Modeling and numerical approximation of multi-component anisothermal flows in porous media

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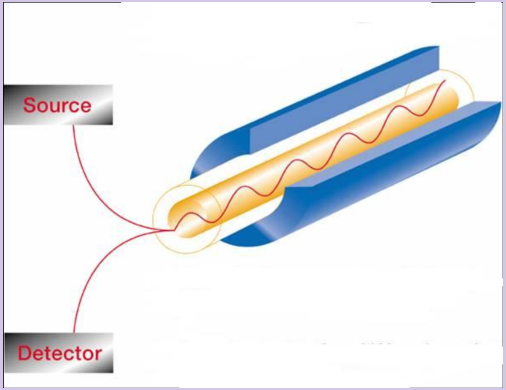
Motivations

Optical fiber

- Send a light source
- Detect a backscattering light
- The time for the backscattered signal gives distance along fiber
- The ratio of wave lengths gives temperature



Possible applications

- Estimate virgin reservoir temperature
- Predict flow profiles and the flow rate of each layer

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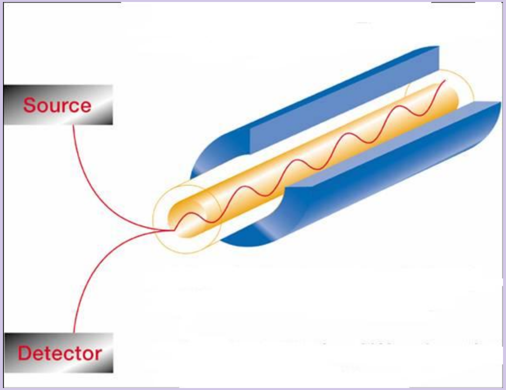

Possible applications

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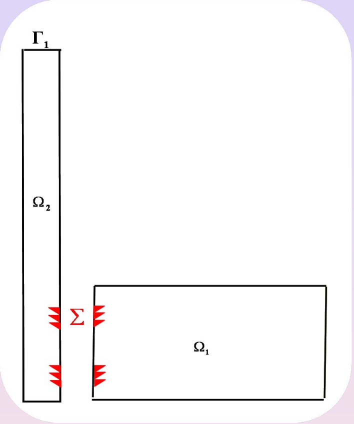
Coupling of monophasic reservoir and wellbore models with heat transfer

- **Resevoir model** :

$$\begin{cases} r\phi \frac{\partial \rho}{\partial t} + div(r\mathbf{G}) = 0 \\ \rho^{-1}(\mu \underline{\mathbf{K}}^{-1}\mathbf{G} + F|\mathbf{G}|\,\mathbf{G}) + \nabla p = -\rho \mathbf{g} \\ r(\rho c)_* \frac{\partial T}{\partial t} + r\rho^{-1}(\rho c)_f \mathbf{G}\cdot\nabla T - div(r\mathbf{q}) - r\phi\beta T\,\frac{\partial p}{\partial t} - r\rho^{-1}(\beta T - 1)\mathbf{G}\cdot\nabla p = 0 \\ \frac{1}{\lambda}\mathbf{q} - \nabla T = 0 \\ \rho = \rho(p, T) \end{cases}$$

- **Wellbore model**

$$\begin{cases} \frac{\partial}{\partial t}(r\rho) + \nabla\cdot(r\rho\mathbf{u}) = 0 \\ \frac{\partial}{\partial t}(r\rho u_r) + \nabla\cdot(ru_r\rho\mathbf{u}) + r\frac{\partial p}{\partial r} - \frac{\partial}{\partial r}(r\tau_{rr}) - \frac{\partial}{\partial z}(r\tau_{zr}) + \tau_{\theta\theta} + r\kappa\rho|\mathbf{u}|u_r = 0 \\ \frac{\partial}{\partial t}(r\rho u_z) + \nabla\cdot(ru_z\rho\mathbf{u}) + r\frac{\partial p}{\partial z} - \frac{\partial}{\partial r}(r\tau_{rz}) - \frac{\partial}{\partial z}(r\tau_{zz}) + r\rho g + r\kappa\rho|\mathbf{u}|u_z = 0 \\ \frac{\partial}{\partial t}(r\rho E) + \nabla\cdot(r(\rho E + p)\mathbf{u}) - \nabla\cdot(r\underline{\tau}\mathbf{u}) - \nabla\cdot(r\lambda\nabla T) + r\rho g u_z = 0 \\ \rho = \rho(p, T) \end{cases}$$



* M. Amara, D. Capatina and L. Lizaik, *Coupling of a Darcy-Forchheimer model and compressible Navier-Stokes equations with heat transfer*, Accepted in SIAM J. Sci. Comp. 2008.
* M. Amara, D. Capatina and L. Lizaik, *Numerical coupling of 2.5D reservoir and 1.5D wellbore models in order to interpret thermometrics*, Int. J. Numer. Meth. Fluids, Vol. 56, No. 8, pp. 1115-1122, 2008.

Outline

- **Physical modeling**
- **Primary and secondary variables**
- **Boundary conditions**
- **Numerical scheme**
- **Numerical simulations**

Physical modeling

- **Three phases (p) : water(w), oil(o) and gas (g)**
- **n_c components: water, heavy hydrocarbons, light hydrocarbons, methan....**
- **n_h hydrocarbon components ($n_h = n_c - 1$)**

	$\bar{w}$	n_1	n_2	...	...	n_h
w	×					
o		×	×	×	×	×
g		×	×	×	×	×

- 3D / Porous media Ω with n_W wells

Gridding

- Cartesian rectangular mesh
- The code is able to interface with any gridding software by reading some necessary informations

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Governing equations

- **Mass conservation equation for each component c :**

$$\mathcal{F}_c = \sum_{p=o,g,w} \left(\frac{\partial}{\partial t}(\phi S_p \rho_p y_{c,p}) + \nabla \cdot (\rho_p \mathbf{u}_p y_{c,p}) \right) = 0$$

$\mathbf{u}_p$ is given by the generalized Darcy law : $\mathbf{u}_p = -k_{rp}\mu_p^{-1}\underline{\mathbf{K}}(\nabla p_p - \rho_p \mathbf{g})$

- **Energy equation :**

$$\mathcal{F}_T = \frac{\partial}{\partial t}\left[\sum_{p=o,g,w} (\phi\, S_p\, \rho_p\, \mathcal{H}_p - p_p) + \left(1-\phi\right)\rho_s \mathcal{H}_s \right] + \sum_{p=o,w,g} \nabla \cdot (\phi S_p \rho_p \mathcal{H}_p \mathbf{u}_p)$$

$$-\nabla \cdot (\lambda \nabla T) + \sum_{p=o,g,w} \mathbf{u}_p \cdot \nabla p_p = 0$$

$\mathcal{H}_p$ enthalpy of phase p
T temperature
λ equivalent thermal conductivity $\qquad \lambda = (\lambda_s)^{(1-\phi)} \times (\lambda_w)^{s_w \times \phi} \times (\lambda_o)^{s_o \times \phi} \times \left(\lambda_g\right)^{s_g \times \phi}$

Take into account convective, diffusive, compressibility and viscous dissipation effects

- **Capillary pressure constraints :**

$$p_{c,ow} = p_o - p_w \quad \textit{(oil-water capillary pressure)}$$
$$p_{c,go} = p_g - p_o \quad \textit{(gas-oil capillary pressure)}$$

Capillary pressures are measured in laboratories

- **Saturation constraint :**

$$\sum_{p=1}^{n_p} S_p = 1$$

- **Component mole fraction constraints :**

$$\sum_{c=1}^{n_c} y_{c,p} = 1 \quad \forall p = w, o, g$$

- **Phase equilibrium relation for each hydrocarbon component c in oil and gas phases:**

$$\mathcal{F}_e = f_{c,o} - f_{c,g} = 0$$

$f_{c,o}$ **and** $f_{c,g}$ **are the fugacities of hydrocarbon component** c **in oil and gas phases respectively, calculated from the Peng Robinson equation of state**

⋆ **Number of equations :**

Type	Number
Mass conservation	$n_h + 1$
Energy equation	1
Capillary pressure constraints	2
Saturation constraint	1
Component mole fraction constraints	2
Equilibrium relation equations	n_h
Total	$2n_h + 7$

Primary and secondary variables

- According to Gibb's phase rule, the number of primary variables is equal to :

$$(n_c + 2 - n_{phase}) + (n_{phase} - 1) \;=\; n_c + 1$$

- Use linear constraint equations to remove two pressures, one saturation and two component mole fractions
  $\longrightarrow$ $2n_h + 2$ number of non-linear equations and variables is left

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- **Multiple choices for the selection of primary variables and equations leading to different models**

Coats Model

- Primary equations are the $n_c + 1$ mass and energy balance equations ($\mathbf{F}_p = \{\mathcal{F}_c, \mathcal{F}_T\}$)
- Equations left over are the secondary equations ($\mathbf{F}_s = \{\mathcal{F}_e\}$)
- Primary variables $\mathbf{X}_p$ are :
  - p_g, T, S_g, S_o, $y_{c,g;\ c=3...n_h}$ when both oil and gaz phases are present
  - p_o, T, S_o, $y_{c,o;\ c=1...n_h}$ when gaz phase is not present
  - p_g, T, S_g, $y_{c,g;\ c=1...n_h}$ when oil phase is not present

- Adjacent gridblocks may have different sets of primary variables
  → need to switch variables when a hydrocarbon phase disappears or reappears
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- After solving primary variables, secondary variables are updated explicitly gridblock by gridblock.

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Boundary conditions

- **On each surface boundary, choice between :**
  1. **mass flow / constant pressure**
  2. **heat flux / constant temperature**

- On the top and the bottom of the reservoir, no flow and the geothermal gradient are imposed

Well treatment

Two types of well control are implemented :

1. Bottom hole pressure
   Reservoir equations will depend only on reservoir variables
2. Constant phase volumetric flow rate
   - An extra well variable p^w
   - An extra well equation based on component mass balance within the wellbore
     Ex : *for a constant oil phase flow rate* q_o^{SP} *, we have :* $\sum_l \sum_p WI_l \lambda_{p,l} \rho_{p,l} (p_{p,l} - p^w) \frac{l^{SP}}{\rho_o^{SP}} - q_o^{SP} = 0$

$\oplus$ well temperature/ null heat flux

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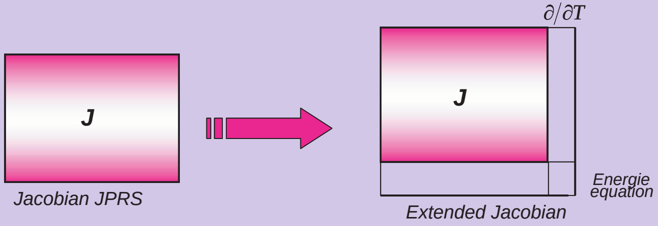
$\oplus$ **well temperature/ null heat flux**

- **Extend an existing isothermal simulator in the reservoir (GPRS General Purpose Reservoir Simulator)**

- **Finite volume scheme : equations integrated over each gridblock V**
- **FIM scheme**
- **Iterative Newton Raphson method :**
$$\begin{aligned} \mathbf{J}\Delta\mathbf{X} &= -\mathbf{E}(\mathbf{X}) \\ \Delta\mathbf{X} &= \mathbf{X}^{n+1} - \mathbf{X}^{n} \text{ and } \mathbf{J} = \frac{\partial \mathbf{E}}{\partial \mathbf{X}}(\mathbf{X}) \end{aligned}$$



- **The non-linear set of equations can be expressed as :**
$$\begin{cases} \mathbf{F}_p(\mathbf{X}_p, \mathbf{X}_s) = 0 \\ \mathbf{F}_s(\mathbf{X}_p, \mathbf{X}_s) = 0 \end{cases}$$

Jacobian matrix can be written as :

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{F}_p}{\partial \mathbf{X}_p} & \frac{\partial \mathbf{F}_p}{\partial \mathbf{X}_s} \\ \frac{\partial \mathbf{F}_s}{\partial \mathbf{X}_p} & \frac{\partial \mathbf{F}_s}{\partial \mathbf{X}_s} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ and } -\mathbf{E} = \begin{bmatrix} -\mathbf{F}_p \\ -\mathbf{F}_p \end{bmatrix} = \begin{bmatrix} \mathbf{M} \\ \mathbf{N} \end{bmatrix}$$

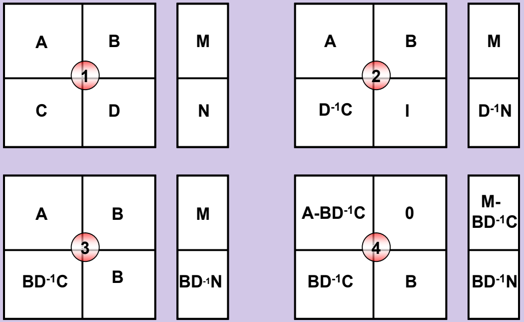
Reduce Full set $\mathbf{F}(\mathbf{X}) = 0$ to Primary set $\mathbf{F}_p(\mathbf{X}_p) = 0$

- **Primary equation set can be extracted and written as :**

$$(A - B\, D^{-1}\, C)\, \Delta\mathbf{X}_p = (\mathbf{M} - B\, D^{-1}\, \mathbf{N})$$

- **After solving primary variables, secondary ones are updated gridblock by gridblock as follows :**

$$\Delta\mathbf{X}_s = (D^{-1}\, \mathbf{N}) - (D^{-1}\, C)\, \Delta\mathbf{X}_p$$



Flash calculation

1. Build relations between secondary and primary variables
   ⟶ This role is only necessary when both hydrocarbon phases exist in a gridblock
2. Check the state of hydrocarbon phases in gridblocks
   - *Phase disappearance for a gridblock with two hydrocarbon phases*
     If either S_o or S_g is negative, the corresponding hydrocarbon phase has disappeared
     ⟶ set the negative saturation to zero and reassign mole fractions
   - *Phase reappearance for a gridblock with only one hydrocarbon phase*
     Do a flash and calculate the tangent plane distance for the current phase
     ⟶if it is less than zero, a second hydrocarbon phase reappear and need to reassign saturations and mole fractions

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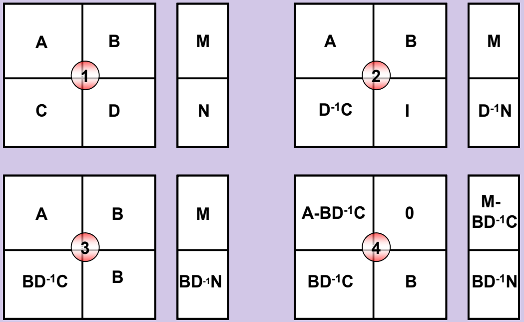

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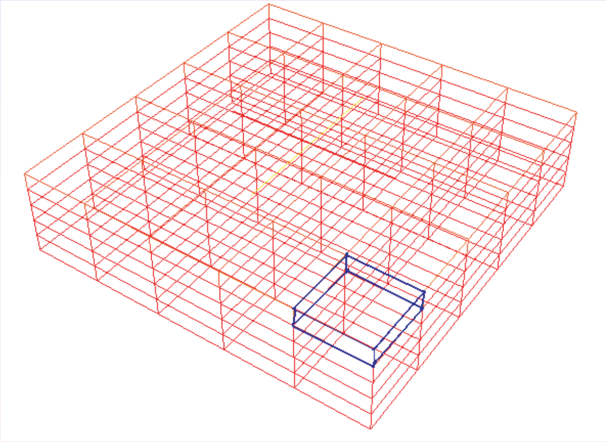
Essential steps of the code

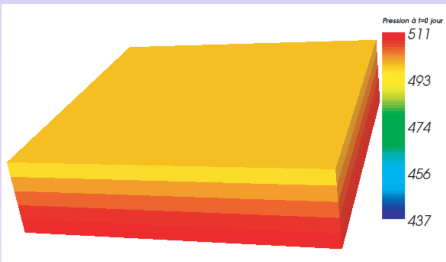
1. Read input data
2. Initialize with initial conditions
   ↪ Assign initial pressure, temperature, saturations
   ↪ Do a flash calculation in order to assign intial mole fractions and define cell status
3. Start time step calculations (the Newton iteration)
   - Calculate gridblock properties
     ↪ For water phase, calculate the thermodynamic properties (Enthalpy, density, viscosity...)
     ↪ Check disapperence or reappearence of hydrocarbon phases and calculate their thermodynamic properties
     ↪ calculate fugacities when both hydrocarbon phases are present
   - Solve the linear system
     ↪ calculate the full jacobian matrix: $\mathbf{J}.\Delta\mathbf{X} = -\mathbf{F}(\mathbf{X})$
     ↪ calculate primary variables: $(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})\Delta\mathbf{X}_p = (\mathbf{M}-\mathbf{B}\mathbf{D}^{-1}\mathbf{N})$
     ↪ update secondary variables: $\Delta\mathbf{X}_s = (\mathbf{D}^{-1}\mathbf{N}) - (\mathbf{D}^{-1}\mathbf{C})\Delta\mathbf{X}_p$
   - Perform Newton update : $\mathbf{X}^{n+1} = \mathbf{X}^n + \Delta\mathbf{X}$
   - Check convergence, do another iteration if necessary
4. Print results, increment time ang go to step 3

Comparison with isothermal GPRS

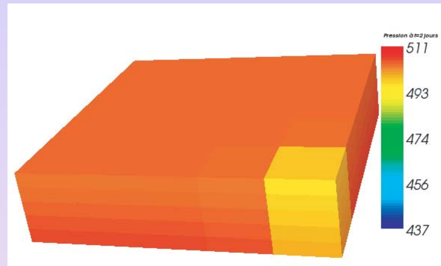
- **Reservoir with dimensions** $5000ft \times 5000ft \times 50ft$
- **Three components: methan** CH_4**, butan** C_4H_{10} **and heptan** C_7H_{16}
- **Production for 50 days by imposing a BHP of** $300\ psi$



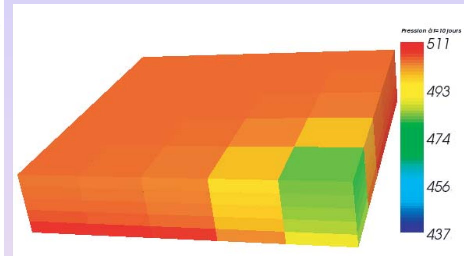
Behaviour of the pressurre during 50 days production



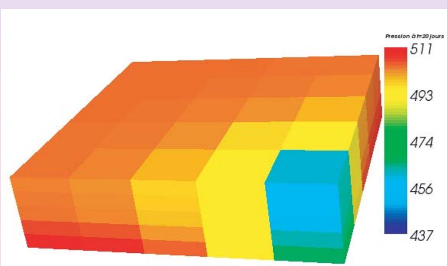
(a) Pressure at t=0 day



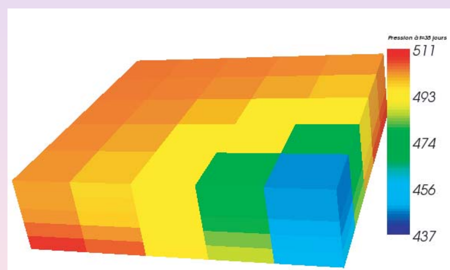
(b) Pressure at t=2 days



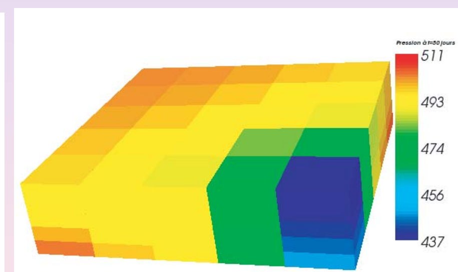
(c) Pressure at t=10 days



(d) Pressure at t=20 days

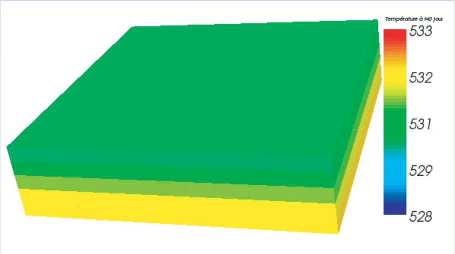


(e) Pressure at t=35 days

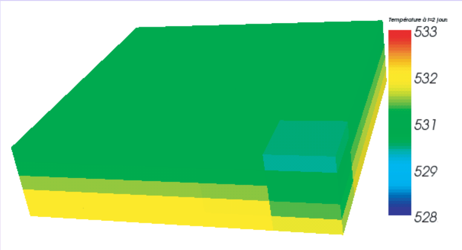


(f) Pressure at t=50 days

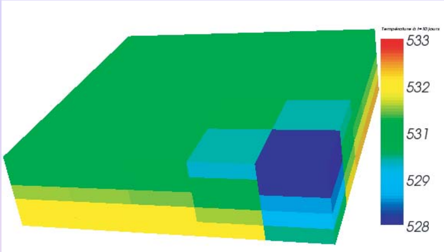
Behaviour of the temperature during 50 days production



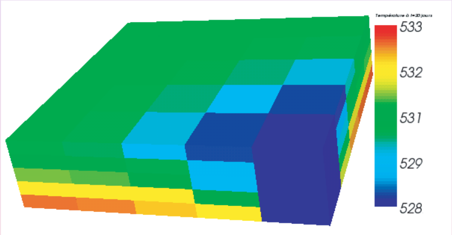
(g) Temperature at t=0 day



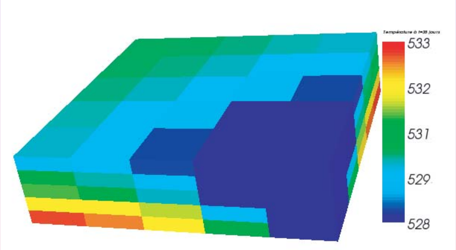
(h) Temperature at t=2 days



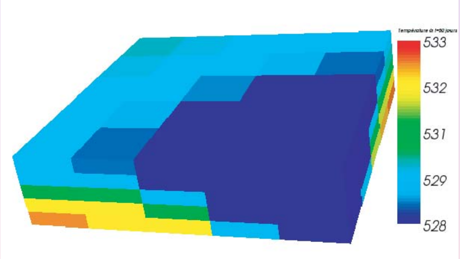
(i) Temperature at t=10 days



(j) Temperature at t=20 days

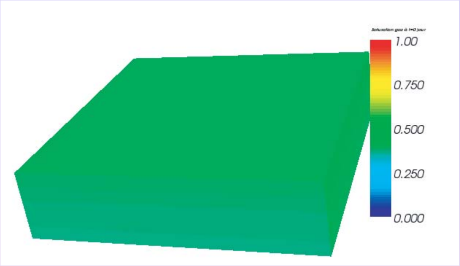


(k) Temperature at t=35 days

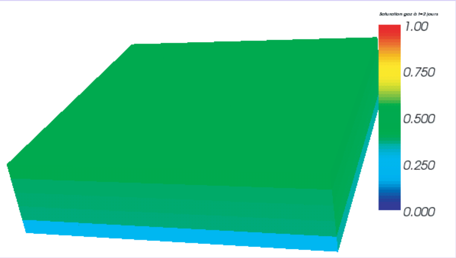


(l) Temperature at t=50 days

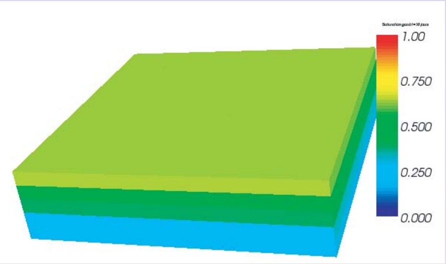
Behaviour of the gas saturation during 50 days production



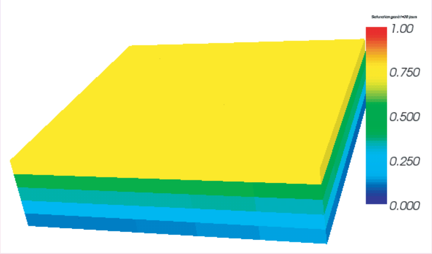
(m) Saturation of gas phase at t=0 day



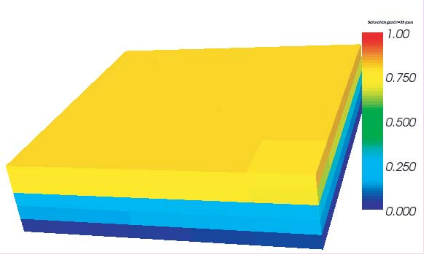
(n) Saturation of gas phase at t=2 days



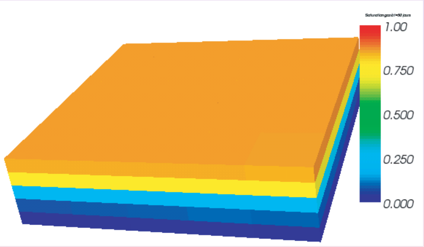
(o) Saturation of gas phase at t=10 days



(p) Saturation of gas phase at t=20 days

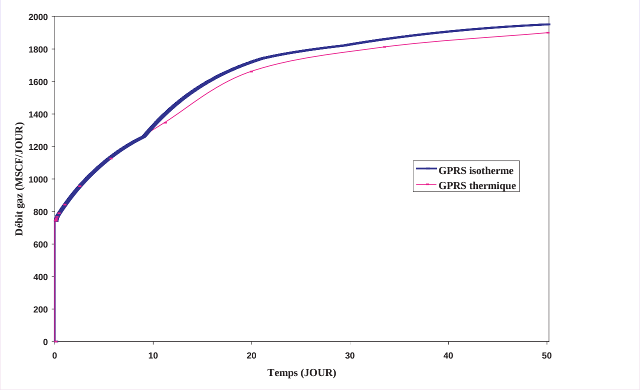


(q) Saturation of gas phase at t=35 days

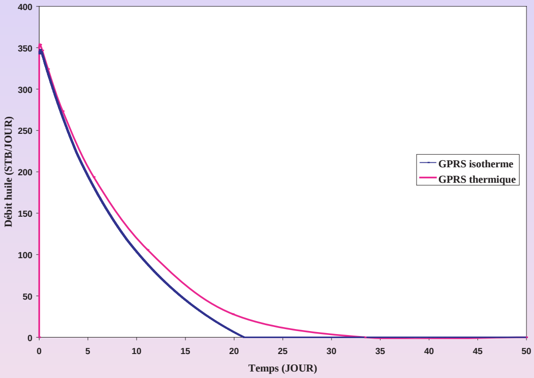


(r) Saturation of gas phase at t=50 days

Comparison of production rates

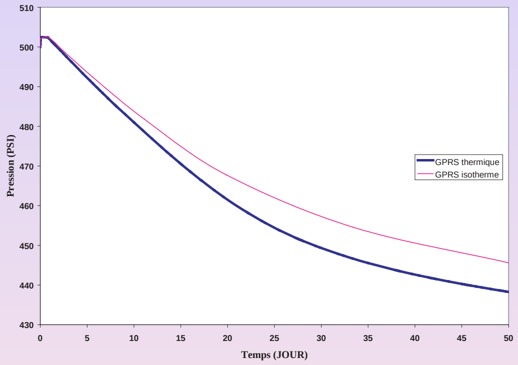


(s) Gas production rate (MSCF/DAY)

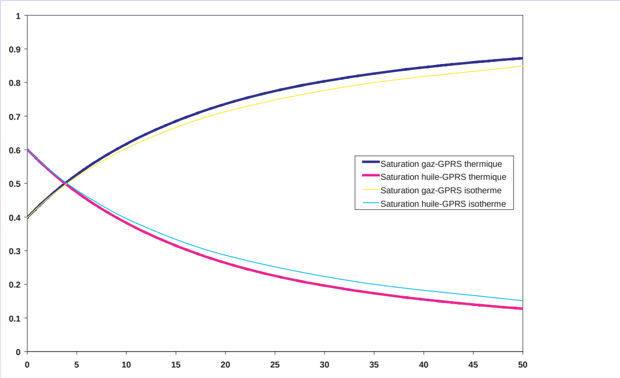


(t) Oil production rate (STB/DAY)

Comparison of pressure and saturations at the well block



(u) Pressure in *psia*

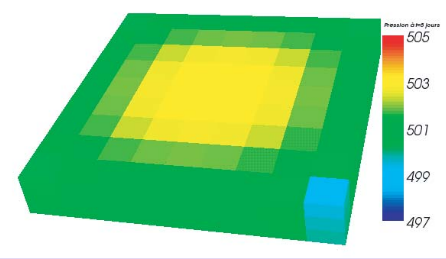


(v) Saturations of oil and gas phases

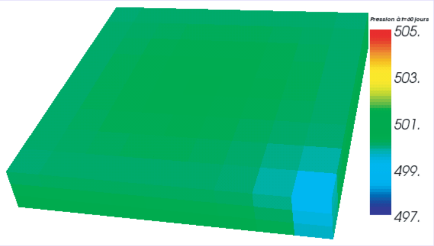
Production of gas for 90 days
Sensibility via boundary conditions

- **Reservoir with dimensions** $9000ft \times 9000ft \times 30ft$
- **Two components: methan** CH_4 **and butan** C_4H_{10}
- **Production for 90 days by imposing constant gas flow rate at the well**

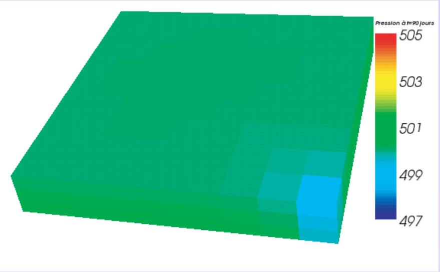
Behaviour of the pressure by imposing constant pressure on the exterior boundary



(a) Pressure at t=5 days

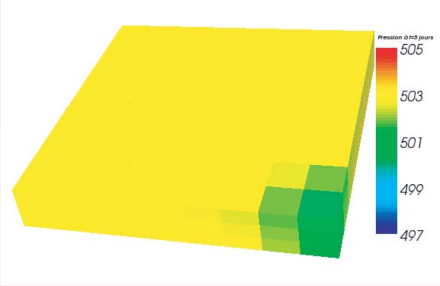


(b) Pressure at t=60 days

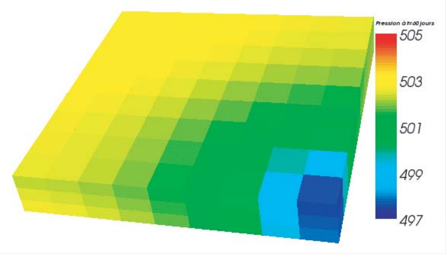


(c) Pressure at t=90 days

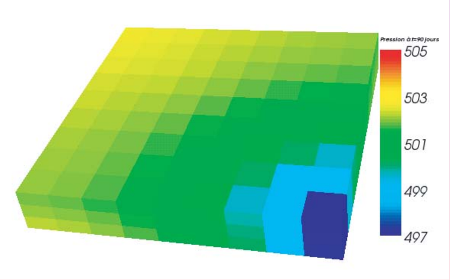
Behaviour of the pressure by imposing no flow on the exterior boundary



(d) Pressure at t=5 days

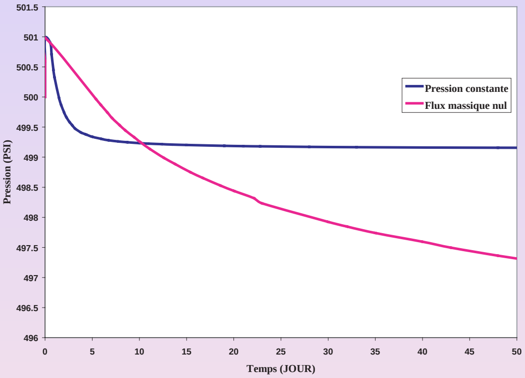


(e) Pressure at t=60 days

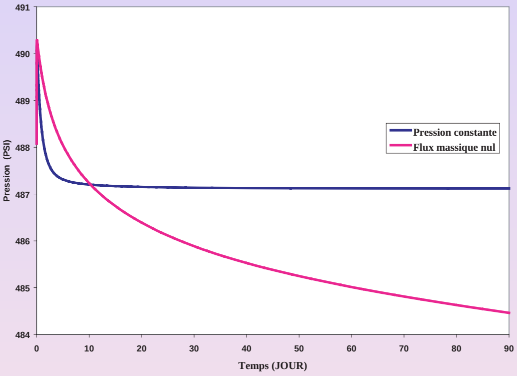


(f) Pressure at t=90 days

Comparison of pressures in the well block and in the well



(g) Pressures in the well block



(h) Pressures in the well

Thank you for your attention