MODELLING OF THERMAL DISPERSION IN HEATED PIPES

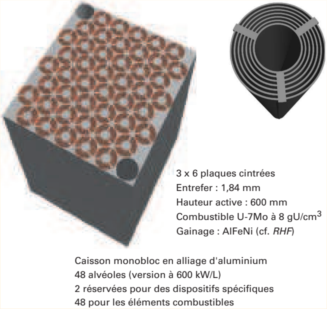
Marie DROUIN [1,2]
Olivier GRÉGOIRE [1] Olivier SIMONIN [2] Augustin CHANOINE [1]

[1] CEA Saclay, DEN/DANS/DM2S/SFME/LETR, 91191 Gif-sur-Yvette, France

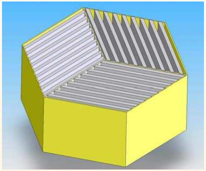
[2] IMFT, UMR CNRS/INP/UPS, Allée du Professeur Camille Soula, 31400 Toulouse, France

Scaling Up and Modeling for Transport and Flow in Porous Media
Dubrovnik, Croatia, 13-16 October 2008

CONTEXT: HEAT EXCHANGERS, NUCLEAR REACTORS



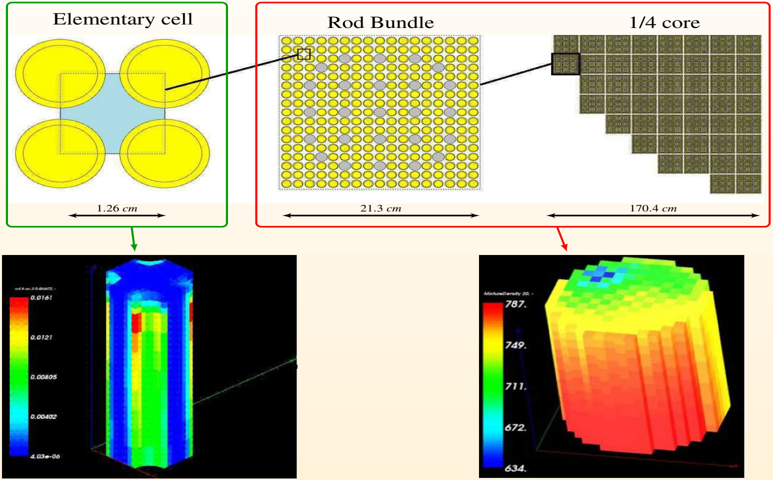
Jules Horowitz Reactor (JHR)



Fast Breeder Reactor (FBR)

Pressurized Water Reactor (PWR)

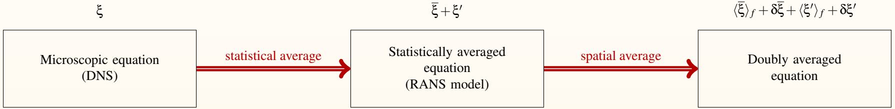
UP-SCALING

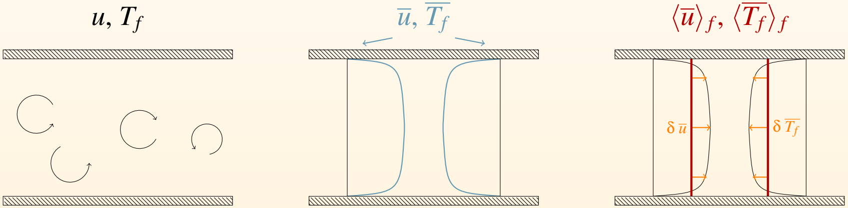


Subchannel fine scale simulation

Macroscale simulation

AVERAGING PROCEDURE





Instantaneous microscopic scale | Statistically averaged microscopic scale | Macroscopic scale

Pedras and De Lemos (*IJHMT*, 2001), Quintard and Whitaker (Transport Porous Med., 1994) A B

HYPOTHESIS

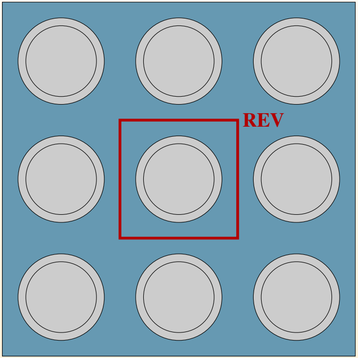
Properties of the flow

- Incompressible flows;
- Constant fluid properties;
- Laminar to high Reynolds number ($Re \sim 10^6$) flows;
- Velocity no-slip condition at the wall.

Properties of the media

- Stratified (flow along the z-axis);
- Spatially periodic;
- The porosity is constant;

$\Longrightarrow$ Heat exchanger study is reduced to a unit cell study.



Statistically averaged temperature equation

Microscopic temperature balance equation

$$\frac{\partial T_f}{\partial t} + \frac{\partial (T_f u_i)}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\alpha_f \frac{\partial T_f}{\partial x_i}\right) + 2\,\alpha_f\ Pr\ \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j} + \frac{Q}{(\rho C_p)_f},$$

Boundary condition on the wall: $\alpha_f \frac{\partial T_f}{\partial x_i} n_i = \frac{\Phi}{(\rho C_p)_f}$.

Satistically averaged temperature equation

$$\frac{\partial \overline{T}_f}{\partial t} + \frac{\partial}{\partial x_i}\left(\bar{u}_i \overline{T}_f\right) = \frac{\partial}{\partial x_i}\left(\alpha_f \frac{\partial \overline{T}_f}{\partial x_i}\right) - \frac{\partial}{\partial x_i} \underbrace{\overline{u_i' T_f'}}_{\text{turbulent heat flux}} + \frac{\overline{Q}}{(\rho C_p)_f},$$

Boundary condition on the wall: $\alpha_f \frac{\partial \overline{T}_f}{\partial x_i} n_i = \frac{\overline{\Phi}}{(\rho C_p)_f}$.

STATISTICALLY AVERAGED TEMPERATURE EQUATION

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Boundary condition on the wall: $\alpha_f \frac{\partial \overline{T}_f}{\partial x_i} n_i = \frac{\overline{\Phi}}{(\rho C_p)_f}.$

where: $-\overline{u_i' T_f'} = \alpha_t \frac{\partial \overline{T}_f}{\partial x_i} = \frac{\nu_t}{Pr_t} \frac{\partial \overline{T}_f}{\partial x_i}.$

SPATIALLY AVERAGED EQUATION OF THE TEMPERATURE

Satistically and spatially averaged temperature equation

$$\frac{\partial \langle \overline{T}_f \rangle_f}{\partial t} + \frac{\partial}{\partial x_i} \langle \bar{u}_i \rangle_f \langle \overline{T}_f \rangle_f = -\frac{\partial}{\partial x_i} \langle \overline{u_i' T_f'} \rangle_f + \frac{\partial}{\partial x_i} \left(\alpha_f \frac{\partial \langle \overline{T}_f \rangle_f}{\partial x_i} \right) + \frac{\langle \overline{Q} \rangle_f}{(\rho C_p)_f}$$

$$+ \underbrace{\frac{\langle \overline{\Phi}\, \delta_\omega \rangle_f}{(\rho C_p)_f}}_{\text{Wall heat transfer}} + \underbrace{\frac{\partial}{\partial x_i} \langle \alpha_f\, \delta \overline{T}_f\, n_i \delta_\omega \rangle_f}_{\text{Tortuosity}} \underbrace{- \frac{\partial}{\partial x_i} \langle \delta \bar{u}_i\, \delta \overline{T}_f \rangle_f}_{\text{Thermal dispersion}}$$

where: $-\langle \overline{u_i' T_f'} \rangle_f \overset{def}{=} \alpha_{t\phi} \frac{\partial \langle \overline{T}_f \rangle_f}{\partial x_i}.$

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- For flows in flat plates, circular or annular pipes, the tortuosity contributions are zero.
- We focus on the analysis and modelization of the dispersion term.

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- For flows in flat plates, circular or annular pipes, the tortuosity contributions are zero.
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ANALYSIS AND MODELIZATION OF THE DISPERSION TERM

- Closure relationship (Carbonell and Whitaker, 1984):

$$\delta \overline{T}_f = \eta_j \frac{\partial \langle \overline{T}_f \rangle_f}{\partial x_j} + \zeta \frac{\langle \overline{\Phi} \delta_\omega \rangle_f}{(\rho C_p)_f},$$

where δ_ω is the Dirac function associated to the wall;

Dispersion term

$$-\frac{\partial}{\partial x_i} \langle \delta \bar{u}_i \, \delta \overline{T}_f \rangle_f = \underbrace{\frac{\partial}{\partial x_i} \left(-\langle \delta \overline{u_i} \, \eta_j \rangle_f \frac{\partial \langle \overline{T}_f \rangle_f}{\partial x_j} \right)}_{\text{passive dispersion}} + \underbrace{\frac{\partial}{\partial x_i} \left(-\langle \delta \overline{u_i} \, \zeta \rangle_f \frac{\langle \overline{\Phi} \delta_\omega \rangle_f}{(\rho C_p)_f} \right)}_{\text{active dispersion}}$$

- Passive dispersion: additionnal macroscopic diffusion term related to the velocity spatial heterogeneities;
- Passive dispersion tensor: $\mathcal{D}^P_{ij} = -\langle \delta \overline{u_i} \, \eta_j \rangle_f$.
- Active dispersion: related to the wall heat flux;
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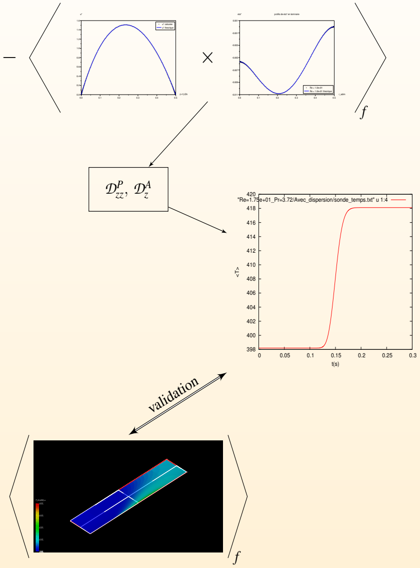
MODELLING APPROACH

- Fine-scale simulations in a unit cell **(1D in a section)** $\rightarrow \overline{u}, \eta, \zeta$;
- Local results are spatially averaged over the unit cell $\rightarrow \mathcal{D}^P, \mathcal{D}^A$;
- A macroscale model is proposed;
- Validation:

Macroscale model ▸ **1D simulation along the $z-$axis**	3D fine-scale simulations ▸ **FLICA-OVAP**



FINE-SCALE SIMULATIONS IN A UNIT CELL

- Spatially periodic porous media:
  - ⇒ Heat exchanger study is reduced to a unit cell study;
- Steady flow along the z axis in plane channels, circular or annular pipes:
  - ⇒ Tortuosity terms are zero,
  - ⇒ We only need to know $\mathcal{D}_{zz}^{P}$ and $\mathcal{D}_{z}^{A}$;
- Simulation results for turbulent flows obtained thanks to $\overline{k}-\overline{\epsilon}$ Chien model;

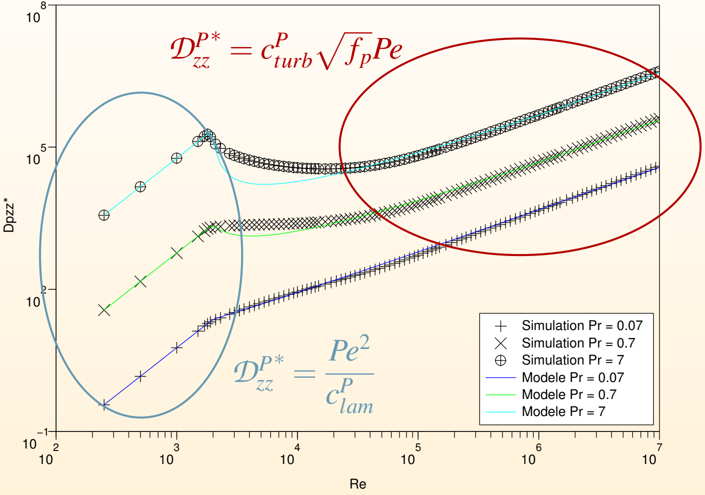
  ▸ Chien model
- Dimensionless formulation:

$$\mathcal{D}_{zz}^{P\,*} = \frac{\mathcal{D}_{zz}^{P}}{\alpha_f}, \quad \mathcal{D}_{z}^{A*} = \frac{\mathcal{D}_{z}^{A}}{D_h}$$

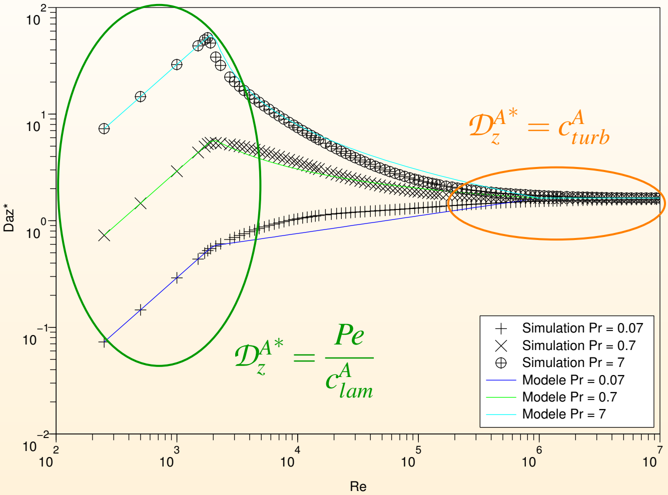
- Péclet number:

$$Pe = Re \times Pr.$$

PASSIVE DISPERSION MODEL

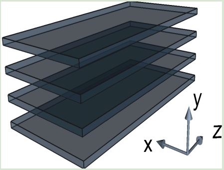


ACTIVE DISPERSION MODEL



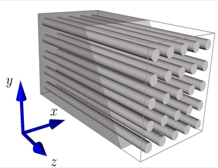
Numerical results obtained for plane channels, circular and annular pipes

Plane channels



$$c^P_{lam} = 840 \,,$$
$$c^A_{lam} = 240 \,,$$
$$c^P_{turb} = 0.62 \,,$$
$$c^A_{turb} = 1.63 \,.$$

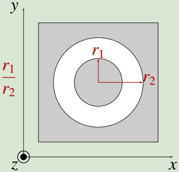
Circular pipes



$$c^P_{lam} = 192 \,,$$
$$c^A_{lam} = 96 \,,$$
$$c^P_{turb} = 1.1 \,,$$
$$c^A_{turb} = 2.1 \,.$$

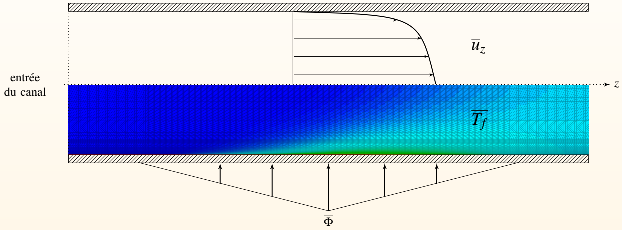
Annular pipes

$$q = \frac{r_1}{r_2}$$



$$c^P_{lam} = 255.2 - 341.5(q^2 - 2q) + 1263.8\left[\ln(1+q) - \frac{q}{2}\right] \,,$$
$$c^P_{turb} = 1.292 - 0.362(q^2 - 2q) - 5.367\left[\ln(1+q) - \frac{q}{2}\right] \,,$$
$$c^A_{lam} = 107.0 - 33.5(q^2 - 2q) + 862.4\left[\ln(1+q) - \frac{q}{2}\right] \,,$$
$$c^A_{turb} = 2.16 - 0.715(q^2 - 2q) - 6.45\left[\ln(1+q) - \frac{q}{2}\right] \,.$$

Wall heat flux heterogeneities



Data

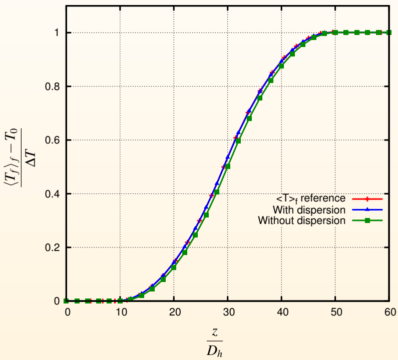
- Plane channel;
- $Pr = 0.74$, $L = 60D_h = 6\ m$;
- $\langle T_f \rangle_f (z=0) = T_0$,
  $\langle T_f \rangle_f (z=L) = T_1$;
- Φ is such that:
  $T_1 - T_0 = \Delta T = 10$;

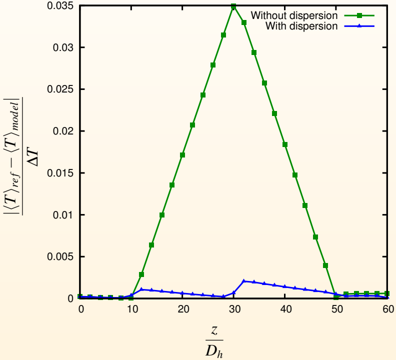
Different Reynolds numbers

- Laminar regime:
  $Re = 175$;
- Intermediate turbulent regime:
  $Re = 7.6 \times 10^4$;
- Asymptotic turbulent regime:
  $Re = 1.14 \times 10^6$.

WALL HEAT FLUX HETEROGENEITIES

- Laminar regime:

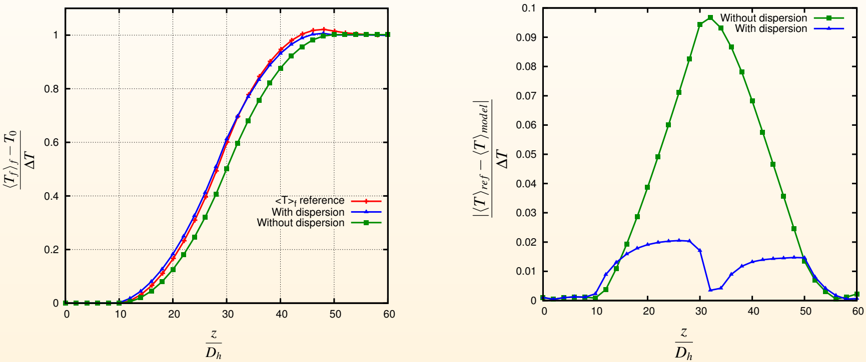




- Passive dispersion is negligeable;
- Wall heat flux heterogeneities ⇒ Active dispersion effects;
- Active dispersion neglected ⇒ $\langle \overline{T}_f \rangle_f$ underestimated.

WALL HEAT FLUX HETEROGENEITIES

- Intermediate turbulent regime:



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WALL HEAT FLUX HETEROGENEITIES

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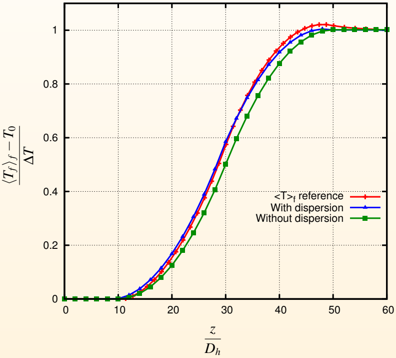
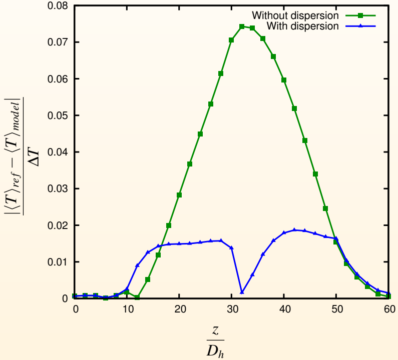
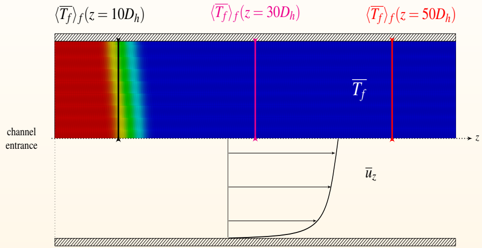
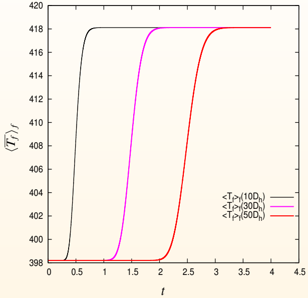




- Passive dispersion is negligeable;
- Wall heat flux heterogeneities $\Rightarrow$ Active dispersion effects;
- Active dispersion neglected $\Rightarrow$ $\langle \overline{T}_f \rangle_f$ underestimated.

EVOLUTION OF A TEMPERATURE JUMP





Data

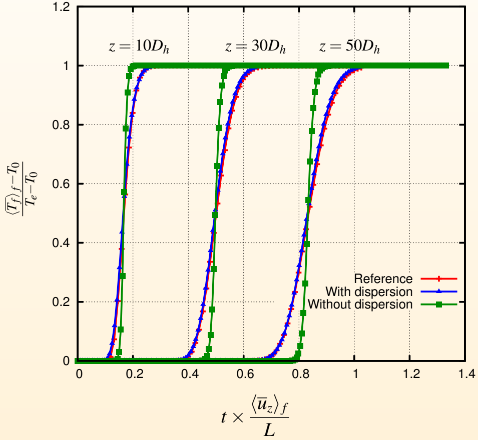
- Plane channel;
- $Pr = 0.74$;
- $L = 60D_h = 6\ m$;
- $\langle \overline{T}_f \rangle_f(t=0,z) = T_0 = 398.2$;
- $\langle \overline{T}_f \rangle_f(t,z=0) = T_e = 418.11$;

Different Reynolds numbers

- Laminar regime: $Re = 175$;
- Intermediate turbulent regime: $Re = 7.6 \times 10^4$;
- Asymptotic turbulent regime: $Re = 1.14 \times 10^6$.

EVOLUTION OF A TEMPERATURE JUMP

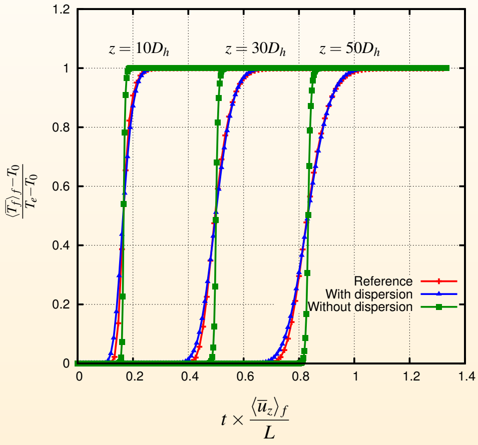
- Laminar regime:



- No wall heat flux
  $\Rightarrow$ No active dispersion effects;
- Passive dispersion: additional macroscopic diffusion term.

Evolution of a temperature jump

- Intermediate turbulent regime:



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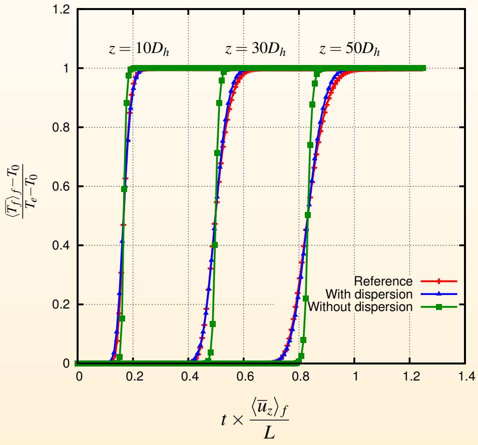
EVOLUTION OF A TEMPERATURE JUMP

- Asymptotic turbulent regime:



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CONCLUSION

- Double averaging procedure puts forward dispersion terms;
- A model is proposed for dispersion in pipes, we relate dispersion coefficients to the Peclet number and the friction coefficent;
- Macroscopic model with dispersion terms gives satisfactory results for steady and transient flows in plane channel, circular and annular pipes;
- Simulations show the importance of dispersion effects for heated flows in pipes with wall heat flux hetrogeneities or temperature jumps;

THANK YOU FOR YOUR ATTENTION

Appendix A: Averages

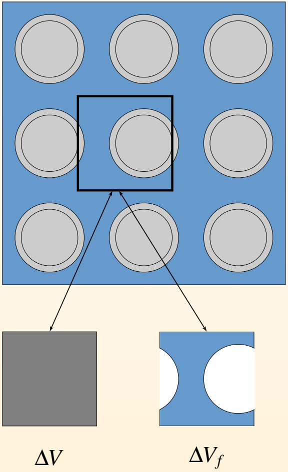
- Classical statistical average: $\xi = \overline{\xi} + \xi'$.
- The spatial average of $\overline{\xi}$ is defined by:

$$\langle \overline{\xi} \rangle_f (\mathbf{x}, t) = \frac{1}{\Delta V_f(\mathbf{x})} \int_{\Delta V_f(\mathbf{x})} \overline{\xi}(\mathbf{y}, t) dV_y .$$

- The spatial decomposition reads:

$$\overline{\xi} = \langle \overline{\xi} \rangle_f + \delta \overline{\xi}.$$

- Porosity: $\phi = \Delta V_f / \Delta V$ where ΔV is the volume of the Representative Elementary Volume (REV).



Back

Statistical average

The statistical average follows the Reynolds axioms:

Linearity $\overline{\lambda\xi+\psi} = \lambda\bar{\xi}+\bar{\psi}$ if λ is a constant;

Idempotence $\bar{\bar{\xi}} = \bar{\xi} \Leftrightarrow \overline{\xi'} = 0$;

Commutative property with the differential operators $\overline{\dfrac{\partial\xi}{\partial t}} = \dfrac{\partial\bar{\xi}}{\partial t}, \quad \overline{\dfrac{\partial\xi}{\partial x_i}} = \dfrac{\partial\bar{\xi}}{\partial x_i}.$

Spatial average

- Linearity;
- $\phi\langle\dfrac{\partial\xi}{\partial x_i}\rangle_f = \dfrac{\partial\phi\langle\xi\rangle_f}{\partial x_i} + \phi\langle\xi\, n_i\, \delta_\omega\rangle_f$;
- $\phi\langle\dfrac{\partial\xi}{\partial x_i}\rangle_f = \phi\dfrac{\partial\langle\xi\rangle_f}{\partial x_i} + \phi\langle\delta\xi\, n_i\, \delta_\omega\rangle_f$.

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Appendix C: $\overline{k}-\overline{\epsilon}$ Chien model (Chien, *AIAA J.*, 1982)

$$\begin{cases} \dfrac{\partial \overline{k}}{\partial t} + \overline{u}_j \dfrac{\partial \overline{k}}{\partial x_j} = -\overline{u_i' u_j'} \dfrac{\partial \overline{u}_i}{\partial x_j} + \dfrac{\partial}{\partial x_j}\left[\left(\nu_f + \frac{\nu_t}{\sigma_k}\right)\dfrac{\partial \overline{k}}{\partial x_j}\right] - \overline{\epsilon} - \overline{\epsilon}_p, \\ \dfrac{\partial \overline{\epsilon}}{\partial t} + \overline{u}_j \dfrac{\partial \overline{\epsilon}}{\partial x_j} = -C_{\epsilon_1} f_1 \frac{\overline{\epsilon}}{\overline{k}} \overline{u_i' u_j'} \dfrac{\partial \overline{u}_i}{\partial x_j} + \dfrac{\partial}{\partial x_j}\left[\left(\frac{\nu_t}{\sigma_\epsilon} + \nu_f\right)\dfrac{\partial \overline{\epsilon}}{\partial x_j}\right] - C_{\epsilon_2} f_2 \frac{\overline{\epsilon}^2}{\overline{k}} - \overline{E}_p. \end{cases}$$

$$\begin{cases} y^+ = \dfrac{y_w u_f}{\nu_f}, & f_\mu = 1 - \exp(-0.0115 y^+), \\ f_1 = 1, & f_2 = 1 - 0.22 \exp(-{Re_t}^2/36), \\ \overline{\epsilon}_p = 2\nu_f\left(\dfrac{\overline{k}}{{y_w}^2}\right), & \overline{E}_p = 2\nu_f\left(\dfrac{\overline{\epsilon}}{{y_w}^2}\right) exp(-y^+/2), \end{cases}$$

$$C_\mu = 0.09, \quad C_{\epsilon_1} = 1.35, \quad C_{\epsilon_2} = 1.8, \quad \sigma_k = 1, \quad \sigma_\epsilon = 1.3,$$

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Model proposed for $\mathcal{D}_{zz}^{P\ *}$

$$\mathcal{D}_{zz}^{P\ *} = \begin{cases} \dfrac{Pe^2}{c_{lam}^P} & \text{if } Re < 2000, \\[2ex] \dfrac{aRe^m}{b + Re^p} & \text{if } 2000 < Re < 10^6, \\[2ex] c_{turb}^P \sqrt{f_p} Pe & \text{if } Re > 10^6, \end{cases}$$

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Appendix E: Active dispersion

Model proposed for $\mathcal{D}_z^{A*}$

$$\mathcal{D}_z^{A*} = \begin{cases} \dfrac{Pe}{c^A_{lam}} & \text{if } Re < 2000, \\ \dfrac{a}{b + Re^p} + m(Pr - Pr_0) & \text{if } 2000 < Re < 10^6, \\ c^A_{turb} & \text{if } Re > 10^6, \end{cases}$$

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APPENDIX F: ANNULAR PIPE (1/4)

We look for dispersion coefficients of the form:

$$\mathcal{D}_{zz}^{P\,*} = \frac{Pe^2}{c_{lam}^P(q)}, \quad \mathcal{D}_z^{A*} = \frac{Pe}{c_{lam}^A(q)},$$

with:

$$c_{lam}^{P,A}(q) = a_1^{P,A} + a_2^{P,A}(q^2 - 2q) + a_3^{P,A}\left(\ln(1+q) - \frac{q}{2}\right),$$

Approximate dispersion coefficients for laminar flows in annular pipes

$$\mathcal{D}_{zz}^{P\,*} = \frac{Pe^2}{255.2 - 341.5(q^2 - 2q) + 1263.8\left(\ln(1+q) - \frac{q}{2}\right)},$$

$$\mathcal{D}_z^{A*} = \frac{Pe}{107.0 - 33.5(q^2 - 2q) + 862.4\left(\ln(1+q) - \frac{q}{2}\right)}.$$

Passive dispersion coefficient:

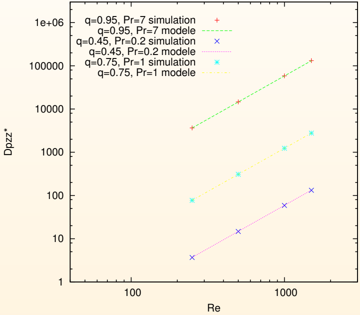

Figure: *Laminar flow in annular pipes: $\mathcal{D}_{zz}^{P\,*}$ vs Re for several q et Pr. Our model matches numerical solutions.*

Active dispersion coefficient:

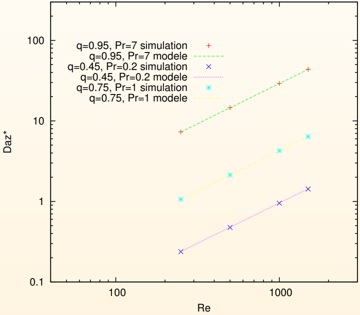

Figure: *Laminar flow in annular pipes: $\mathcal{D}_{z}^{A*}$ vs Re for several q et Pr. Our model matches numerical solutions.*

Appendix F: Annular pipe (3/4)

We look for dispersion coefficients of the form:

$$\lim_{Re\to\infty} \mathcal{D}_{zz}^{P\,*} = c_{turb}^{P}(q)\ \sqrt{f_p}\ Pe, \quad \lim_{Re\to\infty} \mathcal{D}_{z}^{A*} = c_{turb}^{A}(q).$$

with:

$$c_{lam}^{P,A}(q) = a_1^{P,A} + a_2^{P,A}(q^2 - 2q) + a_3^{P,A}\left(\ln(1+q) - \frac{q}{2}\right),$$

Approximate dispersion coefficients in annular pipes for high Reynolds numbers

$$\mathcal{D}_{zz}^{P\,*} = \left\{1.292 - 0.362(q^2 - 2q) - 5.367\left[\ln(1+q) - \frac{q}{2}\right]\right\} \sqrt{f_p} Pe,$$

$$\mathcal{D}_{z}^{A*} = 2.16 - 0.715(q^2 - 2q) - 6.45\left[\ln(1+q) - \frac{q}{2}\right].$$

Appendix F: Annular pipe (4/4)

Passive dispersion coefficient:

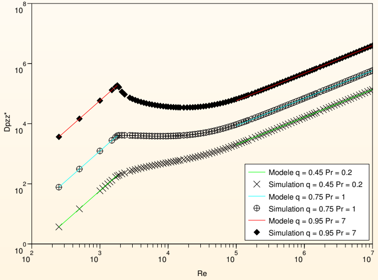

Figure: *Flow in annular pipes: $\mathcal{D}_{zz}^{P\,*}$ vs Re for several q et Pr. Our model matches numerical solutions.*

Active dispersion coefficient:

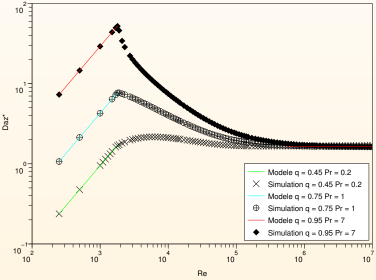

Figure: *Flow in annular pipes: $\mathcal{D}_{z}^{A*}$ vs Re for several q et Pr. Our model matches numerical solutions.*

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