Double penalisation method for porous-fluid problems with applications to flow control

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October 14, 2008

Summary

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- Coupling of Darcy equations with Stokes equations
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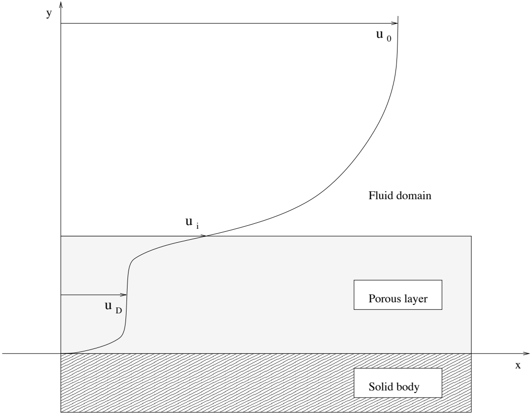
- Drag reduction of a simplified car model
- Conclusions

Targets

- **Computing efficiently the flow in solid-porous-fluid media.**
- To explore a tool to perform simultaneously all computations:

- Low computational tasks;
- Low complexity of the method;
- Useful to solve different industrial problems.

Physical description



We have to solve a problem involving three different media, the solid body, the porous layers and the incompressible fluid.

....physical description

From the solid to the main fluid (Vafai 81, Nield & Bejan 99):

- the boundary layer in the porous medium close to the solid wall has a thickness thickness order of $\mathbf{k^{1/2}}$,
- the homogeneous porous flow with the very low Darcy velocity $\mathbf{u_D}$,
- the porous interface region with the fluid velocity from $\mathbf{u_D}$ to $\mathbf{u_i}$ at the boundary and the thickness about $\mathbf{k^{1/2}}$,
- the boundary layer in the fluid close to the porous frontier that **grows from the interface velocity $\mathbf{u_i}$ instead of zero**,
- the main fluid flow with mean velocity $\mathbf{u_0}$.

Reduction of the porous layer to a boundary condition

From the Darcy law, Beavers and Joseph (1972) derived the *ad hoc* boundary condition

$$\frac{\partial \mathbf{u}}{\partial \mathbf{y}} = \frac{\alpha}{\mathbf{k^{1/2}}}(\mathbf{u_i} - \mathbf{u_D}) \;\; ; \;\; \mathbf{v} = \mathbf{0}$$

with α: a slip coefficient.

Modified boundary condition (Jones 1973)

$$(\frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{y}}) = \frac{\alpha}{\mathbf{k^{1/2}}}(\mathbf{u_i} - \mathbf{u_D}) \;\; ; \;\; \mathbf{v} = \mathbf{0}$$

Normal transpiration (Perot & Moin 1995)

$$\mathbf{u} = \mathbf{0} \;\; ; \;\; \frac{\partial \mathbf{v}}{\partial \mathbf{y}} = \mathbf{0} \;\; or \;\; \mathbf{u} = \mathbf{0} \;\; ; \;\; \mathbf{v} = -\beta \mathbf{p'}$$

with β: the porosity coefficient; $\mathbf{p'} = \mathbf{p} - \mathbf{G(t)x}$: fluctuation of the wall pressure versus the mean pressure gradient.

Coupling of Darcy equations with Fluid equations

Modelling both the porous medium and the flow

$$\frac{\mu_{\mathbf{p}}}{\mathbf{k}}\,\mathbf{U} + \nabla \mathbf{p} = \mathbf{0} \;\; ; \;\; \mathbf{div}\,\mathbf{U} = \mathbf{0}$$

$$\partial_{\mathbf{t}} \mathbf{U} - \nu\, \mathbf{\Delta U} + \nabla \mathbf{p} = \mathbf{0} \;\; ; \;\; \mathbf{div}\,\mathbf{U} = \mathbf{0}$$

Boundary condition at the interface (Das et al. 2002, Hanspal et al. 2006, Salinger et al. 1994)

- Darcy equation as a boundary condition for the fluid
- Beavers & Joseph type condition and Brinkman equation

  Interface velocity continuous with a stress jump

$$\mu_{\mathbf{p}}\left(\frac{\partial \mathbf{u_i}}{\partial \mathbf{y}}\right)_{\mathbf{porous}} - \mu\left(\frac{\partial \mathbf{u_i}}{\partial \mathbf{y}}\right)_{\mathbf{fluid}} = \frac{\gamma}{\mathbf{k^{1/2}}}\mathbf{u_i}$$

  where γ is a dimensionless coefficient of order one.

Penalisation method

Arquis & Caltagirone (88), Angot et al. (99), Kevlahan & Ghidaglia (99).

Brinkman's equation (valid only for high porosities close to one) obtained from Darcy's law by adding the diffusion term:

$$\nabla \mathbf{p} = -\frac{\mu}{\mathbf{k}}\mathbf{\Phi}\mathbf{U} + \tilde{\mu}\mathbf{\Phi}\mathbf{\Delta}\mathbf{U}$$

adding the inertial terms with the Dupuit-Forchheiner relationship, the Forchheiner-Navier-Stokes equations:

$$\rho\,\partial_{\mathbf{t}}\mathbf{U} + \rho\,(\mathbf{U}\cdot\nabla)\,\mathbf{U} + \nabla\mathbf{p} = -\frac{\mu}{\mathbf{k}}\mathbf{\Phi}\mathbf{U} + \tilde{\mu}\mathbf{\Phi}\mathbf{\Delta}\mathbf{U}$$

where $\mathbf{k}$: intrinsic permeability, $\tilde{\mu}$: Brinkman's effective viscosity and $\mathbf{\Phi}$: porosity. As $\mathbf{\Phi}$ is close to $\mathbf{1}$ we have $\tilde{\mu}$ close to $\mu/\mathbf{\Phi}$:

$$\rho\,\partial_{\mathbf{t}}\mathbf{U} + \rho\,(\mathbf{U}\cdot\nabla)\,\mathbf{U} + \nabla\mathbf{p} = -\frac{\mu}{\mathbf{k}}\mathbf{\Phi}\mathbf{U} + \mu\mathbf{\Delta}\mathbf{U}$$

Double penalisation method

Nondimensionalisation using the mean fluid velocity $\overline{U}$ and the obstacles heigh H: $U = U' \, \overline{U} \; ; \; x = x' \, H \; ; \; t = t'/\overline{U}$.

Penalised non dimensional Navier-Stokes equations adding $\mathbf{U}/\mathbf{K}$ to incompressible NS equations ($K = \frac{\rho k \Phi \overline{U}}{\mu H}$ non dimensional permeability coefficient of the medium):

$$
\begin{array}{ll}
\partial_t U + (U \cdot \nabla) U - \dfrac{1}{Re} \Delta U + \dfrac{\mathbf{U}}{\mathbf{K}} + \nabla p = 0 & in \ \Omega_T \\
divU = 0 & in \ \Omega_T \\
U(0,.) = U_0 & in \ \Omega \\
U = U_\infty & on \ \Gamma_D \times I \\
U = 0 & on \ \Gamma_W \times I \\
\sigma(U,p)\, n + \frac{1}{2}(U \cdot n)^-(U - U_{ref}) = \sigma(U_{ref}, p_{ref})\, n & on \ \Gamma_N \times I
\end{array}
$$

Solid: $\mathbf{K} = \mathbf{10^{-8}}$, Fluid: $\mathbf{K} = \mathbf{10^{16}}$, Porous layer: $\mathbf{K} = \mathbf{10^{-1}} \rightarrow$

Specific interpolations needed in the fluid-porous interface.

Outline of the numerical simulation

- Second-order Gear scheme in time.
- The space discretization is performed on staggered grids with strongly coupled equations.
- Second-order centred finite differences are used for the linear terms - The location of the unknowns enforce the divergence-free equation which is discretized on the pressure points.
- The convection terms are approximated by a third order Murman-like scheme.
- The resolution is achieved by a V-cycle multigrid algorithm coupled to a cell-by-cell relaxation procedure. There is a sequence of grids from a coarse 25×10 cells grid to a fine 3200×1280 cells grid for instance.

Applications to passive control (0):
Functionals to be minimized

- As the pressure is computed inside the solid body, the drag and lift forces are computed by integrating the penalisation term on the volume of the body:

$$F_D = -\int_{body} \partial_{x1} p \, dx + \int_{body} \tfrac{1}{Re} \Delta u \, dx \approx \int_{body} \frac{u}{K} \, dx \quad (1)$$

$$F_L = -\int_{body} \partial_{x2} p \, dx + \int_{body} \tfrac{1}{Re} \Delta v \, dx \approx \int_{body} \frac{v}{K} \, dx \quad (2)$$

- Important quantities to quantify the control effect:

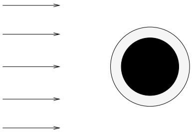
$$C_p = 2(p - p_0)/(\rho |U|^2)$$

$$C_D = \frac{2F_D}{H} \; ; \; C_L = \frac{2F_L}{H}$$

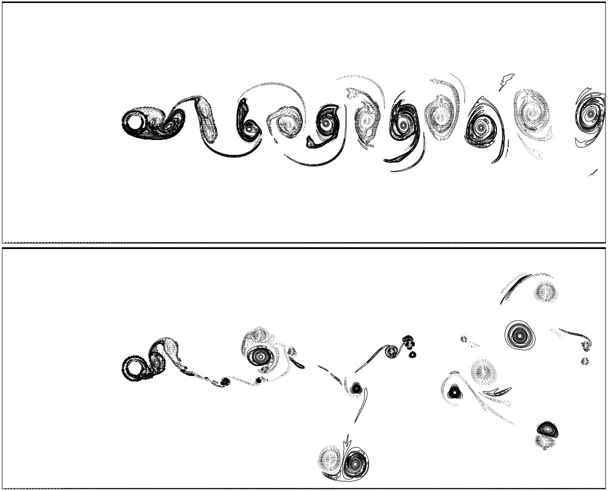
$$C_{Lrms} = \sqrt{\frac{1}{T}\int_0^T C_L^2 \, dt} \; ; \; Z = \frac{1}{2}\int_\Omega |\omega|^2 dx$$

Applications to passive control (1):
Flow control around a riser using a porous ring

- Flow simulation behind a circular bluff body with a size $D = 0.16$, located at the position $(1.1, 1)$ in an open computational domain.
- The pipe is surrounded by a solid (larger diameter), a porous or a fluid sheath (smaller diameter): $\delta D = 0.2$.



- The Reynolds number based on the pipe diameter D is $R_D = 30000$ for the solid case.
- The control target is to reduce the VIV (Vortex Induced Vibrations) around the riser.



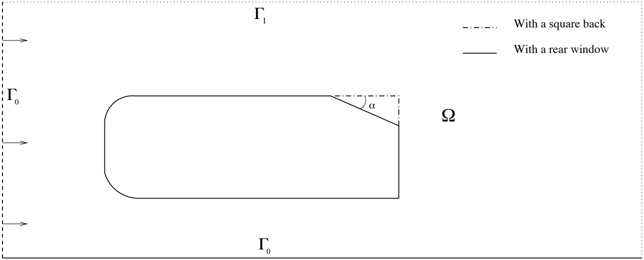
Vorticity field for a fluid (bottom) and a porous (top) sheath for the same time at $\mathbf{R_D = 30000}$.

Mean values of the enstrophy and the drag coefficient and asymptotic value of the CLrms for $\mathbf{R_D = 30000}$.

Grid	K	Enstrophy	Drag	C_{Lrms}
3200×1280	10E-1	291	1.56	0.081
	10E+16	810	1.10	0.293

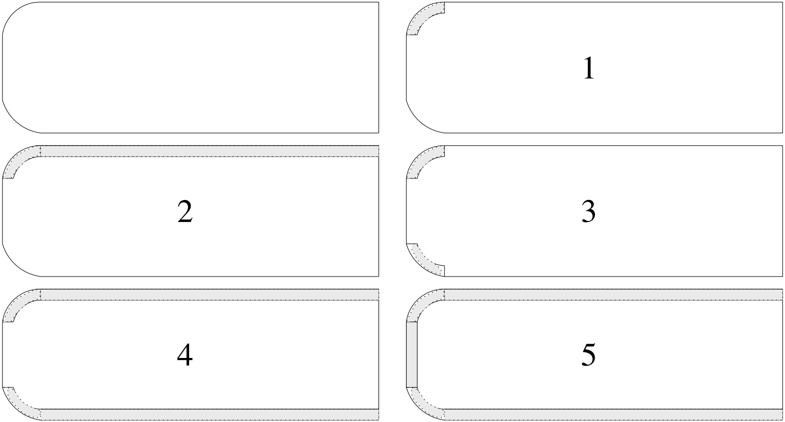
- **A patent in 2004 on the passive control of VIV around riser pipes using porous media with IFP.**

Applications to passive control (2):
Drag reduction for a simplified car model using porous devices (collaboration with Renault)

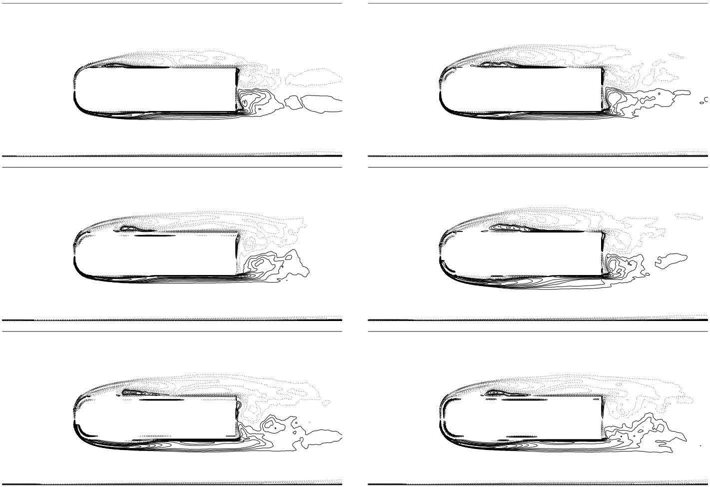


Computational domain for the **Ahmed body** without or with a rear window.

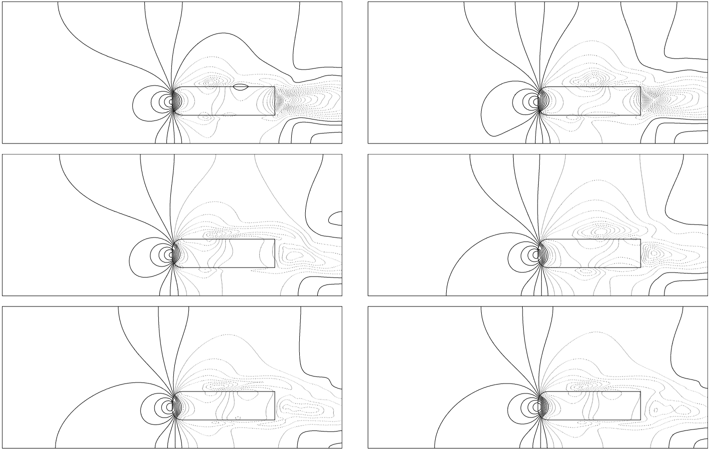
Passive flow control around the square back Ahmed body



From left to right and top to bottom: porous cases 0, 1, 2, 3, 4 and 5 geometries for the square back Ahmed body.



Mean vorticity isolines for the flow around square back Ahmed body on top of a road at $\mathbf{R_L = 30000}$. Cases 0 (top left), 1 (top right), 2 (middle left), 3 (middle right), 4 (bottom left) and 5 (bottom right).



Pressure isolines for the flow around square back Ahmed body on top of a road at $\mathbf{R_L = 30000}$. Cases 0 (top left), 1 (top right), 2 (middle left), 3 (middle right), 4 (bottom left) and 5 (bottom right).

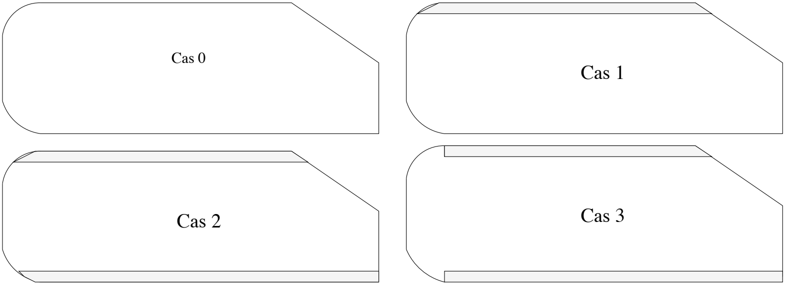
The value and the location of the *minimum pressure* in the close wake of the square back Ahmed body *on top of a road* at $\mathbf{R_L = 30000}$.

	P_{min} value in the wake	P_{min} Location
case 0	-1.636	(10.11 , 1.53)
case 1	-1.758	(10.11 , 1.53)
case 2	**-0.678**	(10.22 , 1.39)
case 3	-0.850	(10.09 , 1.52)
case 4	**-0.540**	(10.89 , 1.34)
case 5	**-0.510**	(10.16 , 1.34)

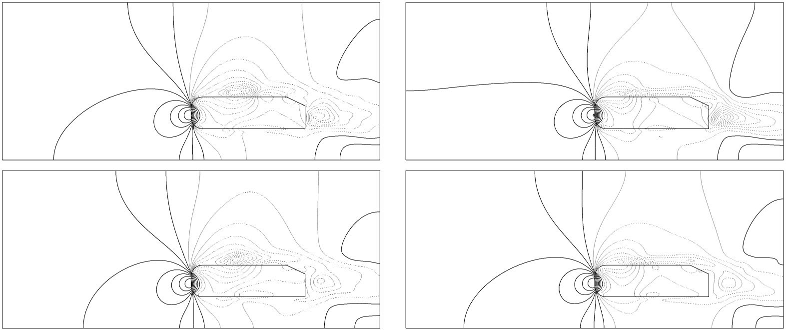
Mean values of the enstrophy and the *drag coefficient* and asymptotic values of C_{Lrms} for square back Ahmed body *on top of a road* at $\mathbf{R_L = 30000}$.

	C_{Lrms}	Z	Up D	Down D	Drag
case 0	0.517	827	0.173	0.343	0.526
case 1	0.545 (+ 5%)	835 (+ 1%)	0.231	0.330	0.567 (+ 8%)
case 2	0.396 (-23%)	592 (-28%)	0.156	0.166	**0.332 (-37%)**
case 3	0.674 (+30%)	732 (-11%)	0.214	0.176	0.391 (-26%)
case 4	0.381 (-26%)	541 (-35%)	0.213	0.139	**0.362 (-31%)**
case 5	0.352 (-32%)	533 (-36%)	0.217	0.127	**0.354 (-33%)**

Flow control around the Ahmed body with a rear window using porous materials



From left to right and top to bottom: cases 0, 1, 2 and 3 geometries for the Ahmed body with a rear window.



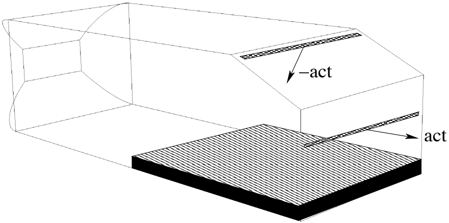
Mean pressure isolines for the flow around the Ahmed body *with a rear window on top of a road* at $\mathbf{R_L = 30000}$. Cases 0 (top left), 1 (top right), 2 (bottom left) and 3 (bottom right).

Mean values of the enstrophy and the *drag coefficient* and asymptotic values of C_{Lrms} for the Ahmed body *with a rear window on top of a road* at $\mathbf{R_L = 30000}$.

	C_{Lrms}	Z	Up D	Down D	Drag
case 0	0.817	726	0.099	0.176	0.282
case 1	0.600 (-27%)	605 (-17%)	0.100	0.190	0.300 (+ 6%)
case 2	0.801 (- 2%)	670 (-18%)	0.093	0.124	0.224 (- 21%)
case 3	0.534 (-35%)	552 (-24%)	0.092	0.151	0.254 (-10%)

$3D$ Control of the body with a rear window

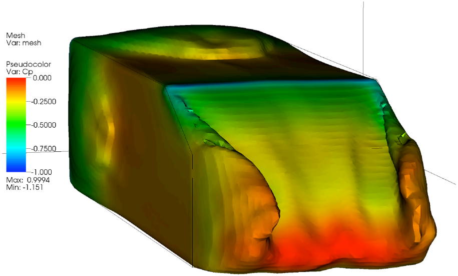
- Passive control with porous surface at the bottom or/and active control with $act = 0.3V_0$:



- Work in progress : study of the fields, computation on finer grids, work with closed-loop control...)

Three-dimensional Ahmed body

- Ahmed body with a rear window (25°) on the top of a road ($h = 0.6$)
- Reynolds number $Re = 30000$
- Isosurface of total pressure coefficient $C_{pi} = 1$ with C_P colors:



Conclusion

- It is shown that the **double penalisation** method handles efficiently the solid-porous-fluid problems.
- Simulations in the three media are accurate and simultaneous.
- Applications with porous interfaces, to implement **passive control** techniques in different industrial area are very promising.
- $3D$ computations to achieve a realistic knowledge of the control around the Ahmed body are in progress.

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