for the deviations**
- **“Closure” = approximate solution of the coupled equations**
  - Quasi-steady solution?
  - *But...historical effect remains through the interface evolution*



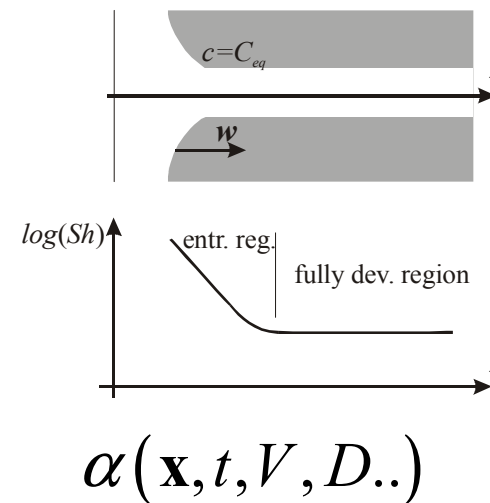


# A simple example: the tube problem (Graetz's problem, $Pe \gg 1$ )

## • Classical Problem

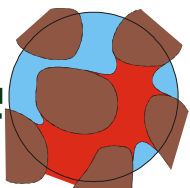


## • Dissolution



Non-local in space and time?

...see Golfier et al. (2001), Pierre et al. (2005)



## 2D and 3D cases (MQ & SW, 94, 99, FG et al., 2002)

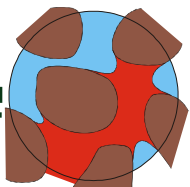
- Representation of the deviations:

$$\tilde{c}_{A\beta} = c_{A\beta} - \langle c_{A\beta} \rangle^\beta = \mathbf{b}_\beta \cdot \nabla \langle c_{A\beta} \rangle^\beta - s_\beta \langle c_{A\beta} \rangle^\beta + \dots$$

- Darcy-Scale equation:

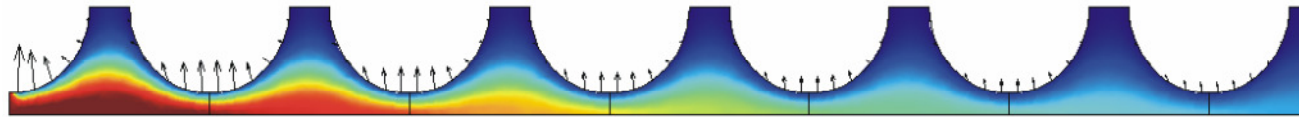
$$\begin{aligned} \frac{\partial}{\partial t} (\varepsilon_\beta \langle c_{A\beta} \rangle^\beta) + \nabla \cdot (\langle \mathbf{v}_\beta \rangle \langle c_{A\beta} \rangle^\beta) - \nabla \cdot [\mathbf{d}_\beta \langle c_{A\beta} \rangle^\beta] - \mathbf{u}_\beta \cdot \nabla \langle c_{A\beta} \rangle^\beta \\ = \nabla \cdot (\mathbf{D}_\beta^* \cdot \nabla \langle c_{A\beta} \rangle^\beta) - \alpha \langle c_{A\beta} \rangle^\beta \end{aligned}$$

- + cell problems → “effective” properties

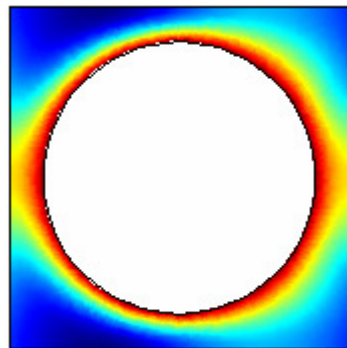


# Comparison with numerical experiments

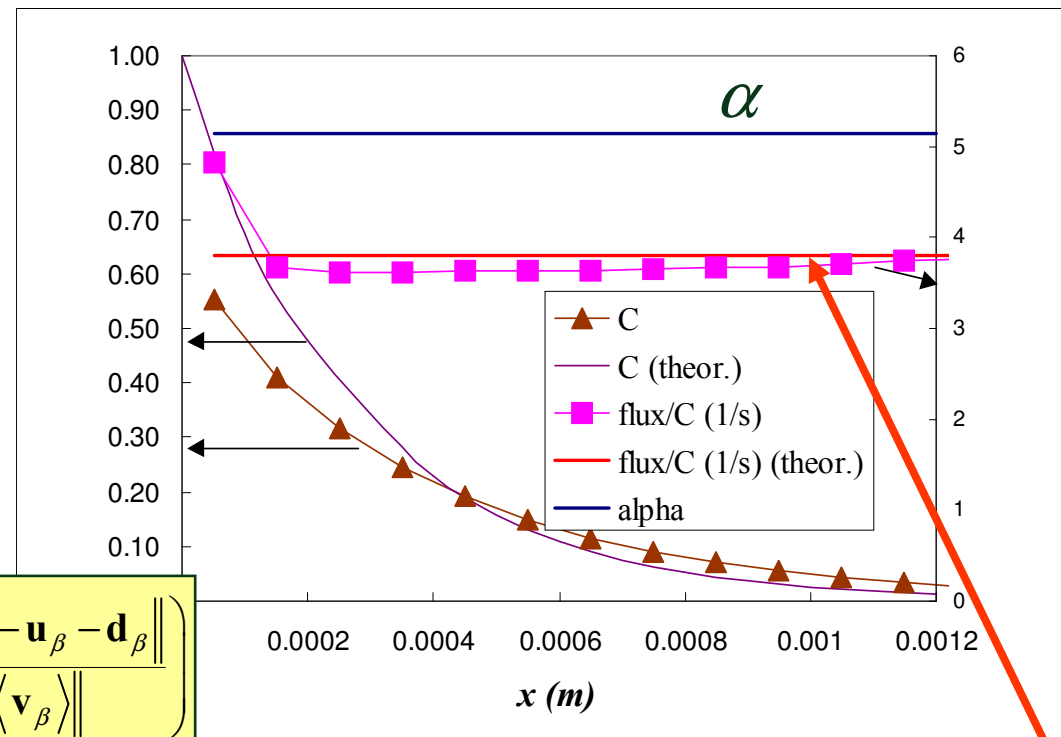
Pe=185



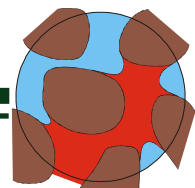
- numerical experiments over several unit cells
- solution of the closure problems over a unit cell



$$\alpha^* = \alpha / \left( \frac{\| \langle \mathbf{v}_\beta \rangle - \mathbf{u}_\beta - \mathbf{d}_\beta \|}{\| \langle \mathbf{v}_\beta \rangle \|} \right)$$



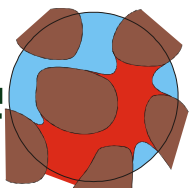
→ need the additional convective transport terms!



## Problem with evolving geometry : Effective Coefficients ( $\alpha$ , $D_\beta$ , $K$ )

- Effective coefficients as a function of the time evolution of the interface (example of direct simul.: Bekri et al.)?
  - $\varepsilon(t)$
  - $\alpha(t, \mathbf{Pe}, \dots) \rightarrow \alpha(\varepsilon, \mathbf{Pe})$
  - $D_\beta(t, \mathbf{Pe}, \dots) \rightarrow D_\beta(\varepsilon, \mathbf{Pe})$
  - $K(t) \rightarrow K(\varepsilon)$

Correlations obtained using: numerical simulation, closure pbs, experiments, ...



# Transport and Dissolution: numerical model

- Dimensionless equations

$$\varepsilon_{\beta} \frac{\partial C'_{A\beta}}{\partial t'} + \mathbf{V}'_{A\beta} \cdot \nabla C'_{A\beta} = \frac{1}{Pe} \nabla \cdot (\mathbf{D}' \cdot \nabla C'_{A\beta}) - Da C'_{A\beta}$$

$$\frac{\partial \varepsilon_{\beta}}{\partial t'} = \frac{(1 - \varepsilon_{\beta})}{\varepsilon_{\beta}} Da N_{ac} C'_{A\beta}$$

+ Darcy-Brinkman

- Dimensionless numbers

$$F = \frac{L_1}{L_2} \quad N_D = \frac{\mathbf{K}}{L_1^2}$$

$$Da = \frac{\alpha l}{|\mathbf{v}_0|}$$

Damköhler

$$Pe = \frac{|\mathbf{v}_0| l}{D}$$

Péclet

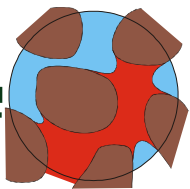
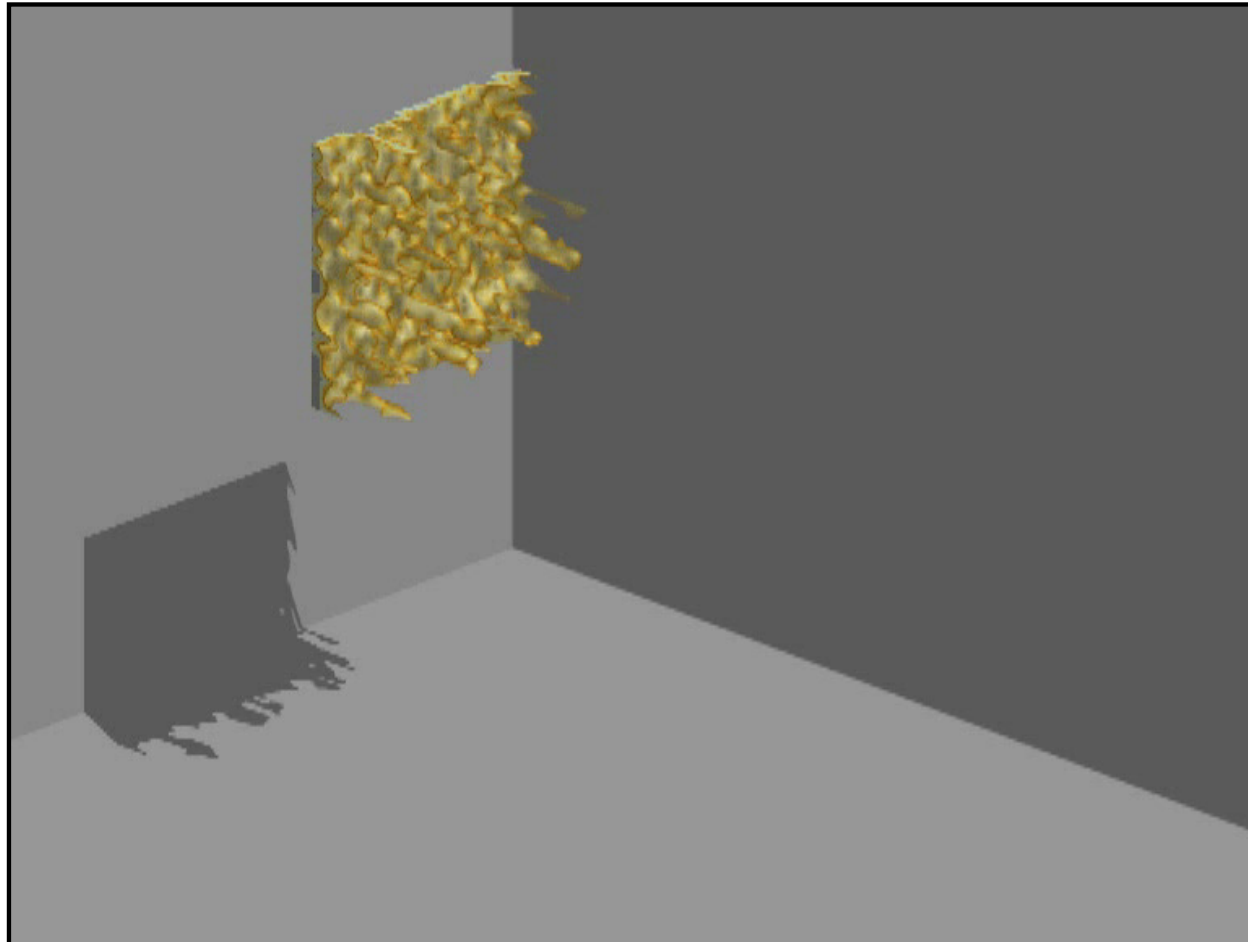
$$N_{ac} = \frac{\varepsilon_{\beta} C_0 \beta}{(1 - \varepsilon_{\beta}) \rho_{\sigma}}$$

Acid capacity number



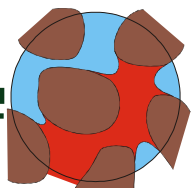
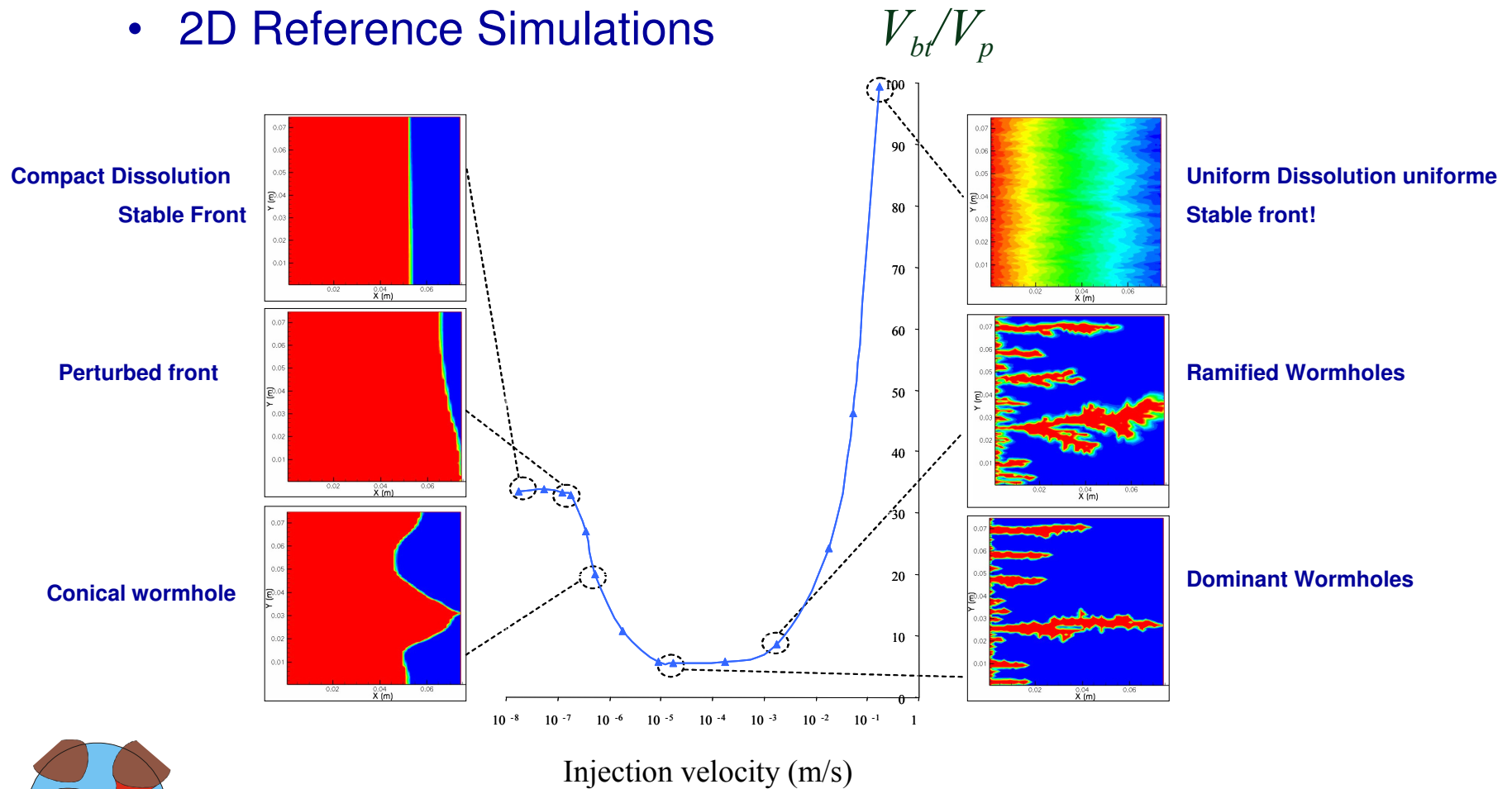
# 3D example

I.C.: random permeability field



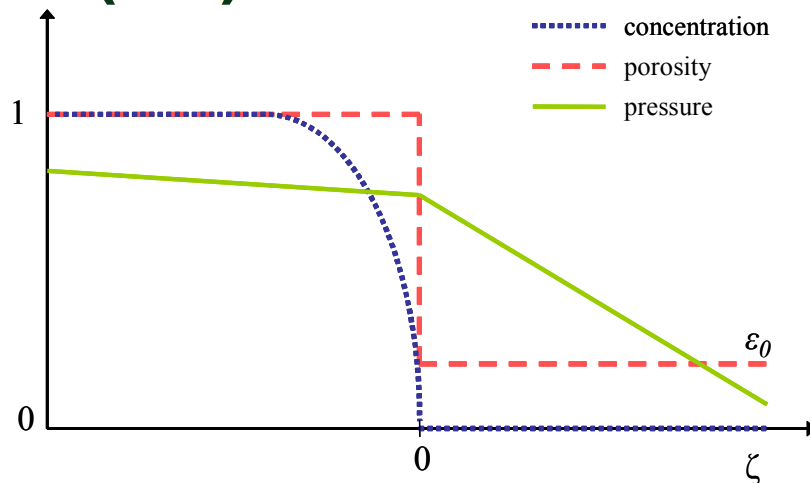
# Stability Analysis

- 2D Reference Simulations



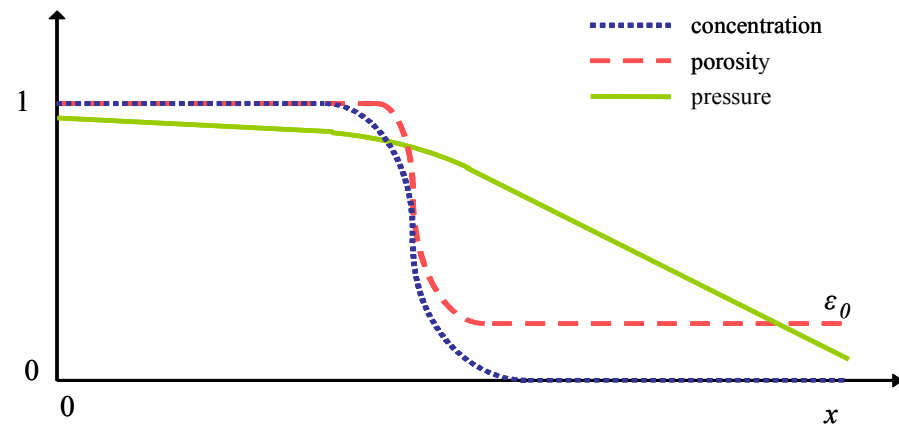
# Stability of the linearised problem (Cohen, 2006)

## • Compact Front (LE)

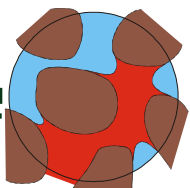


Autonomous Case

## • LNE



2 Cases: autonomous  
and non-autonomous





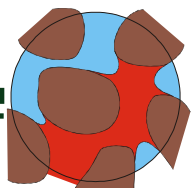
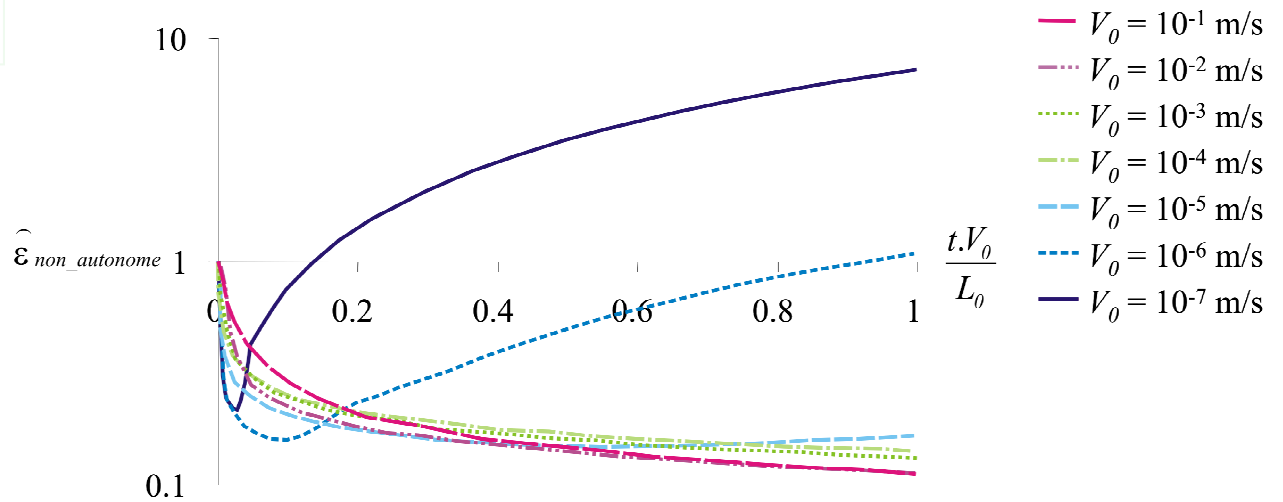
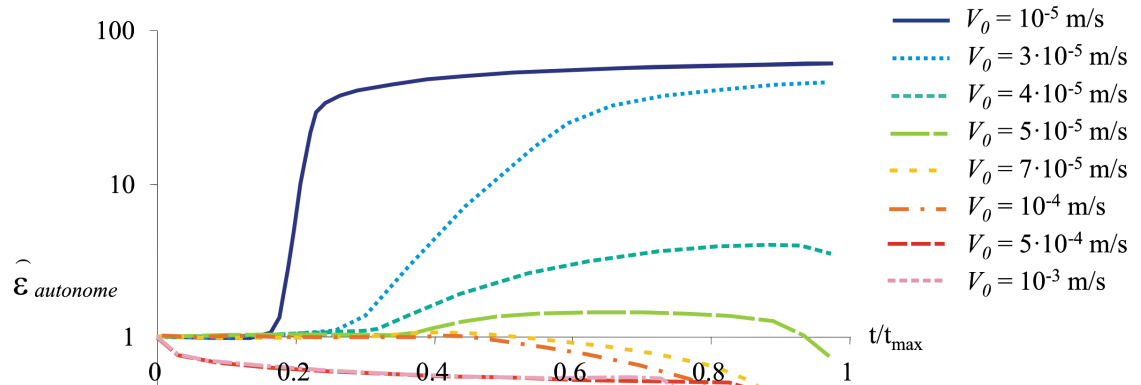
# Case LNE

Amplitude Coefficient

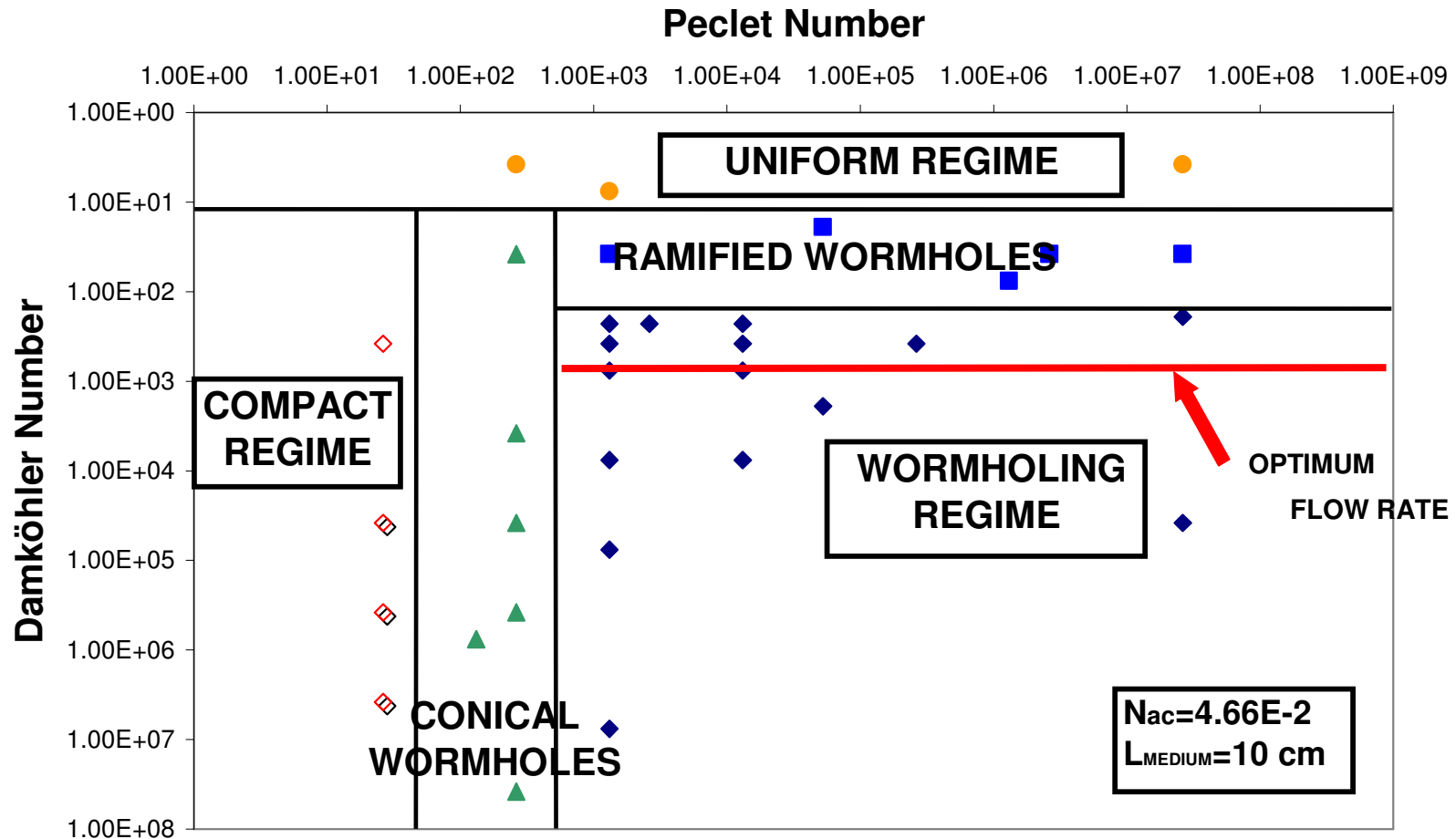
$$\widehat{\mathcal{E}}_{\text{autonome}}(t) = \left( \frac{\int_{\Omega} f_{\varepsilon}^2(t) dz}{\int_{\Omega} f_{\varepsilon}^2(0) dz} \right)^{1/2}$$

Front thickness growth → stabilizing effect

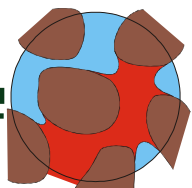
Non-autonomous Case



# Dissolution Diagram (Golfier et al., 2001)



Peclet -Damköhler Diagram

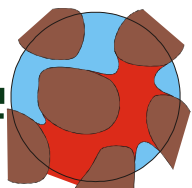
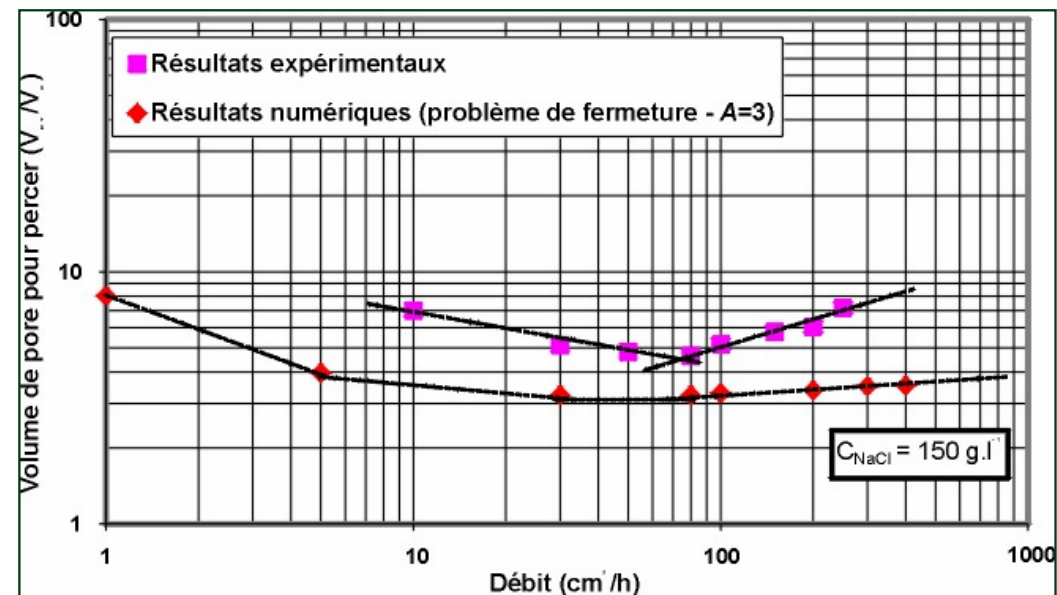


# “Optimum flow rate”

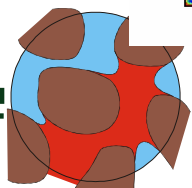
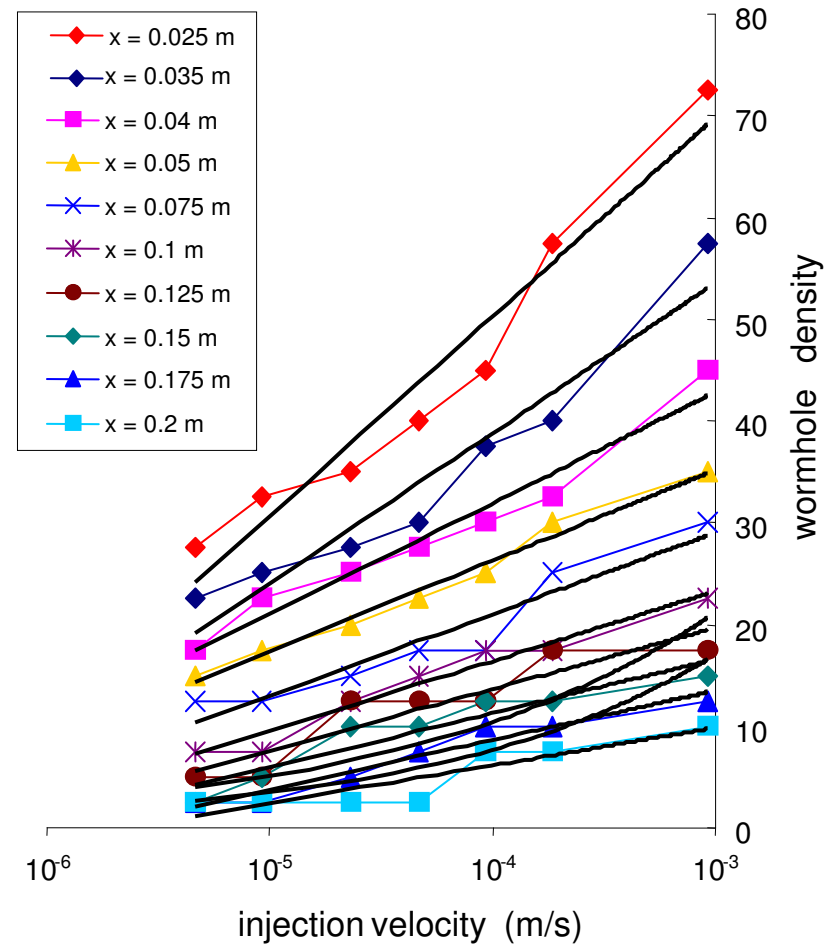
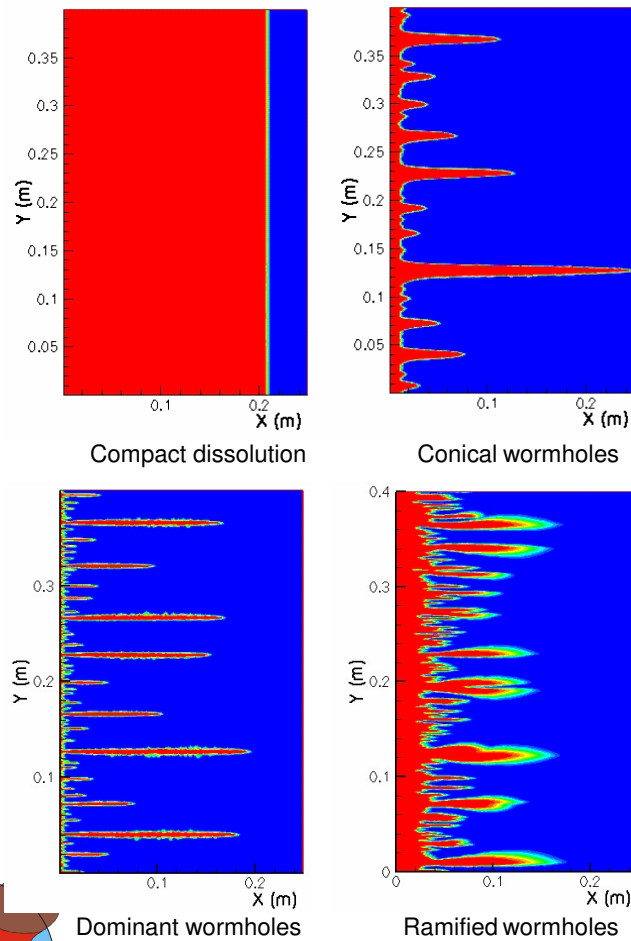
- Need “mainly” to calibrate  $\alpha$

$$\alpha = A \alpha_0$$

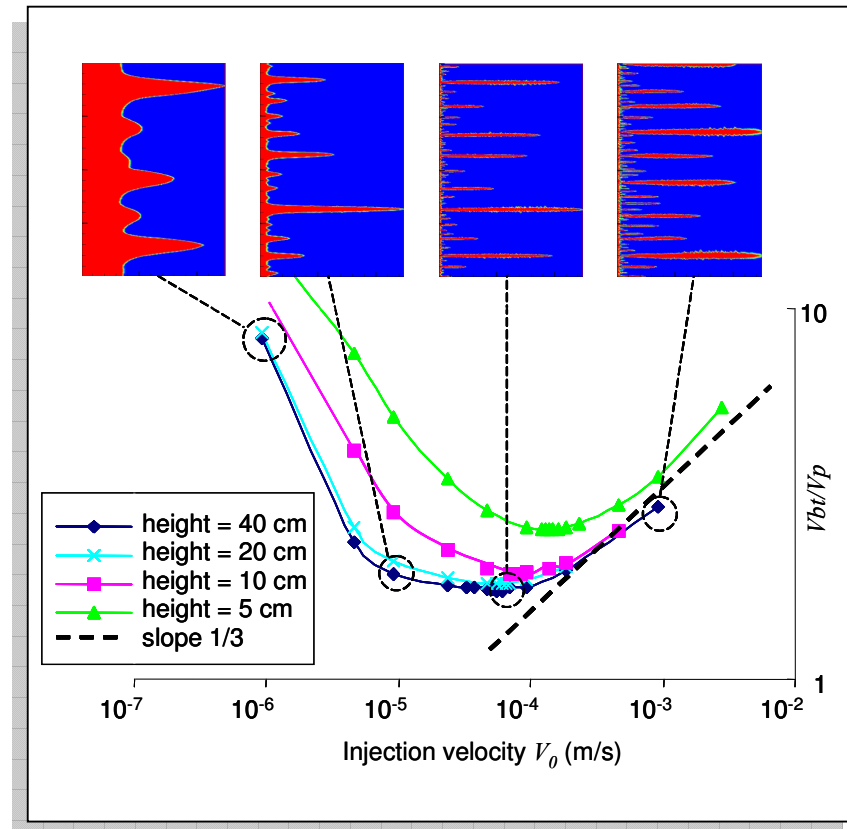
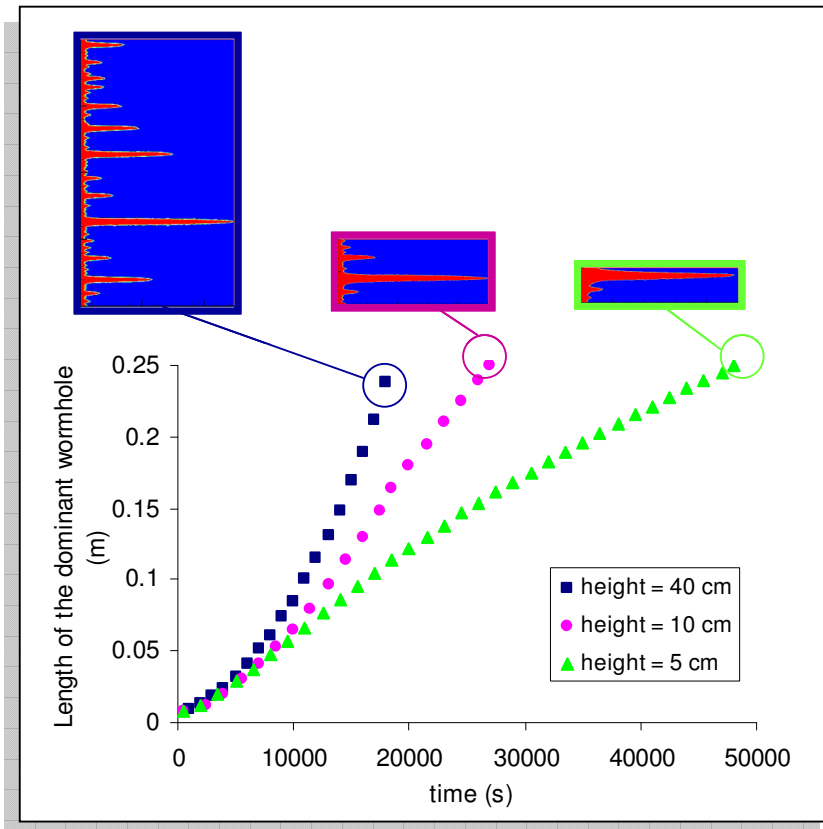
where  $\alpha_0$  is the correlation obtained from the closure problem + simple unit cell



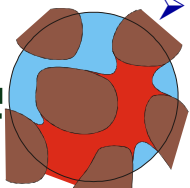
# Confinement Effect: « wormhole competition », Cohen et al. (2007)



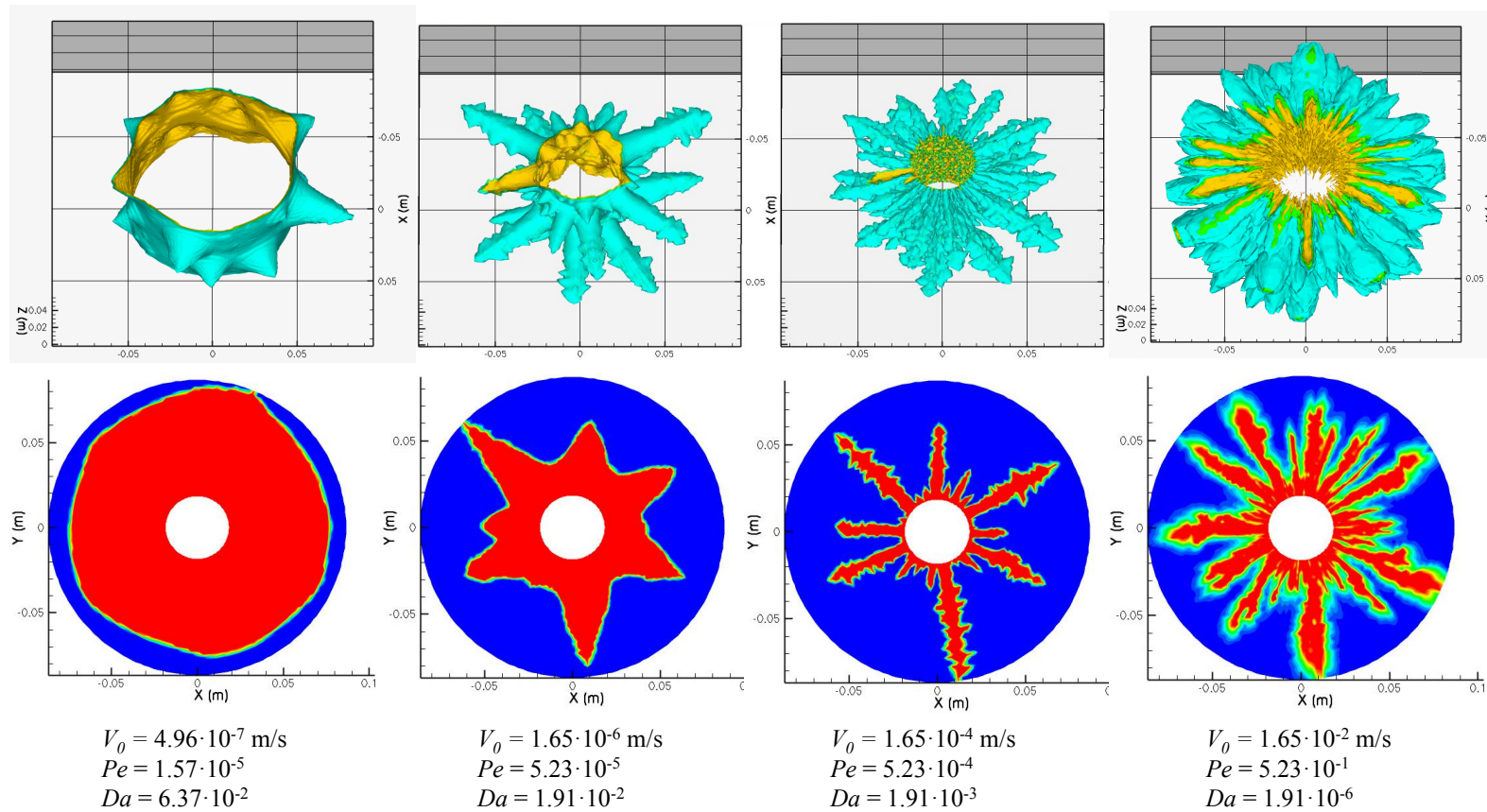
# Confinement (cont.)



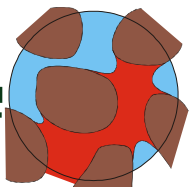
- Dominant wormhole growth rate increases with domain height
- Optimum injection velocity increases with height decreasing



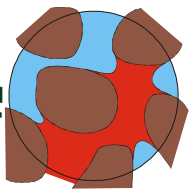
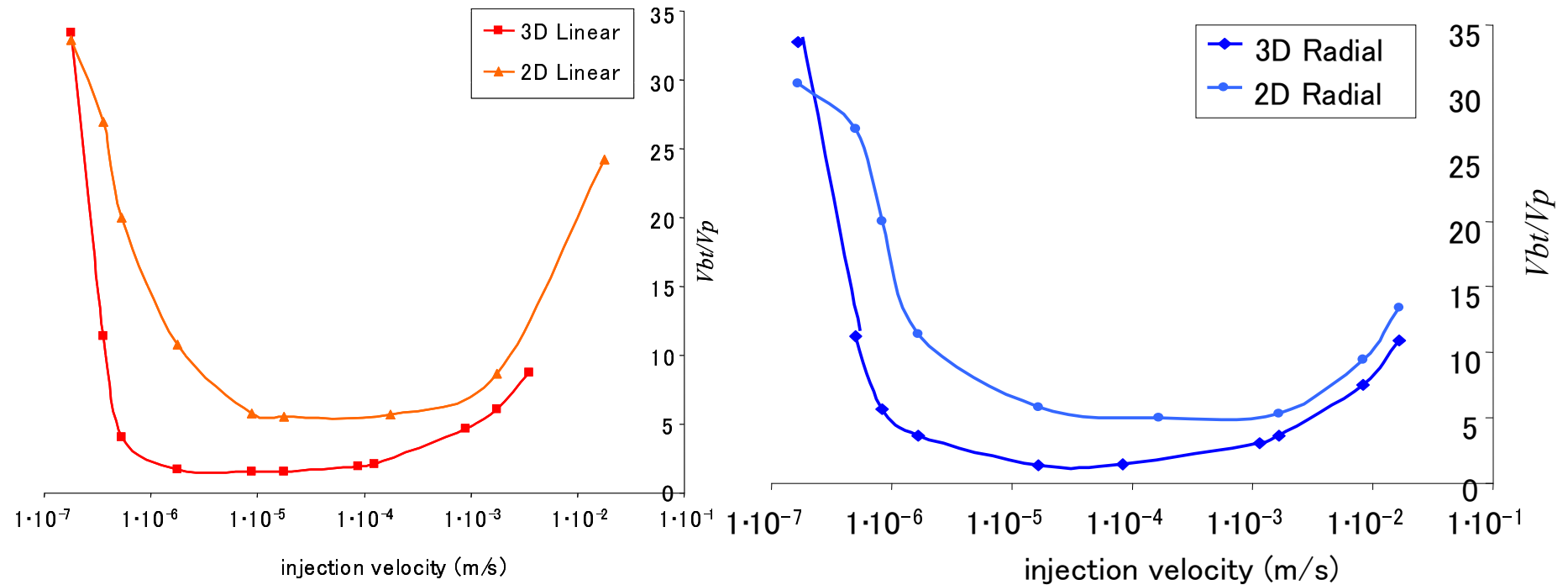
# Effect of geometry: ex. radial



see Cohen, 2007



# Effect of geometry on optimum flowrate

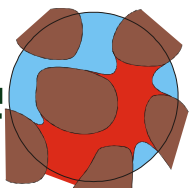


# Extension of LNE models to complex phase diagrams, and multicomponent systems

- **Equilibrium conditions at  $A_{\beta\sigma}$**

$$\begin{bmatrix} \cdot \\ \mu_{\beta i}(\dots, \langle c_{\beta j} \rangle^\beta + \tilde{c}_{\beta j}, \dots) \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \mu_{\sigma i}(\dots, \langle c_{\sigma j} \rangle^\sigma + \tilde{c}_{\sigma j}, \dots) \\ \cdot \end{bmatrix}$$

$$\begin{bmatrix} \cdot \\ \mu_{\beta i}(\dots, \langle c_{\beta j} \rangle^\beta, \dots) \\ \cdot \end{bmatrix} + \begin{bmatrix} \cdot \\ \frac{\partial \mu_{\beta i}}{\partial c_{\beta j}} \Big|_{\langle c_{\beta} \rangle^\beta} \\ \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \tilde{c}_{\beta} \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot \\ \mu_{\sigma i}(\dots, \langle c_{\sigma j} \rangle^\sigma, \dots) \\ \cdot \end{bmatrix} + \begin{bmatrix} \cdot \\ \frac{\partial \mu_{\sigma i}}{\partial c_{\sigma j}} \Big|_{\langle c_{\sigma} \rangle^\sigma} \\ \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \tilde{c}_{\sigma} \\ \cdot \end{bmatrix}$$





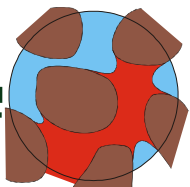
# Expression for the mass exchange terms

$$\begin{bmatrix} \cdot \\ \mu_{\beta} |_{\langle c_{\beta} \rangle^{\beta}} \\ \cdot \end{bmatrix} + J_{\beta} \begin{bmatrix} \tilde{c}_{\beta} \end{bmatrix} = \begin{bmatrix} \cdot \\ \mu_{\sigma} |_{\langle c_{\sigma} \rangle^{\sigma}} \\ \cdot \end{bmatrix} + J_{\sigma} \begin{bmatrix} \tilde{c}_{\sigma} \end{bmatrix}$$

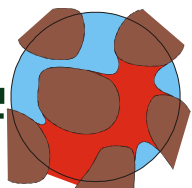
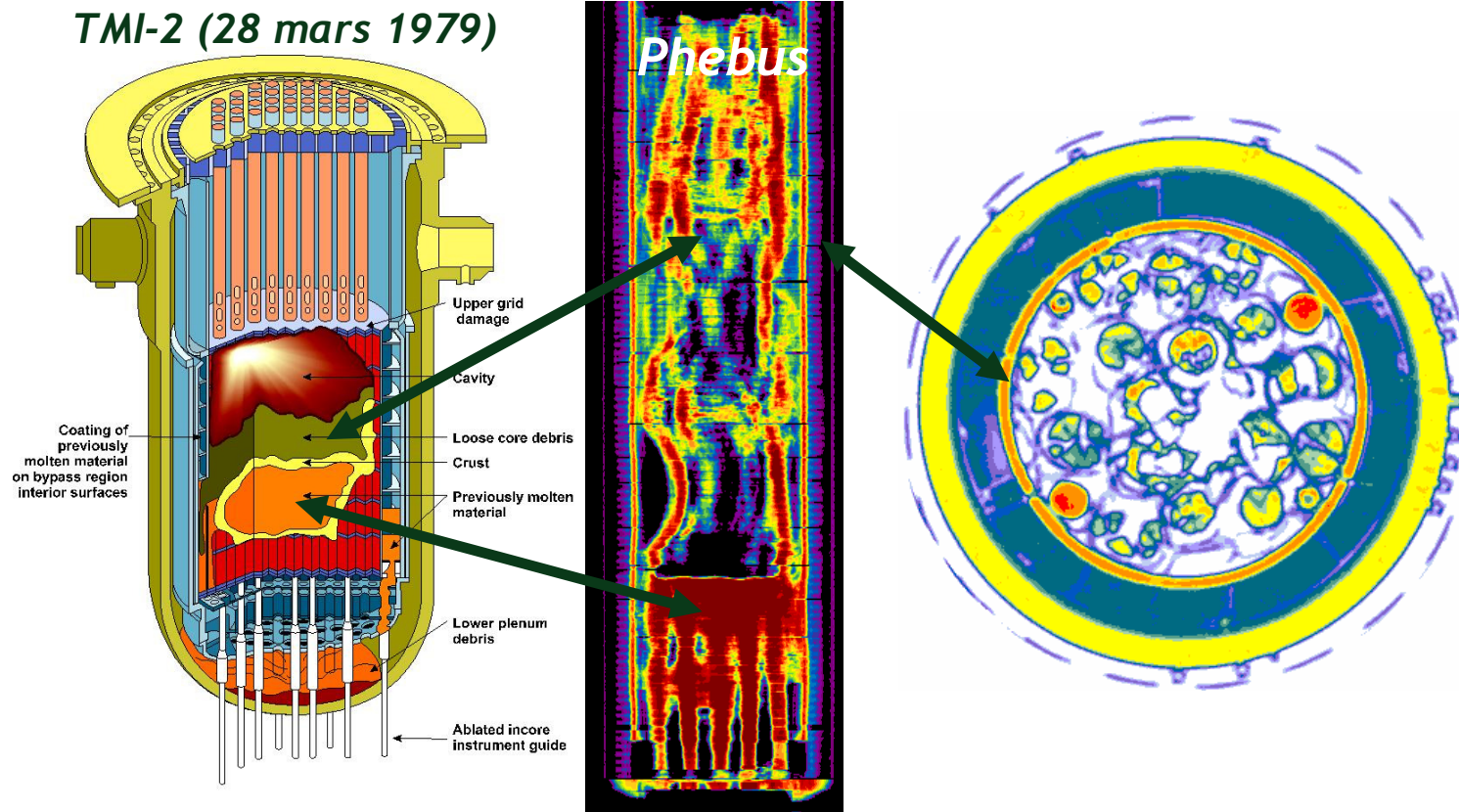
- **Case 1: diagonal  $J_{\beta}$  and  $J_{\sigma}$**

exchange term for species  $i$  :  $\mu_{\beta i} |_{\langle c_{\beta} \rangle^{\beta}} - \frac{J_{\sigma i}}{J_{\beta i}} \mu_{\sigma i} |_{\langle c_{\sigma} \rangle^{\sigma}}$

- **General case?**



# Example: $ZrO_2 / Zr$ (Belloni, 2008)



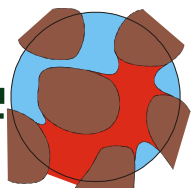
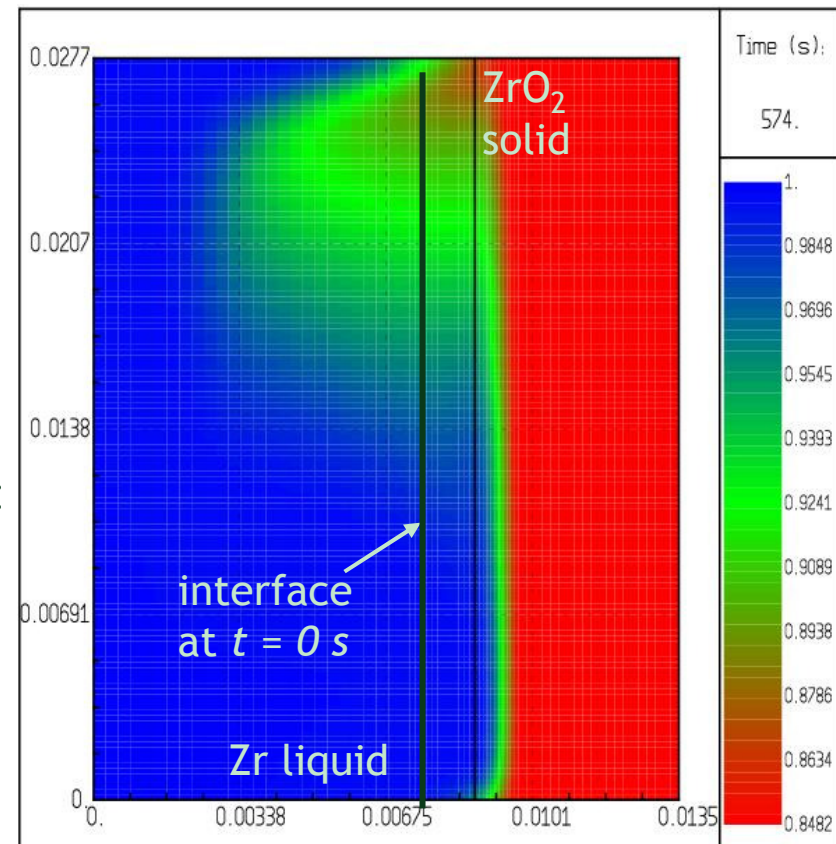
# Example: $\text{ZrO}_2$ / Zr (Belloni, 2008)

*Zr mass fraction ( 2373K , 574s)*

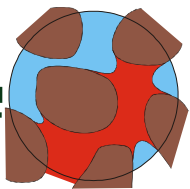
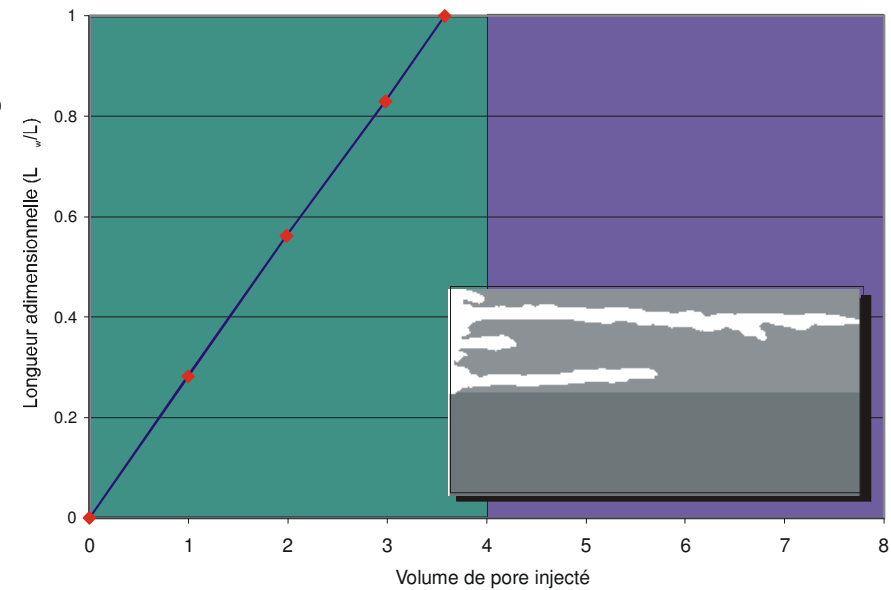
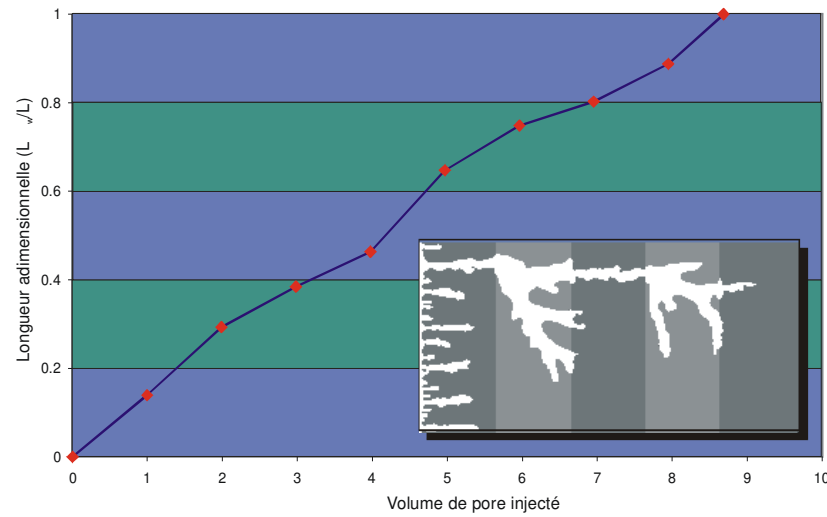
$$\begin{aligned} & \frac{\partial}{\partial t} (\epsilon_l \rho_l \langle C_l \rangle^l) + \nabla \cdot (\epsilon_l \rho_l \langle C_l \rangle^l \langle \mathbf{v}_l \rangle^l) \\ & + C_l^* \frac{\partial}{\partial t} (\epsilon_s \rho_s) = \\ & \nabla \cdot (\epsilon_l \rho_l \mathbf{D}_l \cdot \nabla \langle C_l \rangle^l) + \rho_l h_{ml} (C_l^* - \langle C_l \rangle^l) \\ & + \text{similar equation for } s \end{aligned}$$

+ Averaging of the mass balance BC at  $A_{ls}$ :

$$\begin{aligned} \frac{\partial}{\partial t} (\epsilon_s \rho_s) &= \frac{1}{C_l^* - C_s^*} (\rho_s h_{ms} (C_s^* - \langle C_s \rangle^s) \\ & + \rho_l h_{ml} (C_l^* - \langle C_l \rangle^l)) \end{aligned}$$



# Flow in Heterogeneous Systems



# Core-scale description

- fluid zone/porous zone  
 1D effective medium

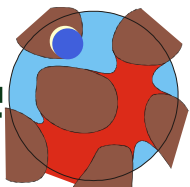
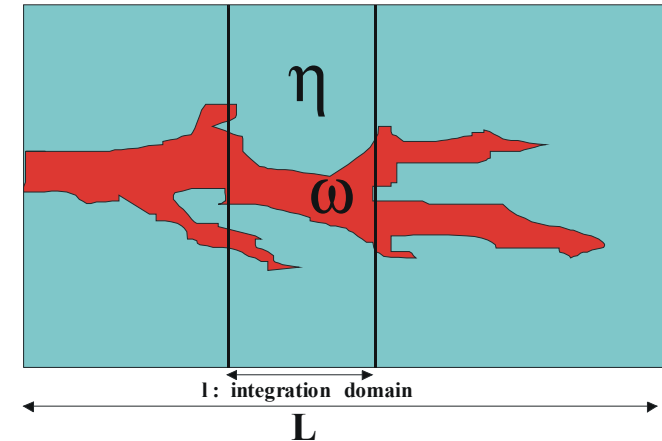
- 1- equation model?

- 1 single equation
- easy to implement
- loss of information

- 2- equation or “double-porosity” model?

- 1 equation for each zone
- *Darcy-scale problem is similar to the pore-scale problem in the case of equilibrium dissolution*

***pb. with non-locality and history effects***



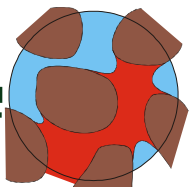
# Core-Scale Volume Fractions: Definitions

- Wormhole volume fraction:  $\varphi_w$
- Core-scale porosity

$$\varepsilon^* = \frac{1}{V_\infty} \int_{V_\infty} \varepsilon dV$$

if Local Equilibrium dissolution:

$$\varepsilon^* = \varphi_w + (1 - \varphi_w) \varepsilon$$



# Ex.: 2-equation model (Golfier et al., 2004, 2006)

## ◆ Flow :

$$\nabla \cdot \mathbf{V}_{\beta}^{\omega} = 0$$

$$\nabla P_{\beta}^{\omega} = -\mu_{\beta} (\mathbf{K}^{\omega})^{-1} \cdot \mathbf{V}_{\beta}^{\omega}$$

$$\nabla \cdot \mathbf{V}_{A\beta}^{\eta} = 0$$

$$\nabla P_{\beta}^{\eta} = -\mu_{\beta} (\mathbf{K}^{\eta})^{-1} \cdot \mathbf{V}_{A\beta}^{\eta}$$

pb. with regional velocities?

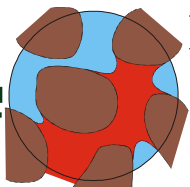
## ◆ Transport and Dissolution :

$$\varphi_{\omega} \frac{\partial C_{A\beta}^{\omega}}{\partial t} + \mathbf{V}_{\beta}^{\omega} \cdot \nabla C_{A\beta}^{\omega} = \frac{1}{Pe} \nabla \cdot (\mathbf{D}^{**} \cdot \nabla C_{A\beta}^{\omega}) - \alpha^{*} C_{A\beta}^{\omega}$$

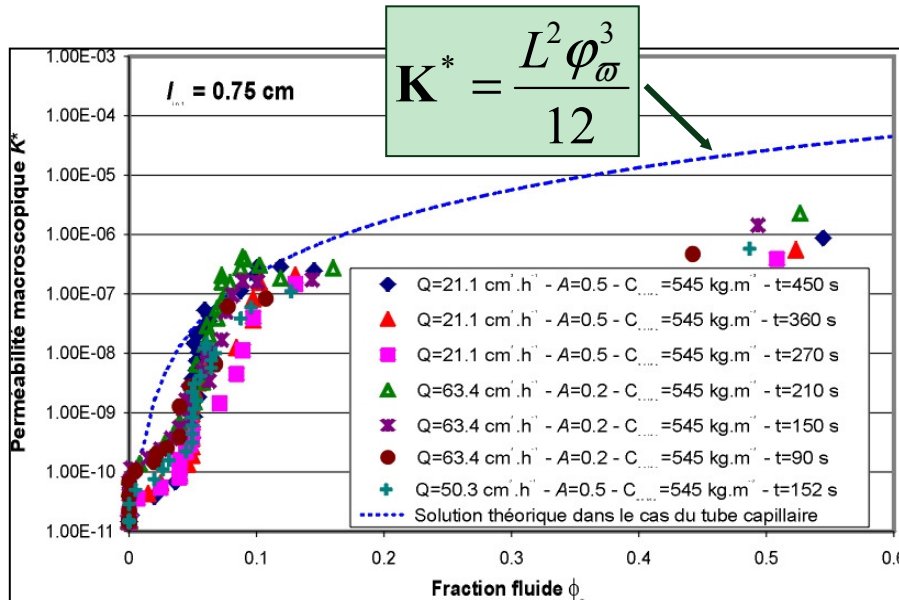
$$\frac{\partial \varphi_{\omega}}{\partial t} = \frac{\beta}{\rho_{\sigma}} \alpha^{*} C_{A\beta}^{\omega}$$

$$C_{A\beta}^{\eta} = 0 \text{ in } \eta\text{-region}$$

**need Darcy-scale local equilibrium!**



# Obtained Correlations: permeability



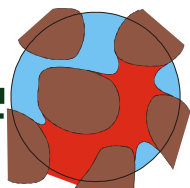
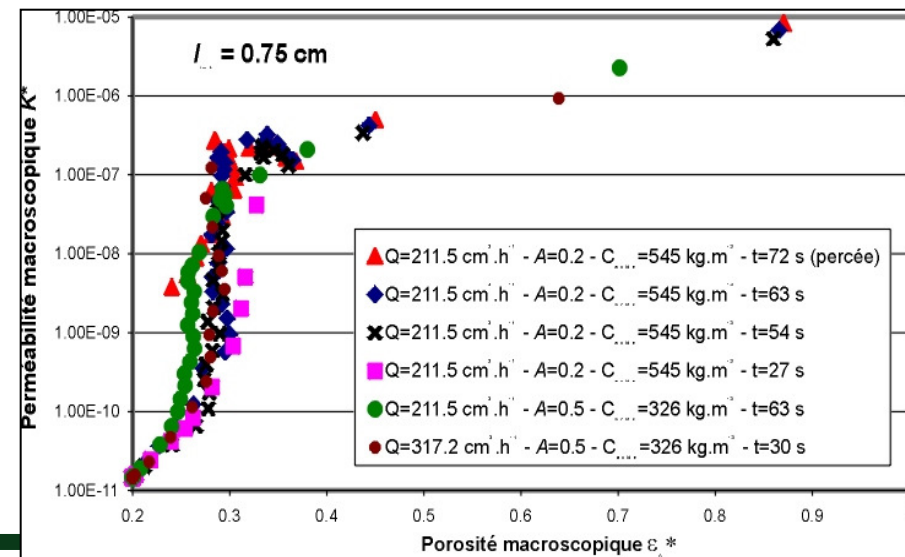
← Wormholing regime

$$K^* = f(\varphi_{\omega})$$

→ Ramified regime

need  $\varepsilon^*_\beta$

$$K^* = f(\varepsilon^*_\beta)$$

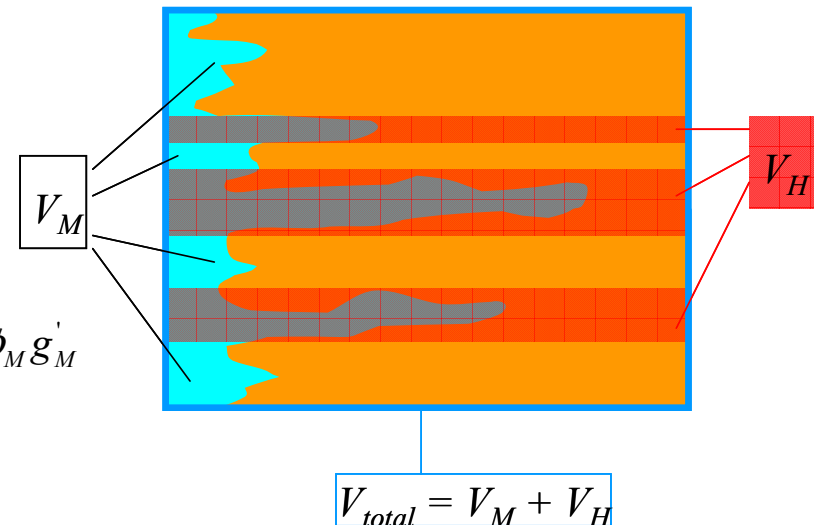




# New Model (Cohen et al., 2006)

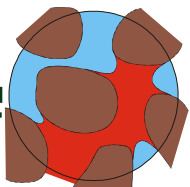
$$\text{Media H} \left\{ \begin{array}{l} \phi_H \varepsilon^H \frac{\partial C'^H}{\partial t} + \nabla \cdot (\phi_H V'^H f'^H) - \psi' C_{H-M} (P'^M - P'^H) = -\phi_H g'_H \\ V'^H = -\mathbf{K}'^H \nabla P'^H \\ \frac{\partial \varepsilon^H}{\partial t} = N_{ac} g'_H \\ \nabla \cdot (\phi_H V'^H) - \psi' (P'^M - P'^H) = 0 \end{array} \right.$$

$$\text{Media M} \left\{ \begin{array}{l} \phi_M \varepsilon^M \frac{\partial C'^M}{\partial t} + \nabla \cdot (\phi_M V'^M f'^M) + \psi' C_{H-M} (P'^M - P'^H) = -\phi_M g'_M \\ V'^M = -\mathbf{K}'^M \nabla P'^M \\ \frac{\partial \varepsilon^M}{\partial t} = N_{ac} g'_M \\ \nabla \cdot (\phi_M V'^M) + \psi' (P'^M - P'^H) = 0 \end{array} \right.$$

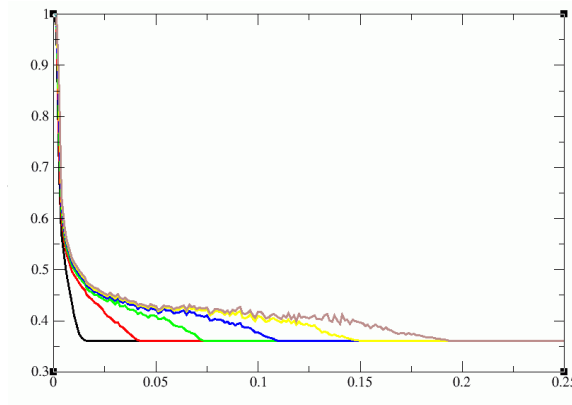
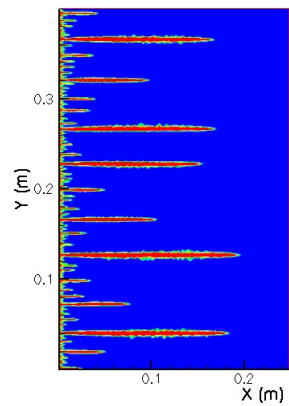


$V_H$  contains dominant wormholes

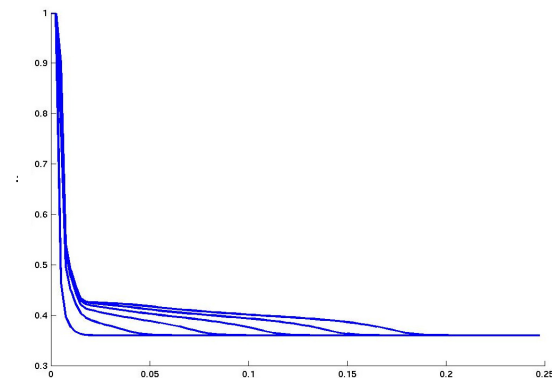
$V_M$  contains face dissolution and short wormholes



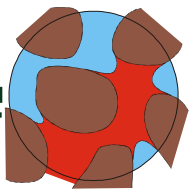
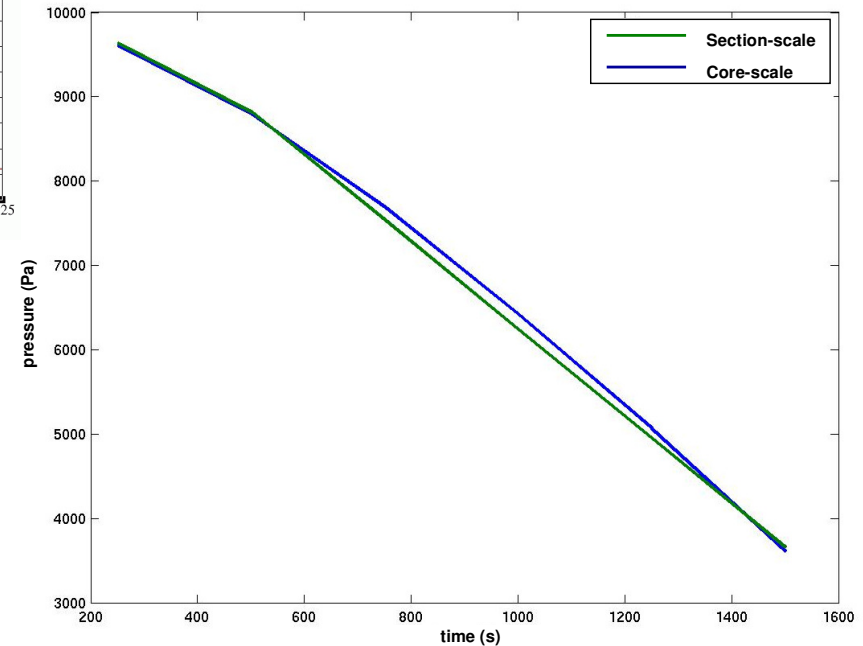
# Validation: example from dominant wormhole regime



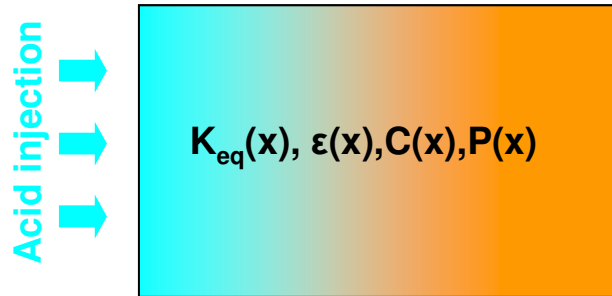
Core-scale simulation



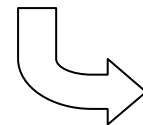
Section-scale simulation



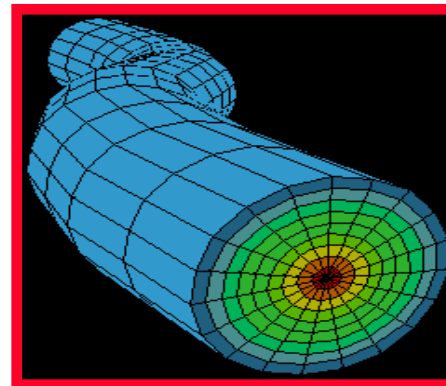
# Reservoir Scale: goals



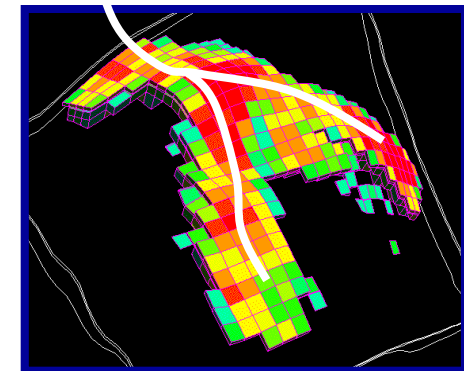
1<sup>st</sup> step : Section-scale –  
dissolution modelling



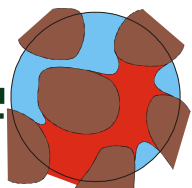
2<sup>nd</sup> step : treatment simulation  
– skin calculation



3<sup>rd</sup> step : introduction of skin  
in simulator reservoir –  
treatment optimisation

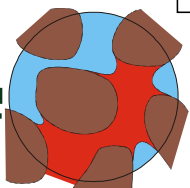
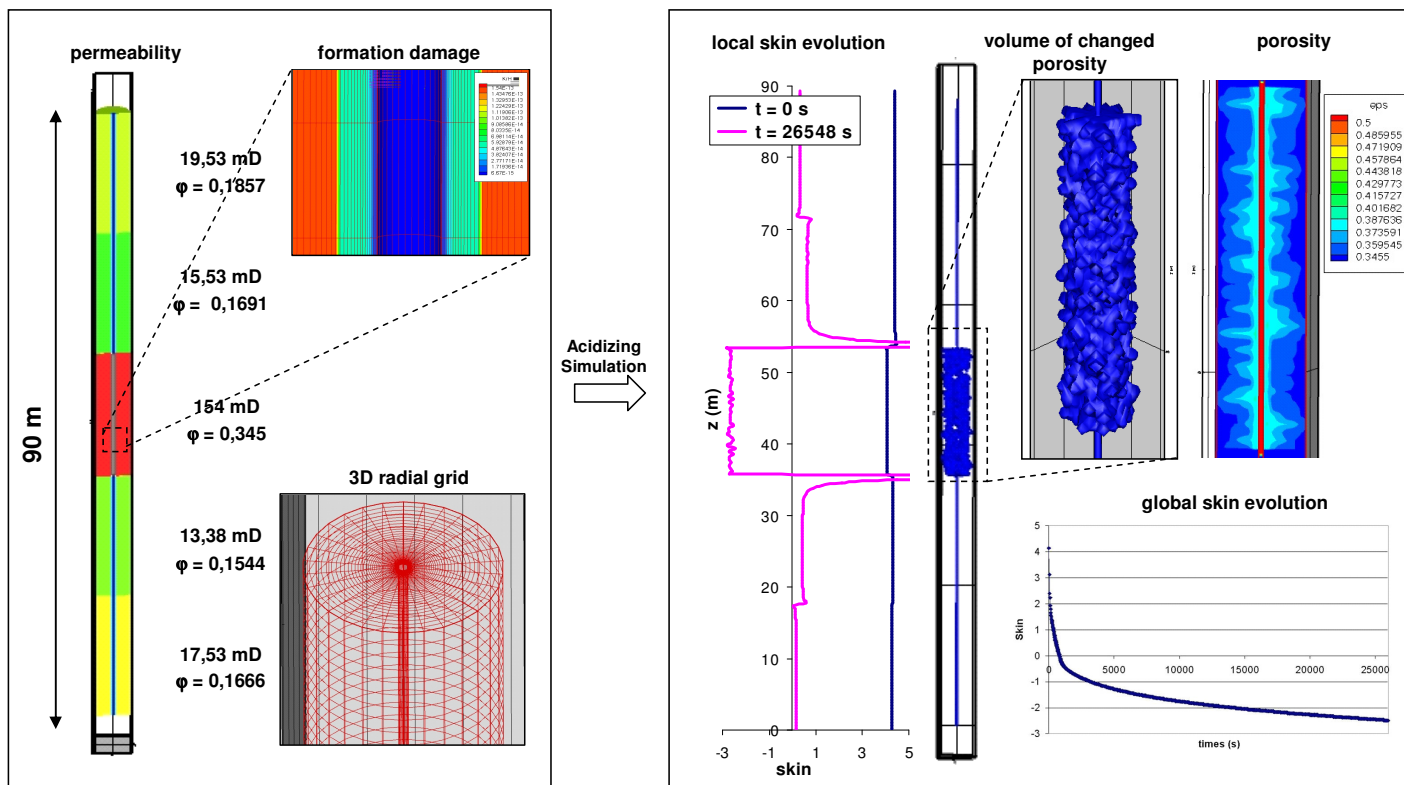


$$Q = \frac{2\pi kh}{B\mu} \frac{\Delta P}{\ln \frac{r_e}{r_w} + S}$$



# Example (Cohen, 2006)

- 3D radial simulation



# Conclusions

- **Effective Surface:**
  - If not limit cases, or if no steady-state: → DNS?
  - Coupling with instabilities?
- **Darcy-scale models:**
  - LNE model has potential for representing instabilities with a minimum of parameters
  - Coupling with strong heterogeneities?
- **Reservoir-scale models?**

