Scaling Up for Modeling of Transport and Flow in Porous Media

A conference in Honor of Alain Bourgeat

A Fully Equivalent Global Pressure Formulation for Three-Phases Compressible Flows .

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Summary

- The three-phases immiscible compressible equations
- Why a global pressure ?
- Equivalent global pressure reformulation: TD Condition.
- An example of Global Capillary Pressure function
- TD-interpolation of two-phase data: compatibility condition
- Conclusions

Notations (1 = water , 2 = oil , 3 = gas)

- dependant variables :

$$\begin{cases} S_j &= S_j(x,t) &= reduced\ saturation,\ 0 \le S_j \le 1\ , \\ P_j &= P_j(x,t) &= pressure, \\ \varphi_j &= \varphi_j(x,t) &= volumetric\ flow\ vector\ at\ reference\ pressure. \end{cases}$$

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- fluids and rock data :

$$\begin{cases} B_j &= B_j(p_j) &= \rho_j/\rho_j^{\mathrm{ref}} = \textit{volume factor}, \\ d_j &= d_j(p_j) &= B_j/\mu_j = \textit{phase mobility}, \\ \phi &= \phi(x, P_{pore}) &= \textit{porosity}, \\ K &= K(x) &= \textit{absolute permeability}, \\ kr_j &= kr_j(s_1, s_3) &= \textit{phase relative permeability}, \\ g & &= \textit{gravity constant}\ , \\ Z &= Z(x) &= \textit{depth}. \end{cases}$$

Three Phases Equations

- conservation laws :

$$\frac{\partial}{\partial t}\{\phi\, B_j(P_j)\, S_j\} + \nabla \cdot \varphi_j = 0 \quad , \quad j = 1, 2, 3.$$

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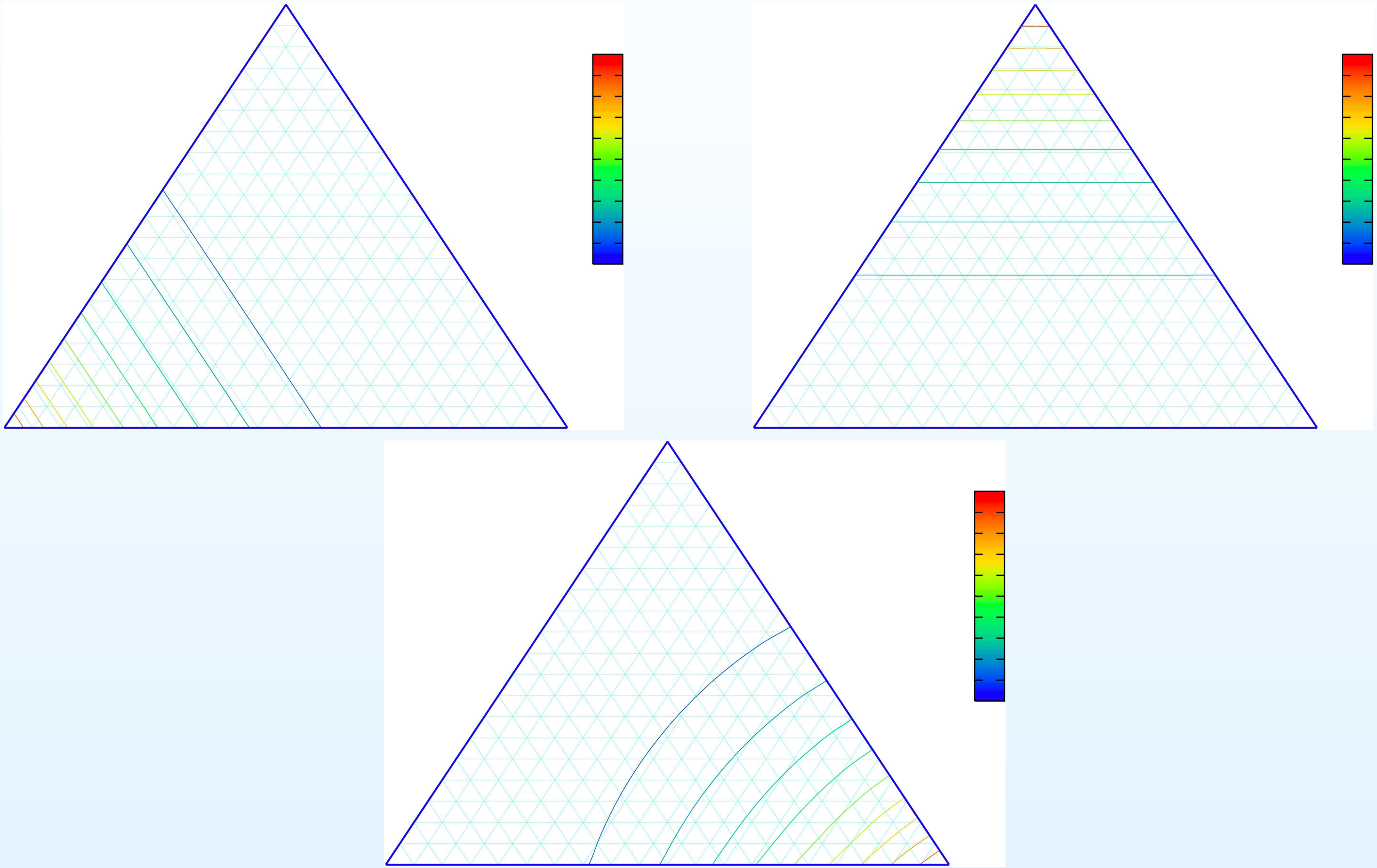
- capillary pressure law :

$$\begin{cases} P_1 - P_2 &= P_c^{12}(S_1)\ , \\ P_3 - P_2 &= P_c^{32}(S_3)\ , \end{cases}$$

An example of three-phase relative permeabilities



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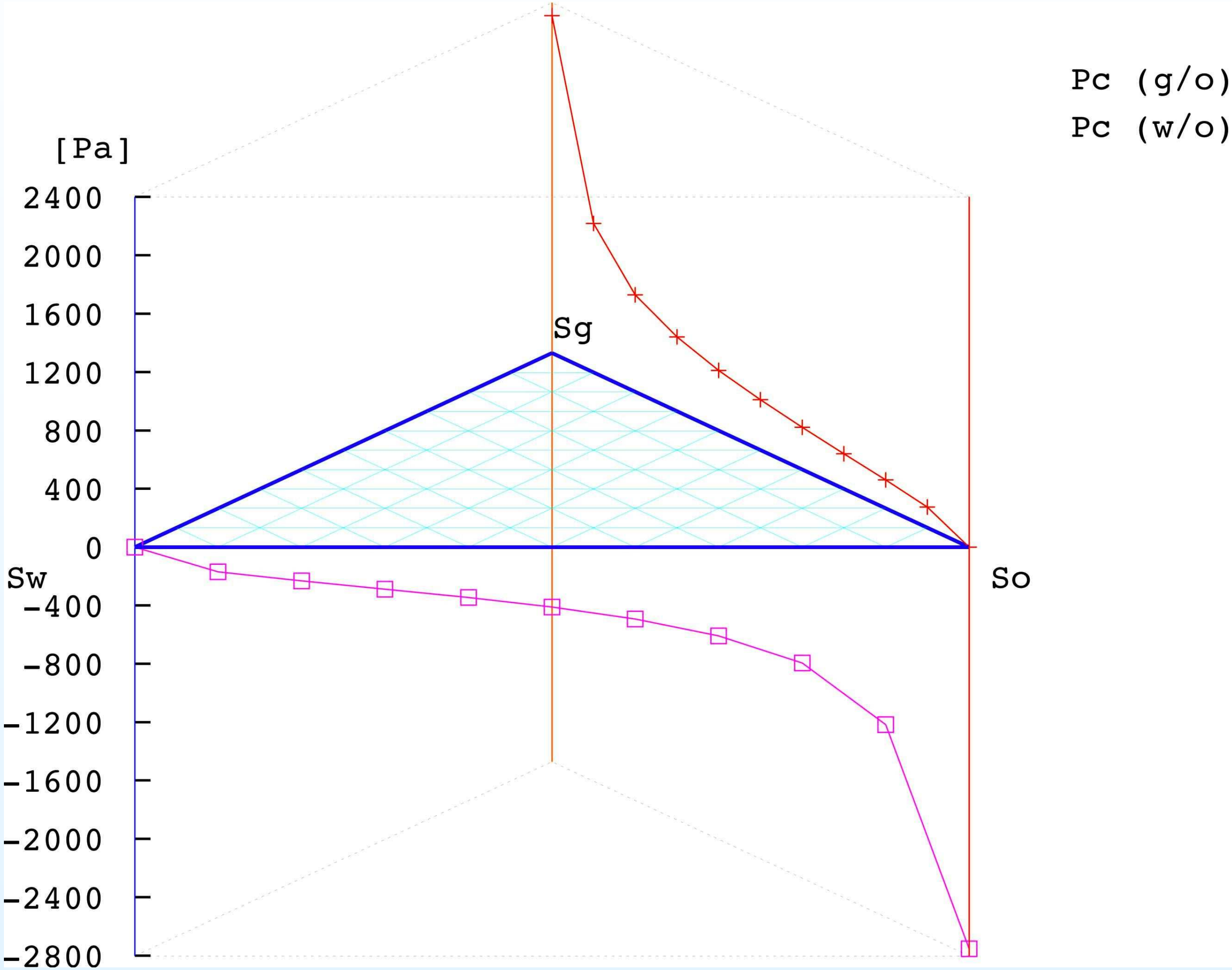

Classical resolution : “pressure” equation

$$\frac{\partial}{\partial t}\{\phi\sum_{j=1}^{3} B_j\, S_j\} + \nabla\cdot q = 0\ ,$$

where q is the global volumetric flow vector:

$$q = \sum_{j=1}^{3}\varphi_j = -K\lambda\{\nabla P_2 + f_1\nabla P_c^{12} + f_3\nabla P_c^{13} - \rho g\nabla Z\}$$

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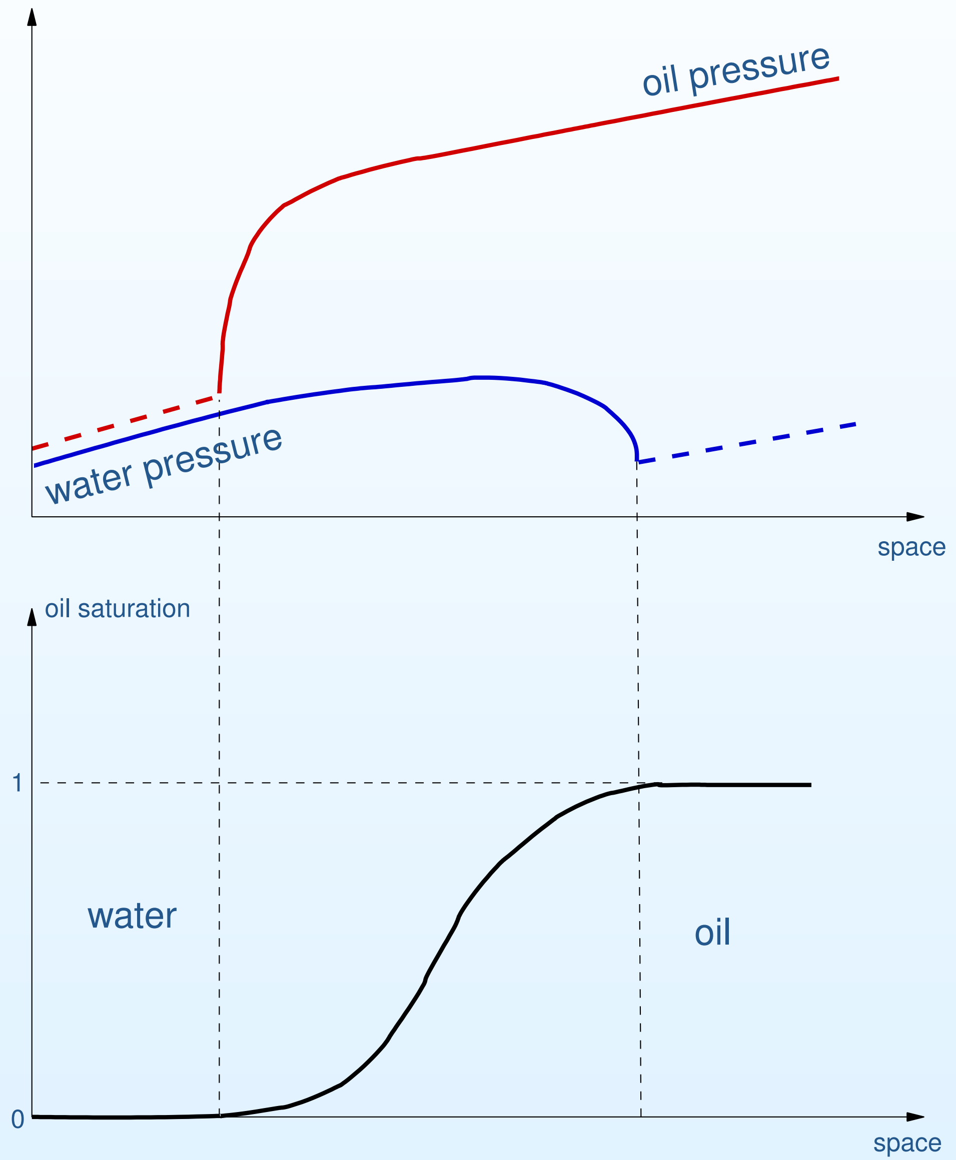
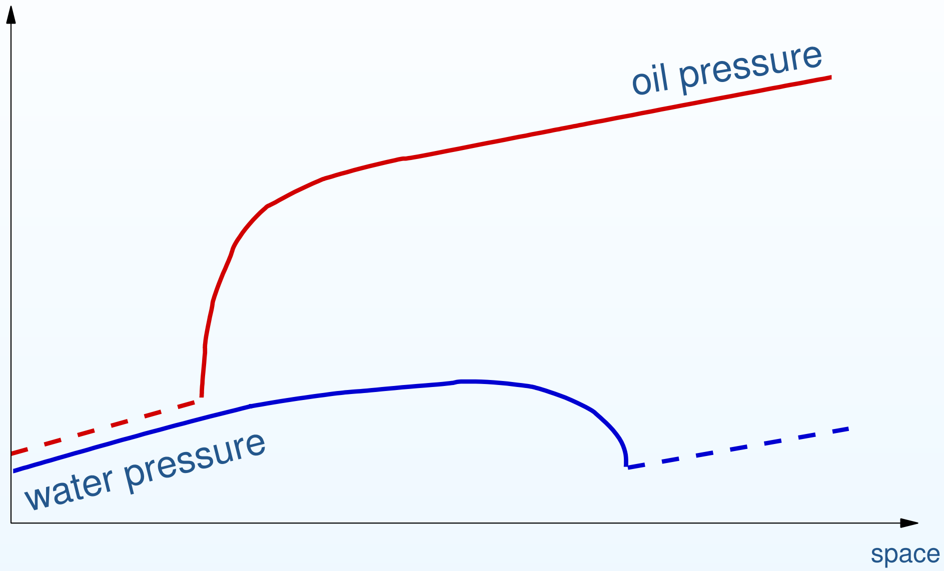
Solve for the oil pressure P_2 ?

Behaviour of individual phase pressures

Case of a two-phase water-oil flow :
oil and water pressures are singular near front boundary



Let us have a dream...



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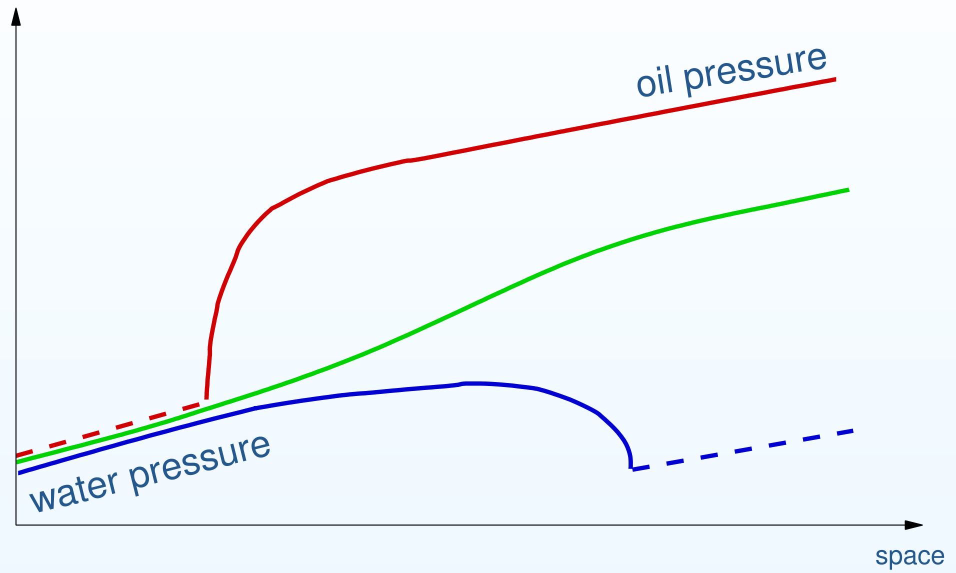

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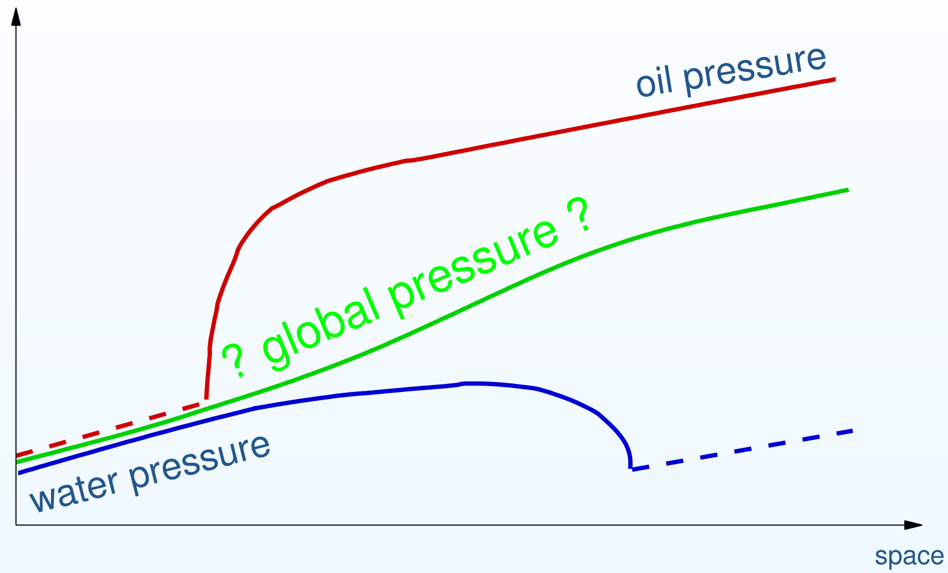

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and P governs the global volumetric flow vector q :

$$q = -Kd\{\nabla P - \rho g \nabla Z\} \quad ?$$

(where $d(s_1, s_3, p) = \lambda(s_1, s_3, p_2)$)

Searching for a Global Pressure P

- how to replace : $q = -K\lambda\{\nabla P_2 + f_1 \nabla P_c^{12} + f_3 \nabla P_c^{13} - \rho g \nabla Z\}$

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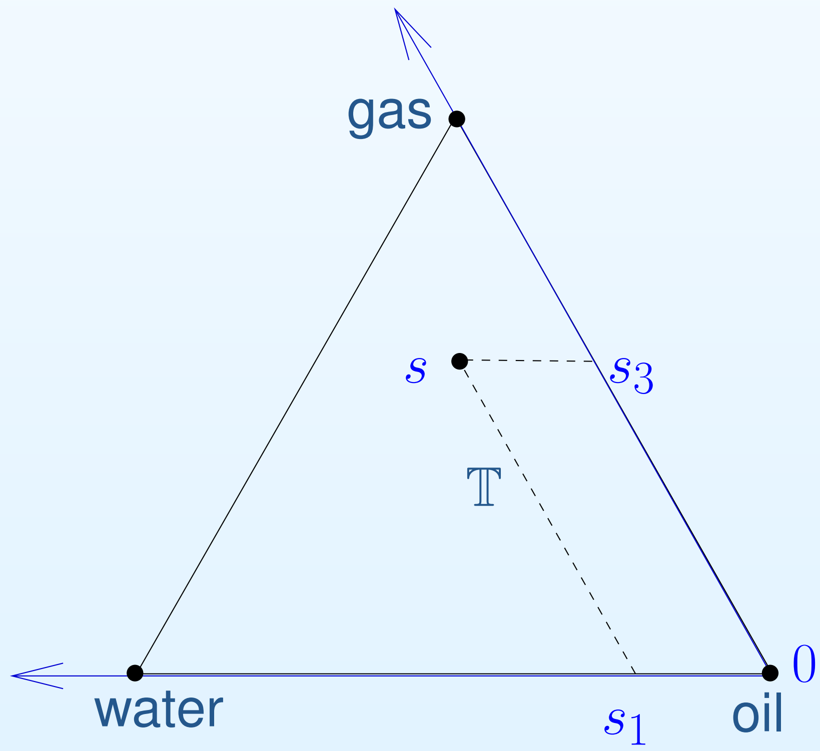

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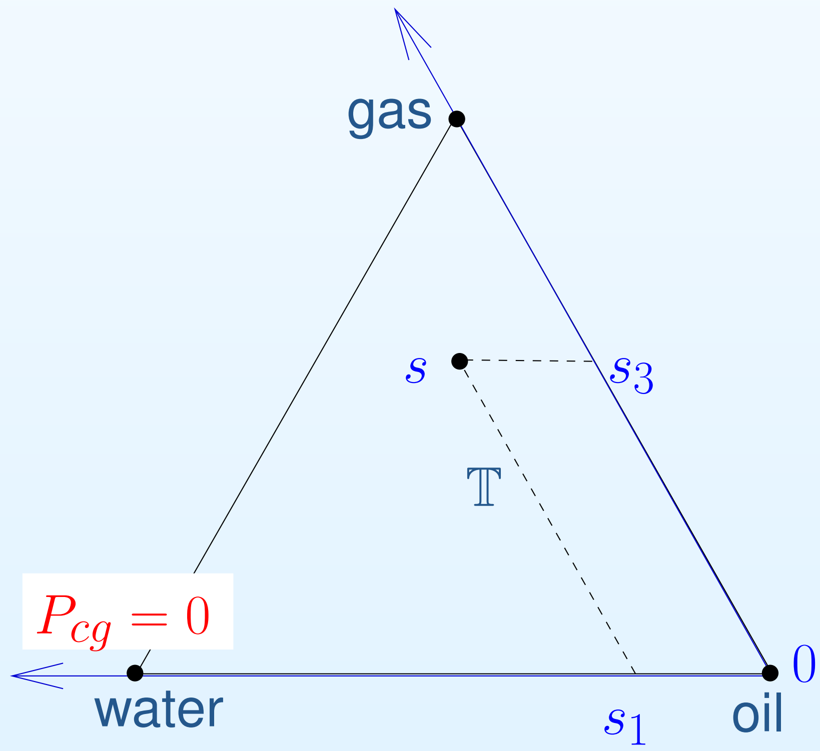

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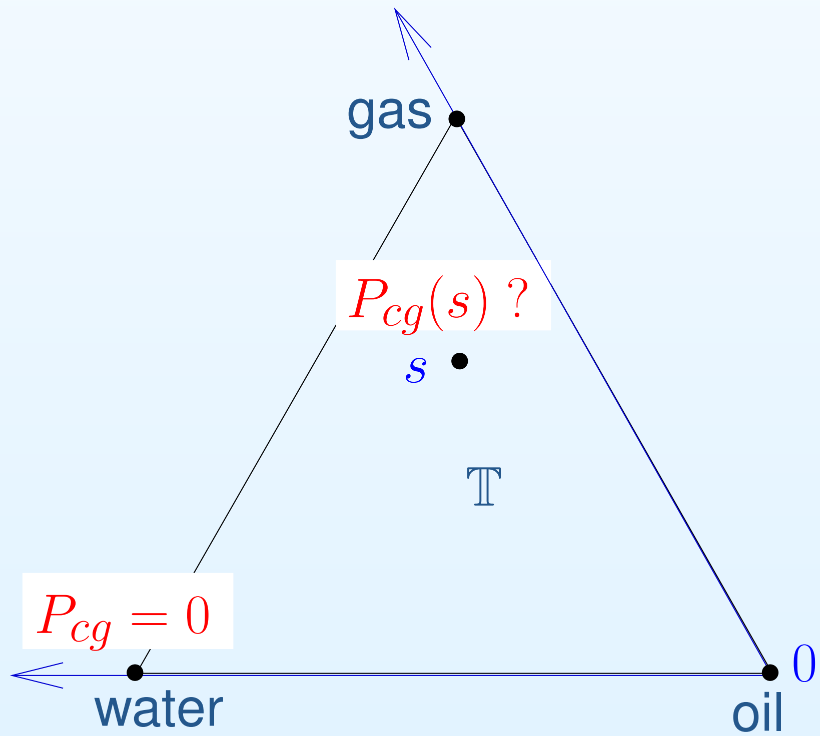

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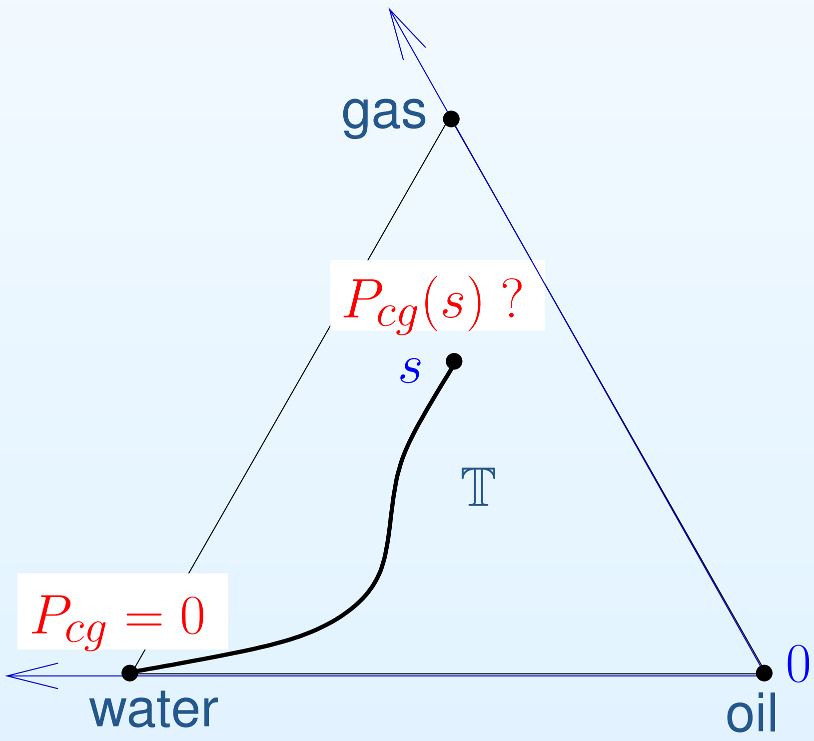

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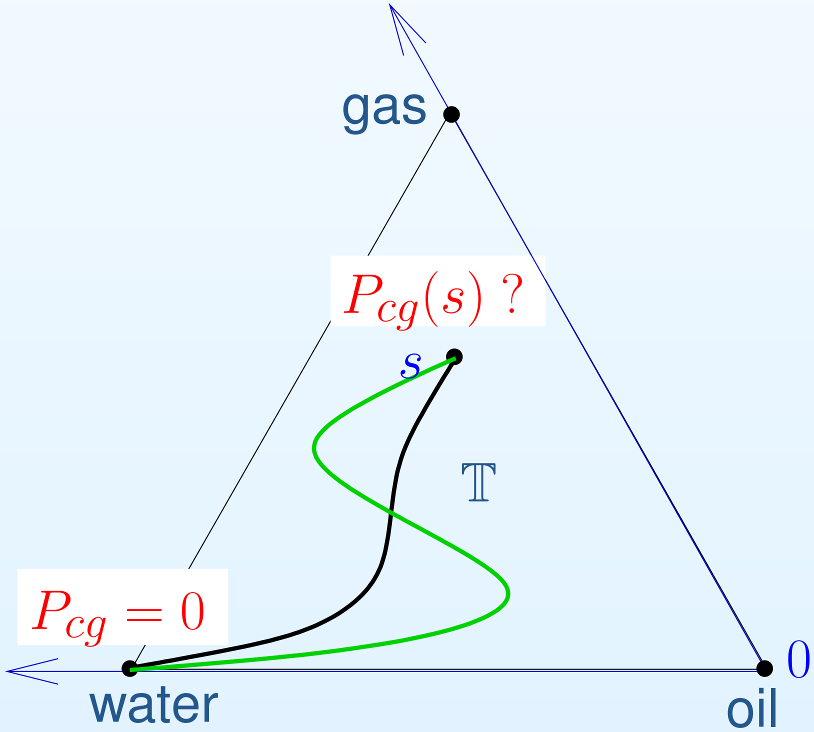

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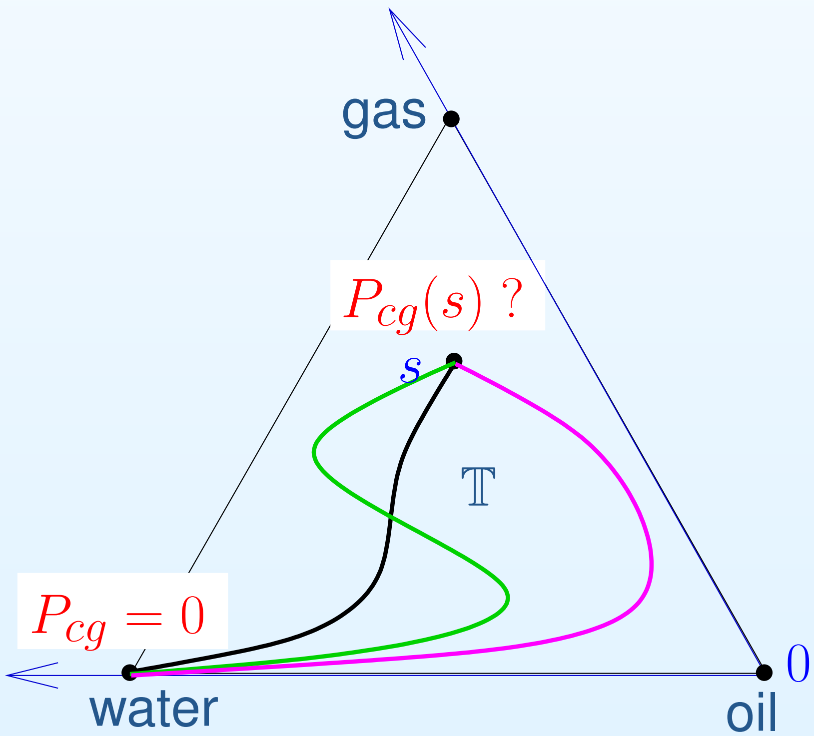

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- A natural parametrization of TD-three-phase data is made of :

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$$\nu_j(s,p) = \partial P_{cg}/\partial s_j(s,p) \Big/ \, dP_c^{j2}/ds_j(s_j) \quad , \quad j = 1,3$$

$$\begin{cases} kr_j(s,p) &= \nu_j(s,p)d(s,p)/d_j\big(p - P_{cg}(s,p) + P_c^{j2}(s_j)\big) \quad j = 1,3 \, , \\ kr_2(s,p) &= (1 - \nu_1(s,p) - \nu_3(s,p))d(s,p)/d_2\big(p - P_{cg}(s,p)\big) \, , \end{cases}$$

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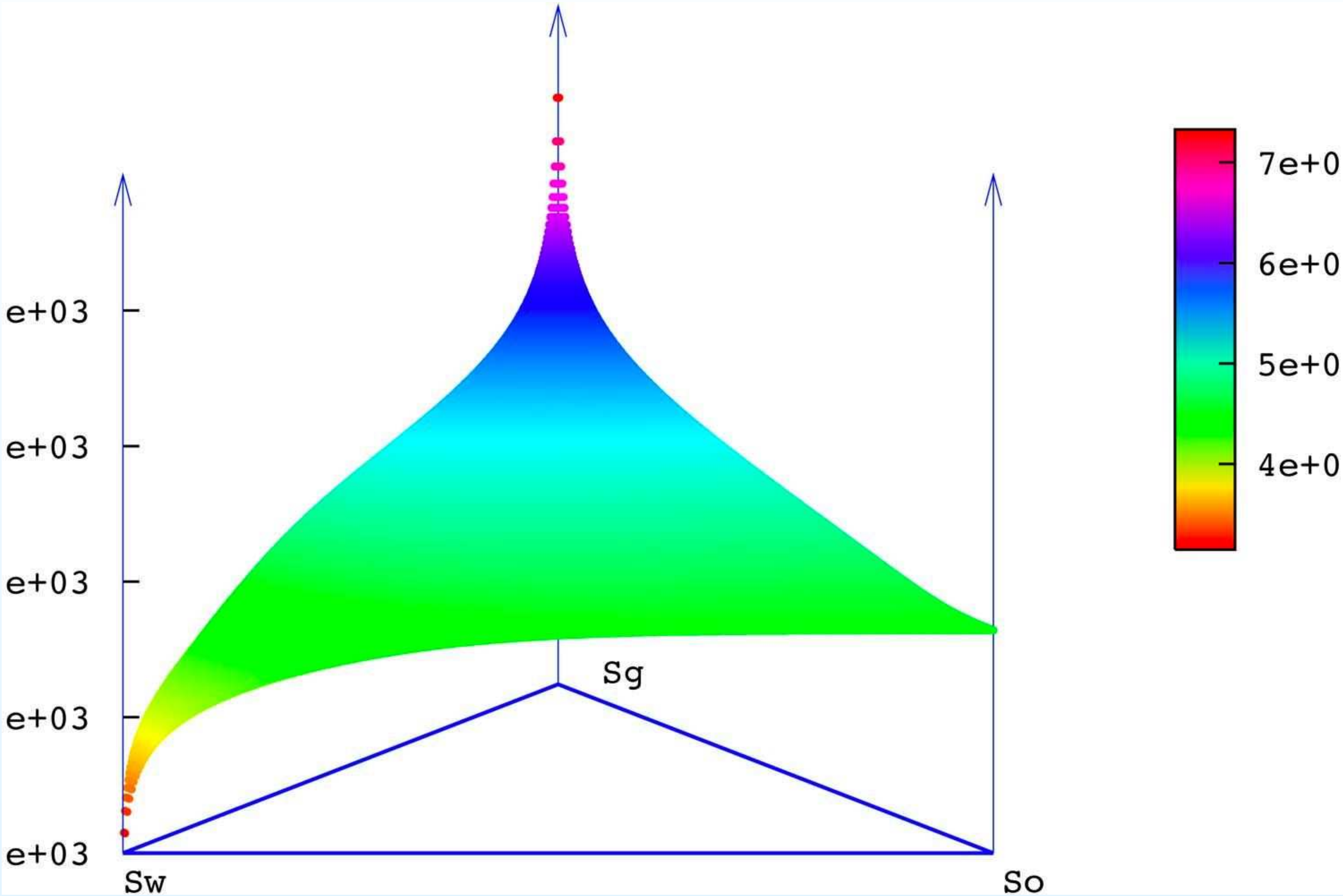
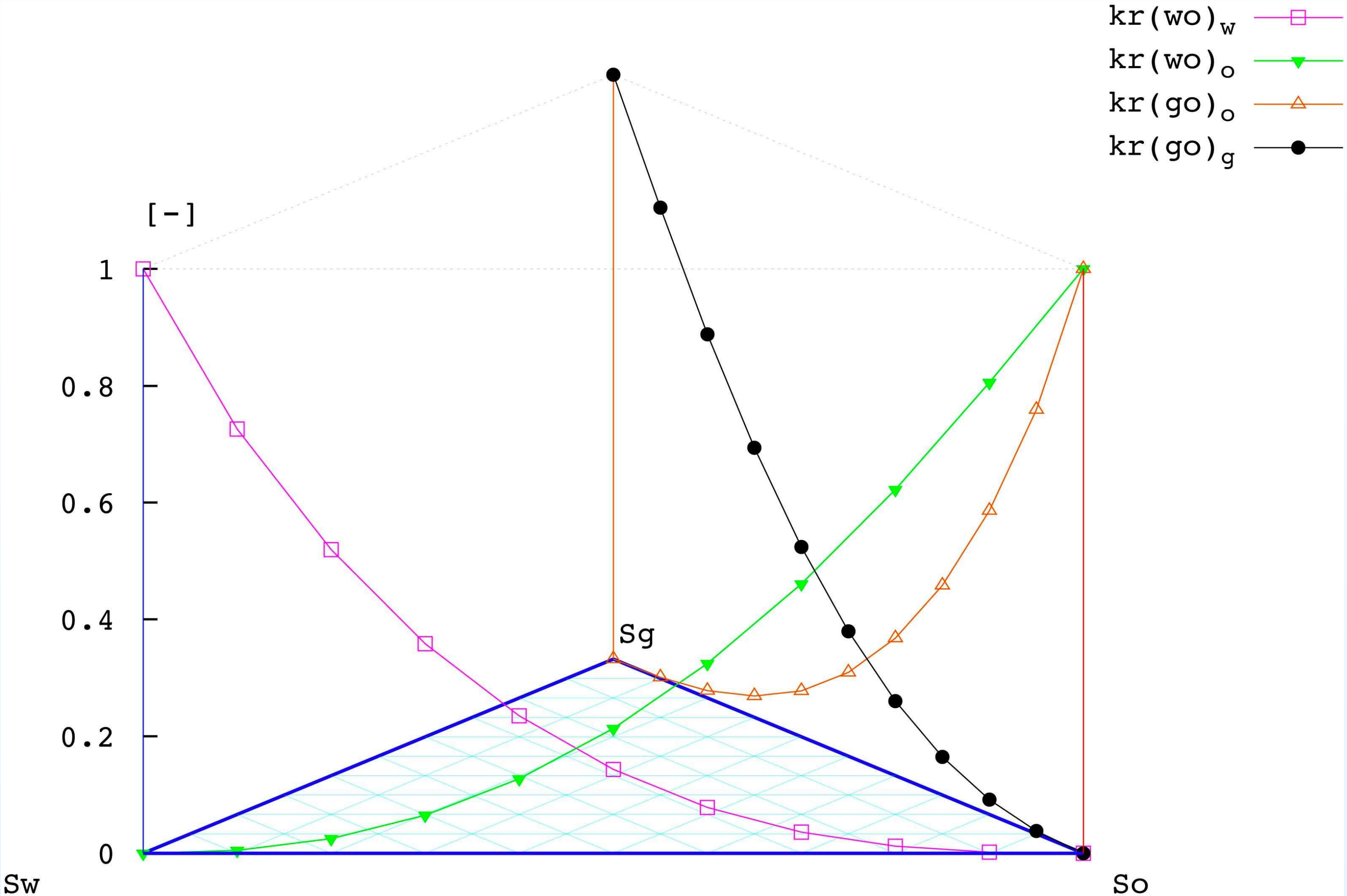
- by construction, TD-three-phase data satisfy :

$$\frac{\partial \nu_1}{\partial S_3}(s,p)\frac{dP_c^{12}}{dS_1}(s_1) = \frac{\partial \nu_3}{\partial S_1}(s,p)\frac{dP_c^{32}}{dS_3}(s_3) \, ,$$

An example of global capillary pressure function



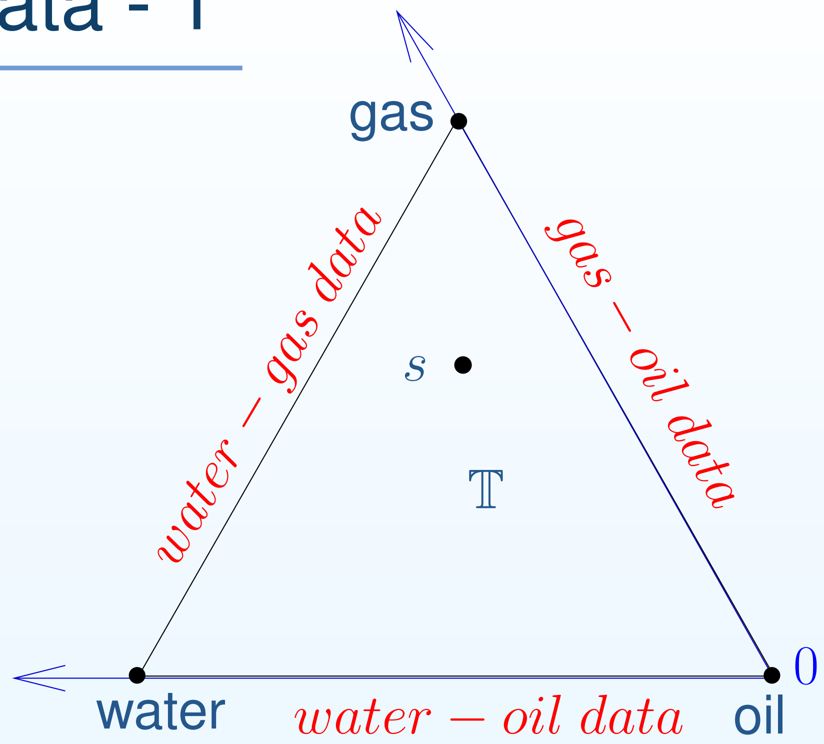
TD-interpolation of two-phase data - 0



TD-interpolation of two-phase data - 1

- **Given :** two phase data on $\partial\mathbb{T}$:

  $kr(s)$, $P_c(s)$ for each pair of fluids,
  $d_j(p_j) =$ fluid mobilities , $j = 1, 2, 3$.



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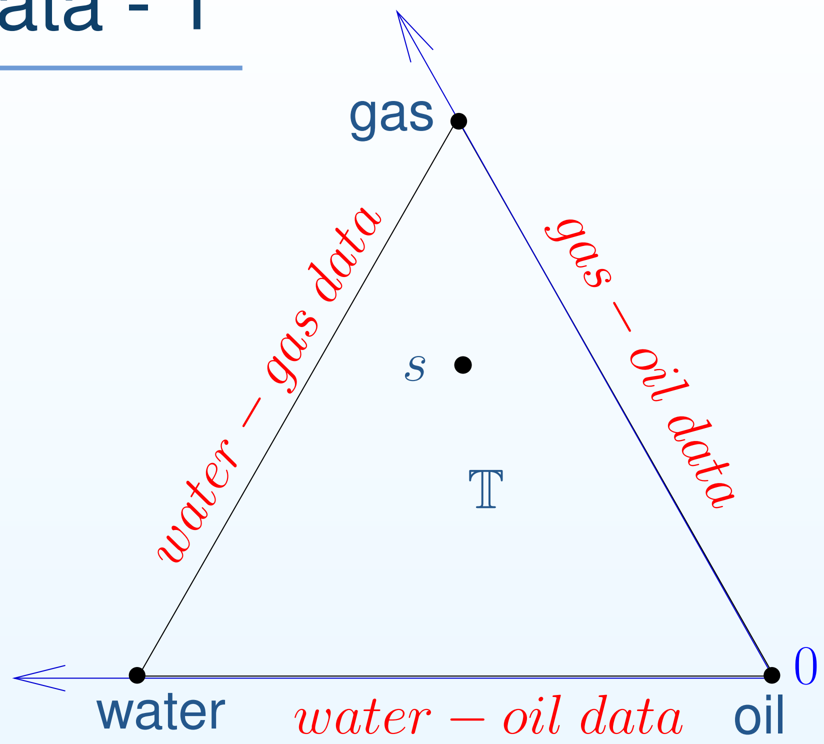
- **Find**, for each global pressure level p, two functions :

  $$s \in \mathbb{T} \rightsquigarrow P_{cg}(s, p) \; , \; s \in \mathbb{T} \rightsquigarrow d(s, p)$$



TD-interpolation of two-phase data - 1

- **Given :** two phase data on $\partial\mathbb{T}$:

  $kr(s)\ ,\ P_c(s)$ for each pair of fluids,
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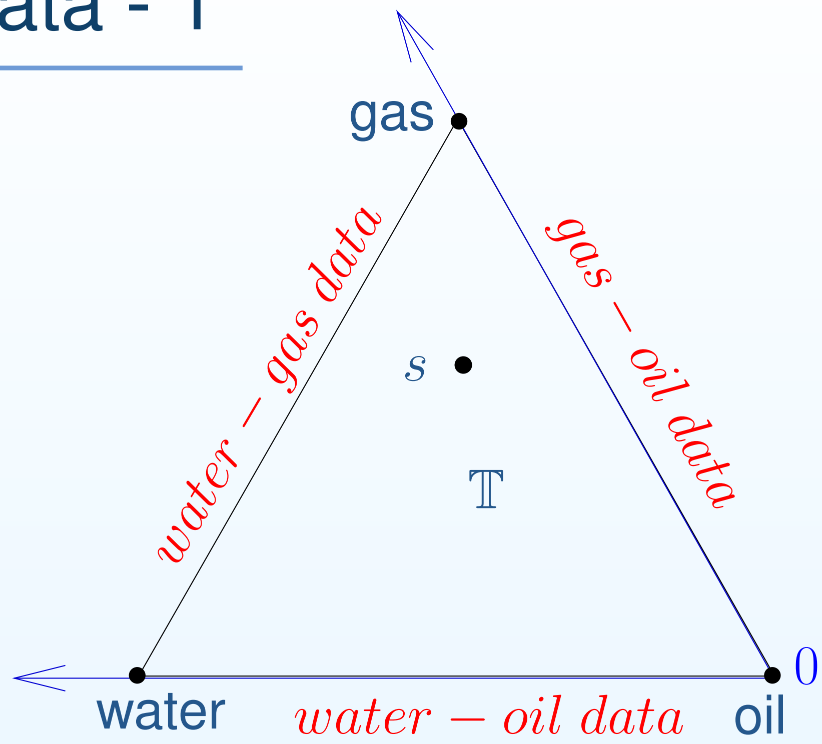

whose associated fractional flows :

$$\nu_j(s,p) = \partial P_{cg}/\partial s_j(s,p) \Big/ dP_c^{j2}/ds_j(s_j) \quad , \quad j = 1, 3$$

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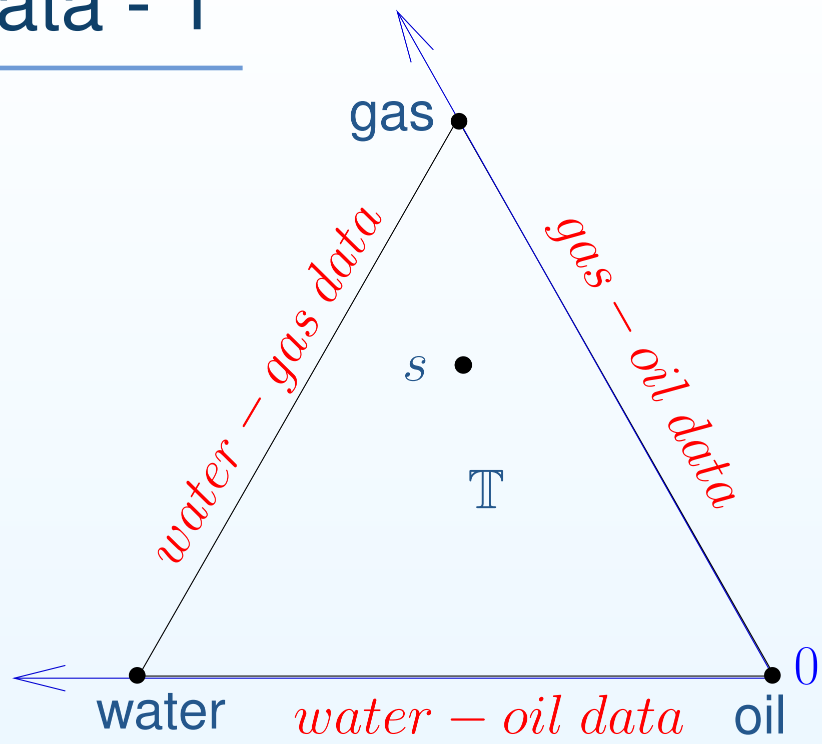

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produce relative permeabilities :

$$\begin{cases} kr_j(s, p) & = & \nu_j(s, p) d(s, p)/d_j\big(p - P_{cg}(s, p) + P_c^{j2}(s_j)\big) \quad j = 1, 3 \ , \\ kr_2(s, p) & = & (1 - \nu_1(s, p) - \nu_3(s, p)) d(s, p)/d_2\big(p - P_{cg}(s, p)\big) \ , \end{cases}$$

which coincide with the given two-phase data on $\partial\mathbb{T}$.

TD-interpolation of two-phase data - 2

$$\begin{cases} \dfrac{\partial P_{cg}}{\partial S_1}(s,p) & = & f_1(s, p - P_{cg}(s,p))\,\dfrac{dP_c^{12}}{dS_1}(s_1)\ , \\ \dfrac{\partial P_{cg}}{\partial S_3}(s,p) & = & f_3(s, p - P_{cg}(s,p))\,\dfrac{dP_c^{32}}{dS_3}(s_3)\ , \end{cases}$$

if the fractional flows

$f_1(s, p_2)\ ,\ f_3(s, p_2)$

are known along a curve

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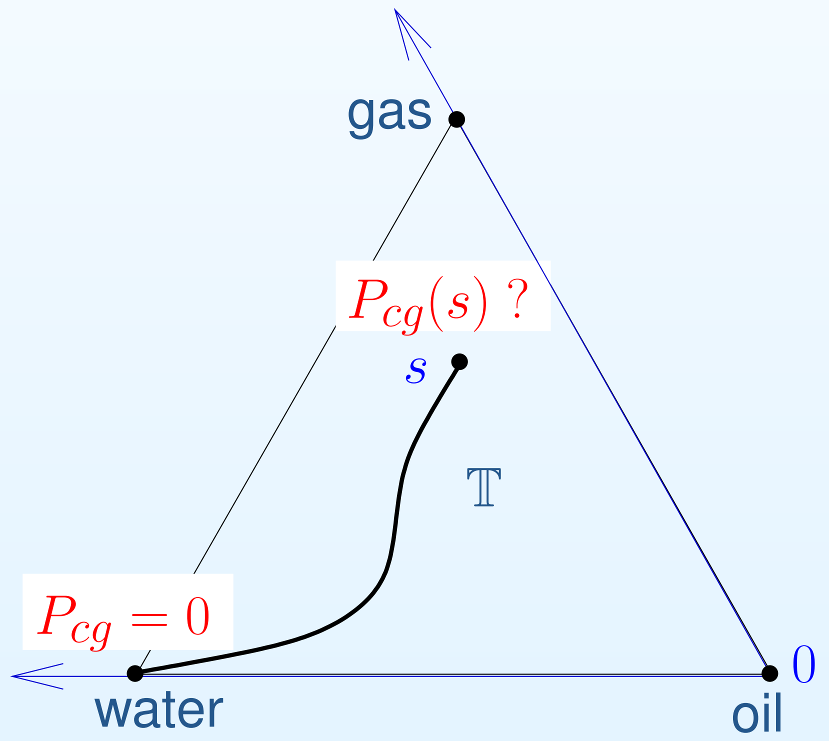

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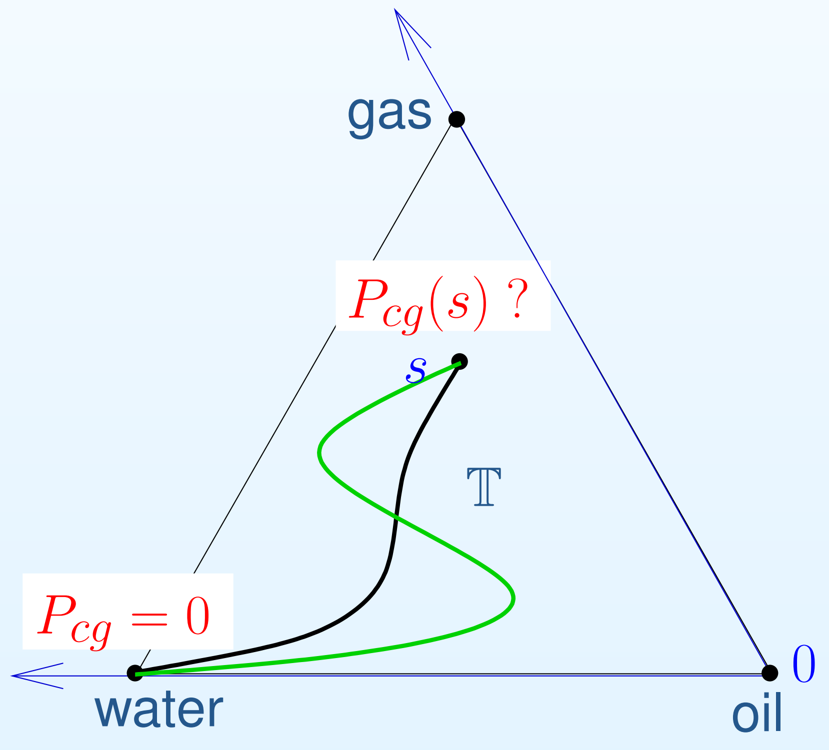

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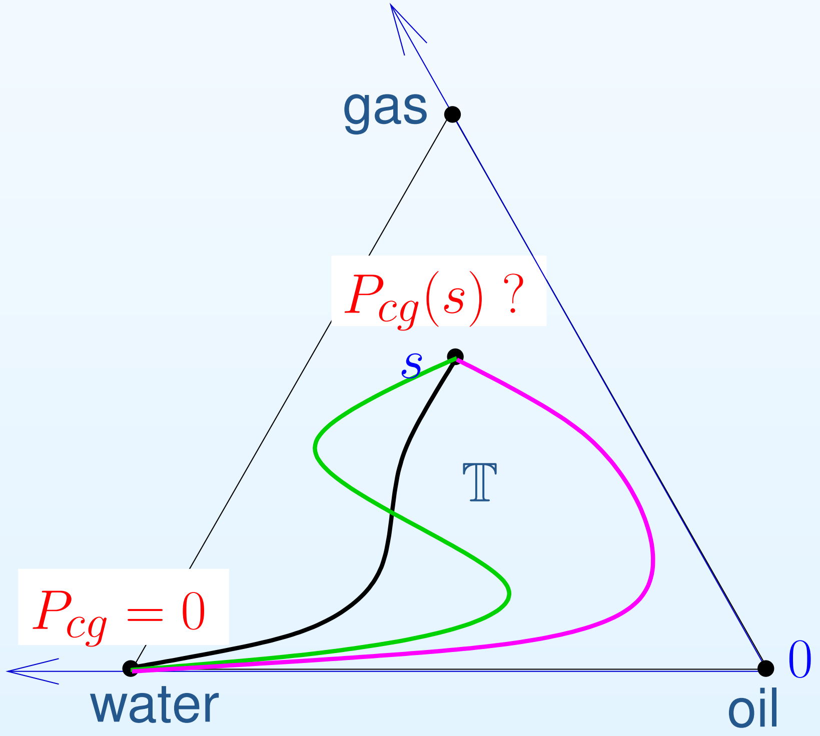

TD-interpolation of two-phase data - 3

Let p $=$ given global pressure level.

- Determination of P_{cg}^{data} on $\partial\mathbb{T}$:
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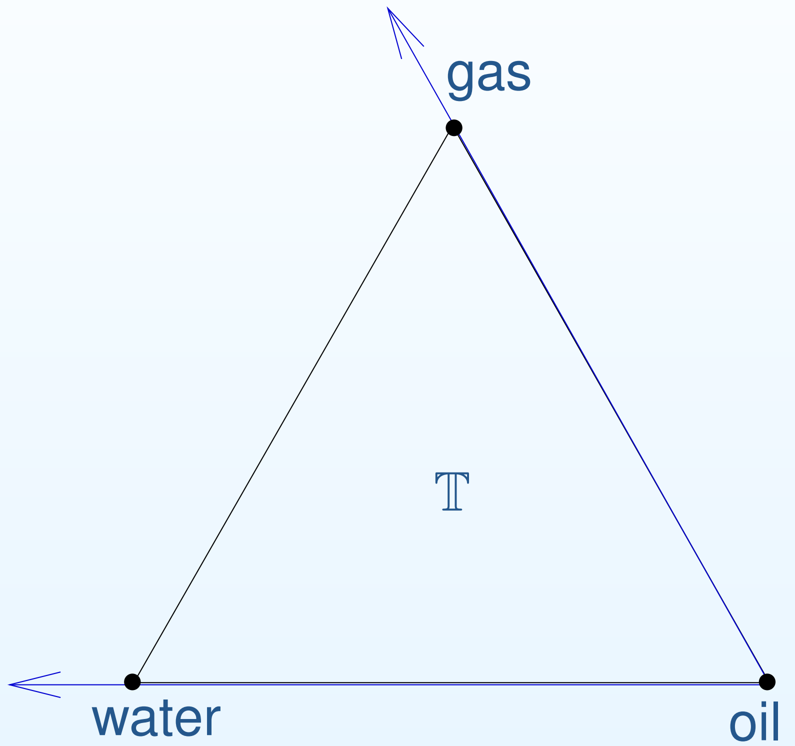

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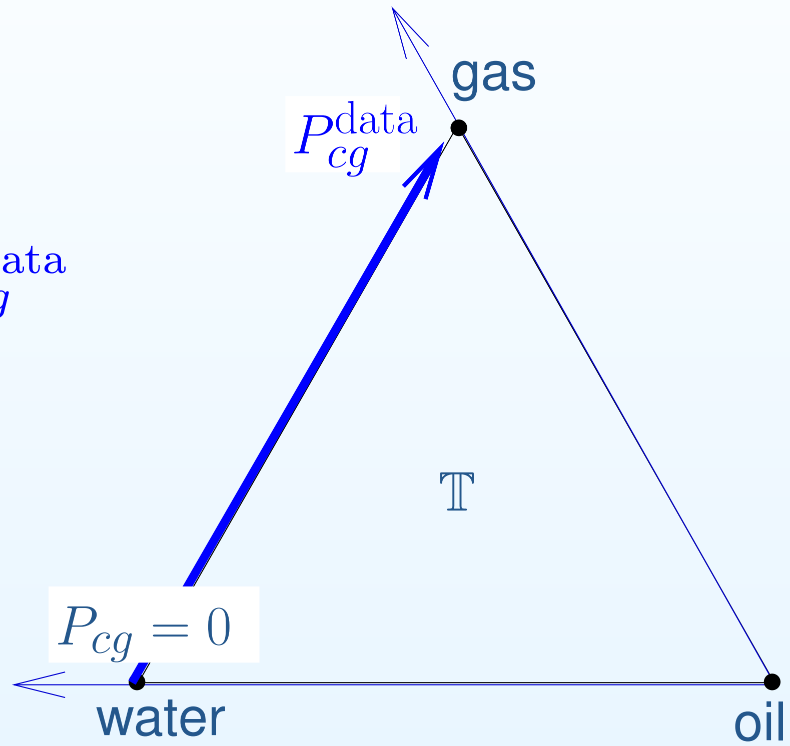

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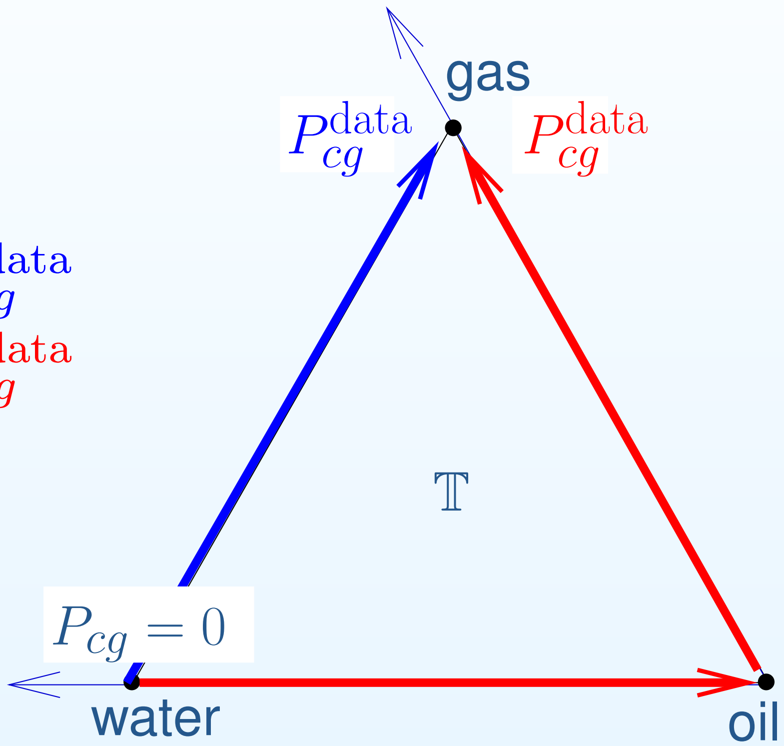

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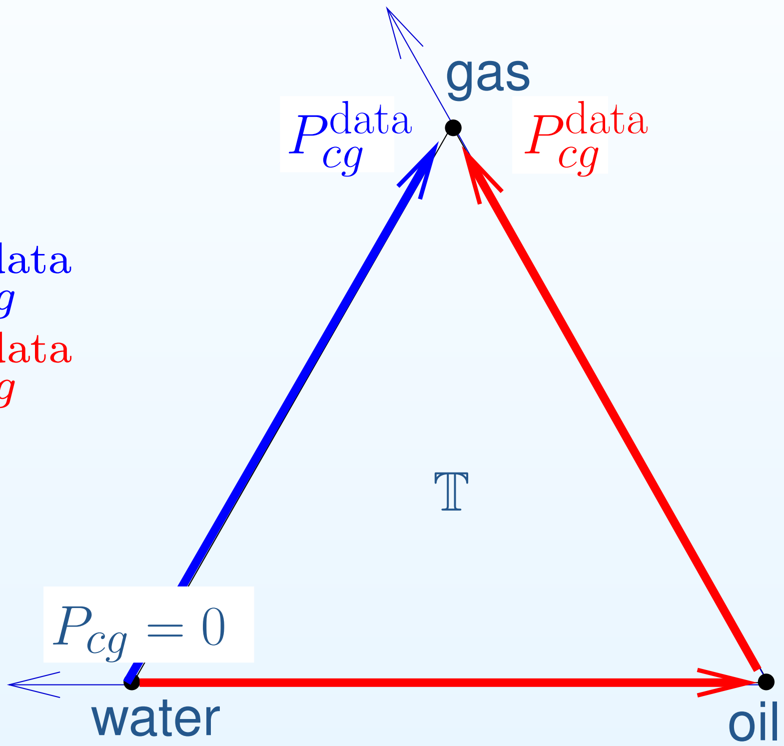

- TD-compatibility condition :

$$P_{cg}^{\mathrm{data}}(\mathrm{gas}) = P_{cg}^{\mathrm{data}}(\mathrm{gas})$$

or, in term of fractional flows
at global pressure p :

$$\int_0^1 (\nu_1^{12,\mathrm{data}} - \nu_1^{13,\mathrm{data}}) \frac{dP_c^{12}}{ds_1} + \int_0^1 (\nu_3^{32,\mathrm{data}} - \nu_3^{31,\mathrm{data}}) \frac{dP_c^{32}}{ds_3} = 0 \; .$$

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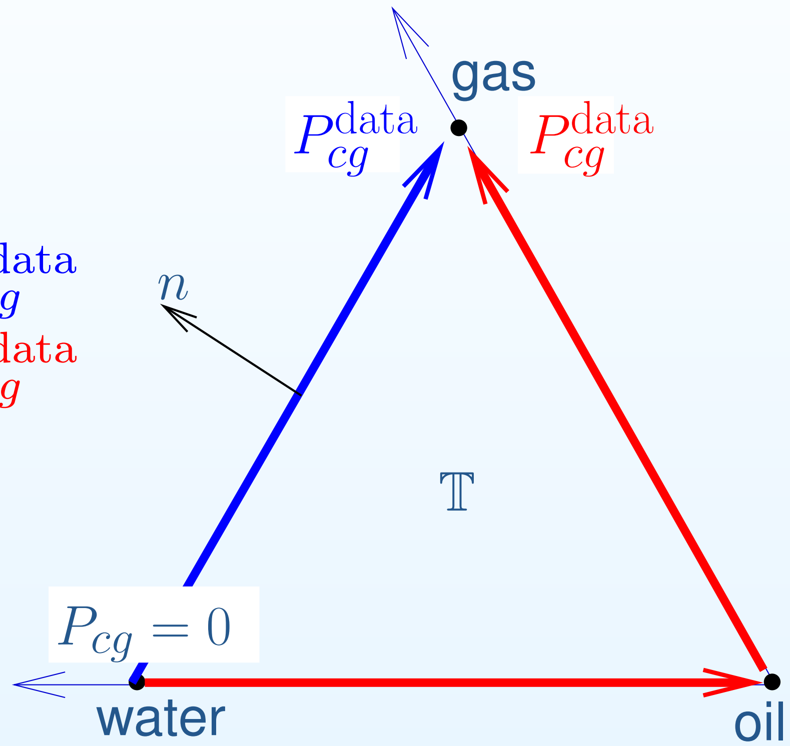

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- Determination of $(\partial P_{cg}/\partial n)^{\text{data}}$ on $\partial\mathbb{T}$: use ν_1 and ν_3 .

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- Determination of d^{data} on $\partial\mathbb{T}$ at global pressure p :

$$d^{\mathrm{data}} = \begin{cases} kr_1^{12}\, d_1(p - P_{cg}^{\mathrm{data}} + P_c^{12}) + kr_2^{12}\, d_2(p - P_{cg}^{\mathrm{data}}) & \text{(water-oil)} \\ kr_1^{13}\, d_1(p - P_{cg}^{\mathrm{data}} + P_c^{12}) + kr_3^{13}\, d_3(p - P_{cg}^{\mathrm{data}} + P_c^{32}) & \text{(gas-water)} \\ kr_3^{32}\, d_3(p - P_{cg}^{\mathrm{data}} + P_c^{32}) + kr_2^{32}\, d_2(p - P_{cg}^{\mathrm{data}}) & \text{(gas-oil)} \end{cases}$$

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- by optimization : use for example $kr_1^{\text{Stone}}(s)$, $kr_3^{\text{Stone}}(s)$ as targets.

- Finite element parameterization : reduced HCT for P_{cg}, P^1 for d.

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- Let the three sets of water-oil, gas-oil and water-gas two-phase data satisfy the TD-compatibility condition.

- Then they can be interpolated by TD-three-phase data by chosing, for each global pressure level p :
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