REPRESENTATION THEORY XVIII

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Organizers:

Dražen Adamović, Tomoyuki Arakawa, Andrej Dujella,
Matija Kazalicki, Hrvoje Kraljević, Antun Milas,
Filip Najman, Pavle Pandžić, Mirko Primc, David Vogan

ABSTRACTS OF TALKS

Real Groups Section

There is no such thing as an Arthur Packet

Jeffrey Adams, University of Maryland

In the 1980s Jim Arthur made a series of deep conjectures concerning automorphic forms and the trace formula. In the case of a reductive group over a local field, he conjectured the existence of certain sets of unitary representations, generalizing the (conjectural) L-packets of Langlands. Although these conjectures have had an enormous influence on the field, they remain open except in some special cases (in the p-adic case the same holds for L-packets). There has been a lot of progress, by a number of authors, on these conjectures in the past few years. I will (try to) give an overview of recent work, focussing on the case of real groups.

The title refers to the fact that, although the term "Arthur Packet" is widely used, there are various definitions of these in different settings, and no clear general picture has emerged. This is especially true of the special case of "Unipotent Arthur Packets" and the corresponding "Unipotent Representations", which continue to be the subject of intensive study.

Noncubic Dirac operators for finite-dimensional modules

Spyros Afentoulidis, Charles University, Prague

Dirac operators in Representation Theory were used by Parthasarathy in 1972 as invariant first order differential operators acting on sections of homogeneous vector bundles over symmetric spaces G/K such that their squares give a sort of Laplacian. Parthasarathy used these operators to obtain realizations of an important family of G-representations, the discrete series representations.

In the late 90's, Kostant suggested a similar Dirac operator, called Kostant cubic Dirac operator, in the case of more general homogeneous spaces G/H having the same good properties as Parthasarthy's one. Kostant cubic Dirac operator turns out to be a particular member of a family of Dirac operators introduced by Slebarski in a series of papers in the late 80's and, provided that G/H is not symmetric, it is the only member of this family which squared gives the Laplacian.

In a series of lectures in MIT in 1997, Vogan introduced an algebraic analogue of Parthasarthy's Dirac opearator, defined a Dirac cohomology of irreducible $(\mathfrak{g}, K)$-modules X using this operator and conjectured a relation between the Dirac comology of X and its infinitesimal character, proved by Huang and Pandzic in 2001.

In this talk, once the above notions are developed, we define algebraic analogues for the other operators of Slebarski's family. Studying these algebraic Dirac operators, we will deduce results for the corresponding geometric ones.

This work is part of my Phd thesis supervised by Prof. Salah Mehdi at the University of Lorraine.

Jacquet Tensors

Dubravka Ban, Southern Illinois University

Let G be a split reductive p-adic group. The category of admissible p-adic Banach space representations of G is equivalent to the corresponding category of finitely generated Iwasawa modules, via the duality map $V \mapsto V'$. The parabolic induction has a nice description on the dual side. In this talk, we will define certain tensors on Iwasawa modules, which are intended to play the role of Jacquet modules. We will describe some properties of Jacquet tensors and show how they can be applied to the study of principal series representations.

Generalisations of Dirac operators for graded affine Hecke algebras

Kieran Calvert, University of Manchester

Starting with an outline of Dirac operators for graded affine Hecke algebras, I will highlight a couple of their best features; Dirac cohomology and Vogan's morphism. Building on these, I will define a family of operators that

exhibit these features. Under a small condition on the Weyl group, the full unitary dual of a graded affine Hecke algebra is covered by the 'Dirac series' for this family of operators.

On the Lefschetz principle for $GL(n, \mathbb{C})$ and $GL(n, \mathbb{Q}_p)$

Kei Yuen Chan, University of Hong Kong

The Harish-Chandra Lefschetz principle predicts what holds for real groups is also true for p-adic groups. For instance, Barbasch establishes a correspondence between K–types for real groups with W–types for p–adic groups to match up the actions of intertwining operators.

Building on work of Arakawa-Suzuki and Ciubotaru-Trapa, we construct an exact functor from the category of Harish-Chandra modules of $GL(n, \mathbb{C})$ to the category of finite-dimensional modules of graded Hecke algebra of type A. This functor preserves parabolically induced modules, standard modules, irreducible modules, unitary modules and Dirac series. It also links a Bernstein-Zelevinsky type functor to tensor decomposition problems. As an application, we generalize some particular instances of the classical tensor product for finite-dimensional representations to some Harish-Chandra modules.

The talk is based on the preprint arXiv:2305:15766, joint with Daniel Wong.

Transgression in the relative Weil algebra

Karmen Grizelj, University of Zagreb

The transgression map is an effective tool for obtaining primitive invariants. For a semisimple Lie algebra $\mathfrak{g}$ and its Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ the image of the transgression is the space of primitive $\mathfrak{k}$-invariants in the exterior algebra of $\mathfrak{p}$. This is a part of the transgression theorem which will be proved in the case of the spin of the adjoint representation being primary. Also, an explicit formula for the transgression will be presented and explained in an example.

Schubert calculus for irreducible quantum flag manifolds

Andrey Krutov, Institute of Mathematics, Czech Academy of Sciences

Schubert calculus is a remarkable area of mathematics which studies the de Rham cohomology of generalised flag manifolds from a combinatorial point of view. The cohomologies of classical irreducible flag manifolds have remarkable ring strucutre which is closely related to representation theory. For example, in the case of Grassmanianns the multiplication table is given by Littelewood–Richardson coefficients. In this talk we will discuss Schubert calculus for for irreducible quantum flag manifolds, in particular quantum Grassmaninanns, from the point of view of representation theory and will keep track its connection with invariant theory and (quanutm) Howe duality. This is joint work with R. Ó Buachalla (Prague).

Quantizations and Harish-Chandra modules

Ivan Losev, Yale

I will discuss an approach to classification of certain irreducible Harish-Chandra modules (that should be related to unitary representations) as quantizations of singular lagrangian subvarieties in singular symplectic varieties. The approach allows to classify these Harish-Chandra modules in terms of some basic geometric data assuming, roughly speaking, that the lagrangian subvariety in question is not too singular. This is based on some of my solo work, 1605.00592, 1810.07625, the joint monograph with Mason-Brown and Matvieievskyi, 2108.03453, and my work in preparation with Shilin Yu.

The Unitary Dual of a Nilpotent Cover

Lucas Mason-Brown, Oxford

Let G be a connected reductive algebraic group and let $\tilde{O}$ be a finite G–equivariant cover of a nilpotent co-adjoint G–orbit. The filtered quantizations of the ring of regular functions $C[\tilde{O}]$ are relatively well-understood—they are parameterized, up to isomorphism, by orbits of finite group W acting on a finite-dimensional vector space P. Under some mild conditions on $\tilde{O}$, it is reasonable to talk about "unitary" quantizations of $C[\tilde{O}]$. Every unitary quantization of $C[\tilde{O}]$ gives rise to an irreducible unitary spherical representation of the complex group G, and examples suggest that every such representation arises in this fashion (for some $\tilde{O}$). So determining the unitary dual of $\tilde{O}$ seems to be a key step in determining the unitary dual of G. In ongoing work with Ivan Losev and Dmytro Matvieievskyi, we show that P and W are the Cartan subalgebra and Weyl group of a complex connected reductive algebraic group $G(\tilde{O})$ which can be recovered from the geometry of $\mathrm{Spec}(C[\tilde{O}])$. I will sketch the definition of this group and explain how it can help us compute the unitary dual of $\tilde{O}$.

Extended duality and unipotent ideals

Dmytro Matvieievskyi, Kavli IPMU

Let G be a complex reductive algebraic group. In a joint work with Ivan Losev and Lucas Mason-Brown we proposed a new definition of unipotent representations of G. Namely, to any G–equivariant finite cover of a nilpotent coadjoint orbit we assign a finite set of irreducible representations, annihilated by a maximal ideal uniquely defined by this cover that we call a unipotent ideal. These ideals include but are not limited to special unipotent ideals of Arthur and Barbasch-Vogan. An interesting question is to describe unipotent ideals using the data of the Langlands dual group $G^\vee$. Following ideas of Sommers and Achar, we construct a duality map D from the set of pairs $(O^\vee, C)$ consisting of a nilpotent orbit $O^\vee$ in $g^\vee$ and a conjugacy class C in the Lusztig's canonical quotient $\bar{A}(O^\vee)$ to the covers of nilpotent coadjoint G orbits. For the covers in the image we describe the corresponding unipotent ideals and parameterize unipotent representations. The talk is based on a joint project with Lucas Mason-Brown and Shilin Yu.

Recent progress on Kostant's problem

Volodymyr Mazorchuk, Uppsala University

Let $\mathfrak{g}$ be a semi-simple complex finite dimensional Lie algebra. Kostant's problem for a $\mathfrak{g}$-module L asks whether the universal enveloping algebra of $\mathfrak{g}$ surjects onto the algebra of all locally $\mathrm{ad}(\mathfrak{g})$–finite endomorphisms of L. Although the answer to Kostant's problem is known for some special classes of modules (for example, the answer is positive for all Verma modules), no complete answer is known, for example, for simple highest weight modules.

In this talk I will describe some recent progress in understanding the answer to Kostant's problem for simple highest weight modules indexed by fully commutative permutations and for some parabolic Verma modules. Based on a joint work with Marco Mackaay and Venessa Miemietz and another joint work with Shraddha Srivastava.

Quiver representations and double flag varieties

Kyo Nishiyama, AGU

Let G be a connected reductive algebraic group.

Magyar-Weyman-Zelevinsky used Kac's theory of quiver representations to determine if a multiple flag variety $\prod_{i=1}^{N} G/P_i$ (P_i : parabolic subgroup of G) has finitely many G-orbits, and finally they succeeded in classification.

Recently Hiroki Homma introduced a Joint variety of quiver representations to study finiteness of double flag varieties of type AIII. His theory almost settled down the classification of double flag varieties of finite type, however, it is difficult to read off the actual classification.

In this talk, we will explain what is the key ideas to relate the quiver representations with the orbits on double flag varieties after Magyar-Weyman-Zelevinsky and Homma. We finally classifies all the double flag varieties of finite type for symmetric pairs of type AIII. We also classify the orbits on a double flag variety using quivers, and give dimension formulas, closure relations in terms of the representation of quivers.

The talk is based on the on-going joint work with Lucas Fresse in Université de Lorraine, IECL (France).

Ind-varieties of generalized flags: a brief survey

Ivan Penkov, Jacobs University

Ind-varieties of generalized flags are ind-varieties of the form G/P for the ind-group $G = Gl(\infty)$. We will recall the definition and will give a purely algebraic-geometric characterization of these ind-varieties. Then we will describe their automorphism groups: in contrast with the finite-dimensional case, these automorphism groups are much larger than $Gl(\infty)$. We will complete the talk by showing that locally reductive ind-groups nonisomorphic to $Gl(\infty)$ can act transitively on certain ind-varieties of generalized flags.

The PRV product of irreducible unitary highest weight $(\mathfrak{g}, K)$–modules

Ana Prlić, University of Zagreb

Let G be a connected simply connected noncompact simple Lie group with Cartan involution Θ and let K be the subgroup of elements fixed by Θ. Let $\mathfrak{g}_0 = \mathfrak{k}_0 \oplus \mathfrak{p}_0$ be the Cartan decomposition of the Lie algebra of G corresponding to Θ. We drop the subscript 0 to denote the complexifications. We assume that (G, K) is a Hermitian symmetric pair. Then the K-module $\mathfrak{p}$ decomposes into two irreducible submodules, $\mathfrak{p} = \mathfrak{p}^+ \oplus \mathfrak{p}^-$, and each of $\mathfrak{p}^\pm$ is an abelian subalgebra of $\mathfrak{g}$.

Let V_1 and V_2 be irreducible unitary highest weight $(\mathfrak{g}, K)$–modules with top K–types F_1 and F_2. If F is the PRV component of the tensor product $F_1 \otimes F_2$, we shall call the module $U(p^-)F$ the PRV product of V_1 and V_2 and we denote it by $V1 \bullet V2$. In this talk, we will explain how this product can be used for the construction of the unitary highest weight modules in the discrete part of the classification of Enright, Howe and Wallach. This is joint work with Pavle Pandžić, Vladimir Souček and Vit Tuček.

Exceptional theta correspondences for real groups

Gordan Savin, University of Utah

Basic properties of classical theta correspondence, such as admissibility of the lift and uniqueness of its quotient, are proved using a see-saw dual pair and some elementary properties of highest weight modules. Gross and Wallach introduced quaternionic representations as closest to highest weight modules in a certain sense. I will explain an interesting property of quaternionc representations and how to use it, again via a see-saw, to establish basic properties of exceptional theta correspondences.

Matching real and p–adic Kazhdan-Lusztig polynomials

Peter E. Trapa, University of Utah

Let G be a complex reductive algebraic group, and let s denote a semisimple element in its Lie algebra. The centralizer L of s in G acts with finitely many orbits on the eigenspaces of $ad(s)$. The singularities of these orbit closures (encoded in p-adic Kazhdan-Lusztig polynomials, i.e. local intersection cohomology Poincare polynomials) determines the multiplicities of irreducible subquotients in standard induced modules for unramified, and more generally unipotent, representations of p-adic forms of the dual group of G. Meanwhile, if K is a symmetric subgroup of G, the singularities of the K orbit closures in the flag variety of G, encoded in "real" Kazhdan-Lusztig-Vogan polynomials, determines the multiplicities of irreducible subquotients in standard modules for real forms of the dual group of G. Our main result describes a natural hypothesis, which is always satisfied for the classical subgroups of $GL(n)$, that implies that the p-adic polynomials are a subset of the real polynomials. In turn, this matches certain multiplicities in standard modules for real and p-adic groups. This is joint work with Leticia Barchini.

Some genuine small representations of a covering group and their wavefront sets

Wan-Yu Tsai, CYCU, Taiwan

In this talk, we will study some genuine small representations of a nonlinear covering group and the algebraic properties that characterize them. One way to describe these representations is the orbit method. We then discuss a formula used to compute the wavefront set in the real group setting, aiming at generalizing the formula to the p–adic groups. More specifically, we conjecture that this formula can be used to compute the maximal elements in the wavefront set of a theta representation. The latter part is joint work with Fan Gao.

Generalizing endoscopy

David Vogan, MIT

The notion of endoscopic group was created by Langlands, Shelstad, and others beginning in the 1970s, in order to study problems in harmonic analysis on a reductive group G: for example, the nature of *characters* of irreducible representations of G. An endoscopic group H for G is a smaller reductive group, equipped with a natural "endoscopic transfer" map from characters of H to characters of G.

Perhaps the simplest example of such an endoscopic group is the Levi subgroup M of a rational parabolic subgroup $P = MU$. Endoscopic transfer in this case is just the Mackey-Gelfand notion of *parabolic induction* from M to G.

This example of rational parabolic induction is almost never mentioned in the literature on endoscopy, because endoscopy offers little that is new in that case. But I believe that it sheds some light on the nature of the Langlands-Shelstad theory.

I will talk about a (real groups) generalization of endoscopic groups and endoscopic transfer, for which the simplest example is the Levi subgroup L of a theta-stable parabolic; and transfer is *cohomological induction* from L to G. The formalism appears to make sense for any local field, and should lead to a definition of endoscopic transfer once an appropriate local Langlands conjecture is proved.

On the unitary dual of complex spin groups

Daniel Wong, Chinese University of Hong Kong, Shenzhen

A major unsolved problem in representation theory of reductive Lie groups is the classification of unitary representations, i.e. the unitary dual. In this talk, we will discuss some conjectures, along with some (old and new) results related to the unitary dual for real reductive groups. Afterwards, we give a classification of the unitary dual for $\mathrm{Spin}(2n, \mathbb{C})$ treated as a real group. We will highlight some tools used in the proof, and how they can be generalized to the case of $\mathrm{Spin}(2n + 1, \mathbb{C})$.

Non-integral Kazhdan-Lusztig algorithm and an application to Whittaker modules

Qixian Zhao, University of Utah

Let $\mathfrak{g}$ be a complex semisimple finite dimensional Lie algebra, and consider a category of representations of $\mathfrak{g}$ where a Kazhdan-Lusztig algorithm exists for integral regular infinitesimal characters. In this talk, we will discuss a potential approach for extending the integral algorithm to arbitrary non-integral regular infinitesimal characters, using intertwining functors. We will then apply this approach to Whittaker modules and demonstrate the non-integral algorithm there using an explicit example.

Vertex Algebra Section

Homological link invariants from Floer theory

Mina Aganagić, Berkeley

I will describe a theory which generalizes Heegard-Floer theory from $\mathfrak{gl}(1|1)$ to arbitrary Lie (super) algebras. The theory gives rise to homological invariants of links and categorifies quantum group link invariants. The corresponding category of A-branes has many special features which render it solvable explicitly. In this talk, I will describe how the theory is solved, and how homological link invariants arise from it. I will focus on the two simplest cases, the $\mathfrak{gl}(1|1)$ theory itself, and the $\mathfrak{su}(2)$ theory, categorifying respectively the Alexander and the Jones polynomials.

Hilbert Schemes of the points in the plane and quasi-lisse vertex algebras with $\mathcal{N} = 4$ symmetry

Tomoyuki Arakawa (RIMS, Kyoto University)

For each complex reflection group Γ one can attach a canonical symplectic singularity $\mathcal{M}_\Gamma$. Motivated by the 4D/2D duality discovered by Beem et at., Bonetti, Menegheli and Rastelli conjectured the existence of a supersymmetric vertex operator algebra $\mathbf{W}_\Gamma$ whose associated variety is isomorphic to M_Γ. We prove this conjecture when the complex reflection group Γ is the symmetric group S_N, by constructing a sheaf of $\hbar$-adic vertex algebras on the Hilbert schemes of N-points in the plane. In physical terms, the vertex operator algebra $\mathbf{W}_{S_N}$ corresponds, by the 4D/2D duality, to the 4-dimensional ;$N = 4$ super Yang-Mills theory with gauge group SL_N.

On the finite irreducible modules over some conformal superalgebras

Lucia Bagnoli, University of Zagreb

Finite simple conformal superalgebras were completely classified by Fattori and Kac and consist of the following list: $Cur\mathfrak{g}$, where $\mathfrak{g}$ is a simple finite−dimensional Lie superalgebra, $W_n(n \geq 0)$, $S_{n,b}$ and $\tilde{S}_n$ $(n \geq 2, b \in \mathbb{C})$, $K_n(n \geq 0,\ n \neq 4)$, K_4', CK_6. In this talk we present the classification of all the finite irreducible modules over the conformal superalgebra K_4' obtained by their correspondence with finite conformal modules over the associated annihilation superalgebra $\mathcal{A}(K_4')$. This is achieved by a complete classification of singular vectors in generalized Verma modules for $\mathcal{A}(K_4')$. We also show that morphisms between generalized Verma modules can be arranged in infinitely many bilateral complexes. This classification is a joint work with F. Caselli. Then we present a result on the homology of these complexes that provides an explicit realization of all irreducible quotients. Next, we present some recent results on the conformal superalgebra CK_6. The classification of all the finite irreducible modules over the conformal superalgebra CK_6 was obtained by Boyallian, Kac, Liberati and Martinez, Zelmanov with different techniques. We present a result on an upper bound of the degree of singular vectors of finite Verma modules over the exceptional Lie superalgebra $E(1,6)$ that is isomorphic to the annihilation superalgebra associated with the conformal superalgebra CK_6. Then we present the computation of the homology of the first and third quadrants of the complexes of finite Verma modules, that were classified by Boyallian, Kac and Liberati, over the annihilation superalgebra $\mathcal{A}(CK_6) = E(1,6)$. The computation of the homology provides an explicit realization of all irreducible quotients. Finally, we discuss some open problems.

Combinatorial properties of bases of standard modules for twisted affine Lie algebras

Marijana Butorac, Faculty of Mathematics, University of Rijeka

We consider standard modules of rectangular highest weights of twisted affine Lie algebras. In this talk will be presented vertex algebraic methods which we used in the construction of combinatorial bases of standard modules. This talk is based on a joint project with S. Kožić and M. Primc.

Quantum Groups and VOAs

Thomas Creutzig, University of Alberta

If one has a VOA and a quantum group that have categories of representation that are equivalent as abelian categories, then it is natural to wonder if this is in fact an equivalence of braided tensor categories. I will present a theory, based on joint work with S. Lentner and M. Rupert, on how to answer this question.

Chiral de Rham complex and automorphic forms

Xuanzhong Dai, Kyoto University

Chiral de Rham complex constructed by Malikov et al. in 1998, is a sheaf of vertex operator algebras on any complex manifold or nonsingular algebraic variety. For any congruence subgroup Γ, we study the vertex algebra constructed from the Γ-invariant global sections of the chiral de Rham complex on the upper half plane with certain cuspidal conditions. We exhibit a linear basis of the Γ-invariant algebra consisting of liftings of modular forms, and compute its character formula. In 1994, D. Zagier, Y. Manin, and W. Eholzer speculated that the Rankin-Cohen brackets of modular forms should be related to a vertex operator algebra. We demonstrate that the vertex operations of Γ-invariant algebras can be expressed by the Rankin-Cohen brackets.

Integrability of classical affine W-algebras

Alberto De Sole, Sapienza Università di Roma

All classical affine W-algebras $W(g, f)$, where g is a simple Lie algebra and f is its non-zero nilpotent element, admit an integrable hierarchy of bi-Hamiltonian PDEs, except possibly for one nilpotent conjugacy class in G_2, one in F_4, and five in E_8.

Coset construction and Kac-Wakimoto set

Chongying Dong, University of California at Santa Cruz

Initiated by Goddard-Kent-Oliver, coset theory studies the centralizer U^c of a vertex operator subalgebra U in a vertex operator algebra V. Motivated by the earlier work of Kac-Wakimoto on the coset constructions associated with affine vertex operator algebras, we introduce the notion of Kac-Wakimoto set consisting of irreducible V-modules which contain U^c as U^c-modules. The Kac-Wakimoto set is then used to decompose the multiplicity space of an irreducible U-module in an irreducible V-module into a direct sum of irreducible U^c-modules if both U, U^c are regular. This is a joint work with Li Ren and Feng Xu.

Rationality and orthosymplectic Feigin-Semikhatov duality

Justine Fasquel, University of Melbourne

In this talk, we present several rationality results for W-algebras associated to subregular nilpotent elements of the Lie algebra $\mathfrak{so}(2n+1)$. We also discuss how to exploit the Feigin-Semikhatov duality stated by Creutzig-Genra-Nakatsuka for orthosymplectic type together with the spectral flow actions to study the representations of the principal W-superalgebras associated with $\mathfrak{osp}(2|2n)$. This is the continuation of our joint work with Shigenori Nakatsuka (U. Alberta).

The path of hooks

Zachary Fehily, University of Melbourne

A very useful tool in the study of W-algebras are inverse reductions. Originating in the work of A.M. Semikhatov and D. Adamović, these are embeddings involving W-algebras corresponding to the same Lie algebra but different nilpotent elements. Inverse reductions can be utilised to relate the representation theory of the W-algebras involved and in many cases compute data essential to physics applications. In this talk, I will show how to construct inverse reduction embeddings amongst hook-type sl_n W-algebras using free-field realisations and screening operators. This includes the principal/regular, subregular, minimal sl_n W-algebras, and the affine sl_n vertex operator algebra.

Reduction by stages on W-algebras

Naoki Genra, University of Tokyo

Let X be a Poisson variety with a Hamiltonian G-action and H be a normal subgroup of G. Then $X//G$ is obtained by a (Hamiltonian) reduction of $X//H$ by the induced G/H-action under suitable assumptions, called reduction by stages. We apply for the Slodowy slices and show that the Slodowy slice associated to (g, O) is obtained by a reduction of the Slodowy slice associated to (g, O') for a simple Lie algebra g and nilpotent orbits O, O' such that $O > O'$ with some conditions. The quantum cases imply that the finite/affine W-algebras associated to (g, O) are obtained by W-algebras associated to (g, O'), which proves a conjecture of Morgan in finite cases and gives a conjectural generalization of results of Madsen and Ragoucy in affine cases. This is a joint work with Thibault Juillard.

PBW bases of Ising modules

Reimundo Heluani (IMPA)

We describe PBW bases of the unique three irreducible modules of the Virasoro Lie algebra with central charge c = 1/2. We use these bases to find new bi-variable character formulas for these modules and describe new Rogers-Ramanujan-type identities from them. This is a report on the thesis of Diego Salazar Gutierrez (IMPA).

Varieties of vertex operator algebras $L_k(sl_3)$ for non-admissible levels k

Cuipo Jiang, Shanghai Jiao Tong University

For $V = L_k(sl_3)$, where $k = -3 + \frac{2}{2m+1}$ is a non-admissible number, $m \in \mathbb{Z}_+$, we prove that if $m = 0$, then the associated variety $X_V = \overline{(G \cdot \mathbb{C}^*(\alpha_1 - \alpha_2)}$ with dimension 5, and if $m = 1$, then $X_V = \overline{(G \cdot (\mathbb{C}^*(\alpha_1 - \alpha_2) + f_\theta)}$ with dimension 7. For $m \geq 2$, we prove that $\mathcal{N} \subseteq X_V \subseteq \overline{(G \cdot (\mathbb{C}^*(\alpha_1 - \alpha_2) + f_\theta)}$, where $\mathcal{N}$ is the nilpotent cone of sl_3. This is a joint work with Jingtian Song.

Unitary representations of minimal W-algebras

Victor Kac, MIT

To each non-zero nilpotent orbit of a simple finite-dimensional Lie superalgebra $\mathfrak{g}$ with a non-degenerate even invariant bilinear form one associates a simple vertex algebra, called a W-algebra. In the simplest case $\mathfrak{g} = sl(2)$ one gets the Virasoro vertex algebra. For small simple Lie superalgebras one gets all $N = 1, 2, 3, 4$, and big $N = 4$ vertex algebras. I will explain classification of unitary representations of W-algebras, associated to minimal nilpotent orbits of all basic simple Lie superalgebras, which cover the above examples. This is a joint work with P. Moseneder Frajria and P. Papi.

Automorphism groups of cyclic orbifolds of lattice VOAs

Ching Hung Lam, Academia Sinica

In this talk, we discuss the automorphism groups of the cyclic orbifold $V_L^{\hat{g}}$ of a vertex operator algebra associated with a rootless even lattice L and a lift of a fixed-point free isometry $g \in O(L)$. It is clear that the normalizer $N_{\mathrm{Aut}(V_L)}(\langle g\rangle)$ of g stabilizes the orbifold VOA $V_L^{\hat{g}}$ and it induces a group homomorphism

$$f : N_{\mathrm{Aut}(V)}(\langle g\rangle)/\langle g\rangle \to \mathrm{Aut}(V_L^{\hat{g}}).$$

The main question is to determine if there are "extra automorphisms" not induced from $N_{\mathrm{Aut}(V_L)}(\langle g\rangle)$. We will discuss several special cases such that extra automorphisms exist. Some necessary and sufficient conditions as well as some explicit examples will also be discussed.

This talk is based on a joint work with Hiroki Shimakura.

Quasi-lisse vertex (super)algebras

Hao Li, Yau Mathematical Sciences Center

Lisse vertex operator algebras are relatively well-understood. Meanwhile, there are many new examples of quasi-lisse vertex operator (super)algebras appearing in the 4D-2D correspondence. But some of their basic properties are not systematically studied, such as the convergence of their genus 0 and genus 1 n-point correlation functions. We obtain a weaker finiteness condition for the convergence of n-point correlation functions. This would be crucial to understanding the structure of the category of modules for quasi-lisse vertex algebras. We also check this finiteness condition for the affine vertex algebra of type $A_1^{(1)}$ at admissible level. As examples, we obtain the twisted modular linear differential equations satisfied by the characters of twisted modules for some quasi-lisse vertex algebras. We also study the twisted modules of quasi-lisse vertex algebras, and calculate the fusion rules among the twisted modules of affine VOA of type $A_1^{(1)}$ at admissible level.

Universal vertex algebras beyond the W_∞-algebras

Andrew Linshaw, University of Denver

It is well known that the cosets $Com(V^k(gl_n), V^k(sl_{n+1}))for \geq 2$, are all quotients of a universal 2-parameter VOA which is freely generated of type $W(2, 3, 4, ...)$. This VOA admits many other interesting 1-parameter quotients which can be identified (up to an extra Heisenberg field) with the Gaiotto-Rapcak Y-algebras. We consider a similar construction in type C, namely, the coset $Com(V^k(sp_{2n}), V^k(sp_{2n+2})), for \geq 2$. This gives rise to a 2-parameter VOA which is freely generated of type $W(1^3, 2, 3^3, 4, \dots)$, which we expect to be the universal VOA of this type. The universal algebra admits 8 infinite families of 1-parameter quotients, which are analogues of the Gaiotto-Rapcak Y-algebras, and can be realized as cosets of W-(super)algebras. Assuming that the universal algebra has exactly two parameters, which is ongoing work to prove, we present some applications including new rationality results for W-(super)algebras. This is a joint work with Thomas Creutzig and Vlad Kovalchuk.

Extra structure on lattice vertex algebras

Ivan Mirković, Amherst and Zagreb

In the ADE case, lattice vertex algebras have an interpretation as integrable modules for affine Lie groups. A "non-abelian" geometric reconstruction of arbitrary lattice vertex algebras suggests analogous structures for arbitrary lattice vertex algebras . These are not yet understood.

An application of Borcherds Lie algebra to a proof of C_2-cofiniteness

Masahiko Miyamoto, University of Tsukuba

In the recent research on holomorphic vertex operator algebras of central charge 24, to use modular invariance property, most researchers put not only unitary condition on V but also rationality and C_2-cofiniteness. In my recent paper, I proved that rationality comes from the positivity and uniqueness of simple modules. In this talk, I will also show that if $V = \oplus V_n$ is a vertex operator algebra of central charge 24 with positive definite invariant

bilinear form and its character $ch_V(\tau) = \sum \dim V_n q^{n-1}$ coincides with $J(\tau) = q^{-1} + 196884q + ...$ then V is C_2-cofinite by using Borcherds Lie algebra.

Functorial constructions of double Poisson vertex algebras

Anne Moreau, Paris-Saclay University, Orsay

To any double Poisson algebra we produce a double Poisson vertex algebra using the jet algebra construction. We show that this construction is compatible with the representation functor which associates to any double Poisson (vertex) algebra and any positive integer a Poisson (vertex) algebra. We also consider related constructions, such as Poisson reductions and Hamiltonian reductions. This allows us to provide various interesting examples of double Poisson vertex algebras, in particular from double quivers. This is joint work in progress with Tristan Bozec and Maxime Fairon.

On the operad structure of moduli spaces and the consistency of non-chiral conformal field theory

Yuto Moriwaki, Institute of Physical and Chemical Research

Conformal field theory is believed to be consistent (well-defined) if it satisfies the bootstrap equation. In this talk, we will formulate this claim mathematically in the two-dimensional case and give a proof under certain conditions. The consistency of 2d conformal field theory comes down to the representation theory of chiral conformal field theories (vertex operator algebras), and the important step in the proof is to show that the operad of the moduli spaces of punctured Riemann spheres acts topologically on the representation category of vertex operator algebras. From this point of view, the bootstrap equation is the generator of a certain groupoid operad, and the consistency follows. This result is inspired by Huang-Kong's full field algebra, but we believe that our approach makes their result more conceptually comprehensible.

Collapsing chains

Pierluigi Möseneder Frajria, Milano

We will report on the phenomenon that collapsing of minimal W-algebras occurs in long chains that usually end with rational vertex algebras. We will describe the application of such phenomenon on the structure of KL category for affine algebras both in the even as well as the super case.

On Feigin-Tipunin type extension of W-algebras

Shigenori Nakatsuka, University of Alberta

The triplet algebra is an extension of the (1,p)-model of Virasoro algebra, which is a famous example of C2-cofinite but irrational VOA. Feigin and Tipunin gave a construction and generalization of this algebra to the simply-laced principal W-algebras by using VOA bundles over flag varieties. In this talk, we'll generalize their construction for all the W-algebras together with some basic conjectural properties. Then I will explain the case of $sl(2)$. The talk is based on my joint work with Thomas Creutzig and Shoma Sugimoto.

Some results on unitary highest weight modules for minimal W-algebra

Paolo Papi, Sapienza Università di Roma

We shall briefly recall the classification of unitary simple minimal W-algebras and the (partly conjectural) classification of their unitary representations, recently obtained in collaboration with V. Kac and P. Möseneder Frajria. Then we will discuss the following results, joint also with D. Adamović:

- any unitary highest weight module over a universal minimal quantum affine W-algebra at non-critical level descends to its simple quotient;
- defining relations of the unitary simple minimal quantum affine W-algebras and the list of all their irreducible positive energy modules;
- classification of all irreducible highest weight modules for simple affine vertex algebras in the cases when the associated simple minimal W-algebra is unitary.

Whittaker Modules for $\widehat{\mathfrak{gl}}$ and Non-Tensor Product $\mathcal{W}_{1+\infty}$-Modules

Veronika Pedić Tomić, University of Zagreb

We consider Whittaker modules $M_1(\lambda, \mu)$ for the Weyl vertex algebra M (also known as the $\beta\gamma$ vertex algebra). Our previous work, joint with C. H. Lam and N. Yu, established their irreducibility for finite cyclic orbifolds $M^{\mathbb{Z}_n}$. Here, we consider $M_1(\lambda, \mu)$ as modules for the $\mathbb{Z}$-orbifold M^0, which is isomorphic to the vertex algebra $\mathcal{W}_{1+\infty,c=-1} = \mathcal{M}(2) \otimes M_1(1)$. We prove their reducibility as $\widehat{\mathfrak{gl}}$-modules (and hence as M^0-modules) and provide a complete description of their irreducible quotients $L(d, \lambda, \mu)$. We demonstrate that $L(d, \lambda, \mu)$ are generally not tensor product modules for $\mathcal{M}(2) \otimes M_1(1)$. Additionally, we establish that all constructed modules are typical, being irreducible for the Heisenberg–Virasoro vertex subalgebra of $\mathcal{W}_{1+\infty,c=-1}$.

This is joint work with D. Adamović.

Orbifold theory for vertex algebras

Li Ren, Sichuan University

Let V be a simple vertex algebra of countable dimension, G be a finite automorphism group of V and σ be a central element of G. For any finite set $\mathcal{S}$ of inequivalent irreducible σ-twisted modules such that $\mathcal{S}$ is invariant under the action of G, we establish a duality result of Schur-Weyl type for V^G and a semisimple associative algebra $A_\alpha(G, \mathcal{S})$ on the sum of irreducible modules in $\mathcal{S}$. In particular, any σ-twisted module is a direct sum of finitely many irreducible V^G-modules. We also obtain a quantum Galois correspondence for the action of G on V. This is joint work with Chongying Dong and Chao Yang.

Inverse quantum hamiltonian reduction: a primer

David Ridout, University of Melbourne

Quantum hamiltonian reduction refers to a collection of functors that map the module category of a given affine vertex algebra to those of its associated W-algebras. Some of these functors are reasonably well understood and then the representation theory of the W-algebra is accessible. But some are not.

Inverse quantum hamiltonian reduction is a recent discovery that there (sometimes) exist functors in the opposite direction: from a given W-algebra module category to that of another W-algebra, which may be the affine vertex algebra itself. I will give an overview of the simplest example, which connects the module categories of the Virasoro and $\mathfrak{sl}_2$ minimal model vertex operator algebras.

Characters of logVOAs and quantum invariants

Shoma Sugimoto, Faculty of Mathematics, Kyushu University, Fukuoka 819-0386, Japan

In this talk, I will present some recent results on the relationships between the characters of logVOAs and quantum invariants. The relation between the character of $(1, p)$-logVOA and the colored Jones polynomial of the torus link $T(2, 2p)$ was proved by Bringmann-Milas in the case of $\mathfrak{sl}_2$ and recently generalized to the case of $\mathfrak{sl}_r$ by S. Kanade. In this talk, I will present the relationship between the character of (p, p')-logVOA and the colored Jones polynomial of the torus link $T(2p, 2p')$. On the other hand, it is known that by Feigin-Tipunin's geometric realization of $(1, p)$-logVOA, its character can be calculated using the Atiyah-Bott formula. In this talk, I will explain that the character of (p, p')-logVOA can be computed by the same method, and that by repeating the Atiyah-Bott formula, we can

obtain the homological block of Seifert 3-manifold, which is an invariant of 3-manifolds introduced by S. Gukov et al. This presentation is based on the joint work with Kazuhiro Hikami.

VOAs associated with intermediate Lie algebra $E_{7+1/2}$

Kaiwen Sun, MPIM, Bonn

$E_{7+1/2}$ is an intermediate Lie algebra filling the hole between E_7 and E_8 in the Deligne-Cvitanović exceptional series. It was found independently by Mathur, Muhki, Sen in the classification of 2d RCFTs via modular linear differential equations and by Deligne, Cohen, de Man in representation theory. I will talk about some recent progress on the VOAs associated with $E_{7+1/2}$ at higher levels and the VOAs associated with $E_{7+1/2}$ instanton SCFTs. These generalize the previous works of Kawasetsu for $E_{7+1/2}$ at level 1 and of Arakawa-Kawasetsu at level -5. This is based on a joint work with Kimyeong Lee and Haowu Wang.

Relations among relations for standard $C_n^{(1)}$-modules of level $2, 3, \ldots$?

Tomislav Šikić, University of Zagreb

The main topic of the talk is construction of combinatorial bases of standard $C_n^{(1)}$-modules. Special accent of this talk will be devoted to the combinatorial parametrization of leading terms of defining relations for all standard modules for affine Lie algebra of type $C_n^{(1)}$. This parametrization is based on Meurman - Primc result (Memoirs of AMS [MP 1999]) and Primc-Šikić (Journal of Mathematical Physics [PŠ 2016]). The conjecture for the standard modules $L(k\Lambda_0)$ where $n \geq 2$ and $k \geq 2$ will be presented (The Ramanujan Journal [PŠ 2019]).

At the end of talk, will be presented recent result for level 2 standard $C_2^{(1)}$-module [PŠ 2023 (arXiv:2301.11222)] and expectations for larger levels in the case $n = 2$. The main result in this article is a construction of combinatorially parameterized relations among the coefficients of annihilating fields. This construction of relations among relations probably play a key role in a construction of the basis of the maximal ideal of the universal vertex operator algebra $V_{\mathfrak{g}}^k$ for $k = 2$. This talk is based on joint work with Mirko Primc.

New semi-simple categories on affine vertex algebras at non-admissible levels

Ivana Vukorepa, University of Zagreb

Representation theory of simple affine vertex algebra $L_k(\mathfrak{g})$, for arbitrary simple Lie algebra $\mathfrak{g}$ and general level $k \in \mathbb{C}$, is a very important direction in the theory of vertex algebras. Some of the best understood cases are non-negative integer levels $k \in \mathbb{Z}_{\geq 0}$ and so-called admissible levels.

In the present talk we consider special non-admissible levels for $\mathfrak{g} = \mathfrak{sl}_m$. We prove that $KL_k(\mathfrak{sl}_m)$ is a semi-simple, rigid braided tensor category for all even $m \geq 4$, and $k = -\frac{m+1}{2}$. Moreover, all modules in $KL_k(\mathfrak{sl}_m)$ are simple-currents and they appear in the decomposition of conformal embeddings $\mathfrak{gl}_m \hookrightarrow \mathfrak{sl}_{m+1}$ at level $k = -\frac{m+1}{2}$. For this we inductively identify minimal affine W–algebra $W_{k-1}(\mathfrak{sl}_{m+2}, \theta)$ as simple current extension of $L_k(\mathfrak{sl}_m) \otimes \mathcal{H} \otimes \mathcal{M}$, where $\mathcal{H}$ is the rank one Heisenberg vertex algebra, and $\mathcal{M}$ the singlet vertex algebra for $c = -2$. This is joint work with D. Adamović, T. Creutzig and O. Perše.

Orbifold theory of the affine vertex operator superalgebra $L_{\widehat{osp(1|2)}}(k, 0)$

Qing Wang, Xiamen University

We classify the irreducible modules and determine the fusion rules for the orbifold of the simple affine vertex operator superalgebra $L_{\widehat{osp(1|2)}}(k, 0)$ with the positive integer k. This is a joint work with Cuipo Jiang.

Reflective forms on orthogonal groups and their expansions at 1-dimensional cusps

Janik Wilhelm, TU Darmstadt

The Dedekind eta function

$$q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

can be considered an automorphic form on $\mathrm{O}_{1,2}(\mathbb{R})$. In this talk, we classify reflective automorphic forms of singular weight on $\mathrm{O}_{n,2}(\mathbb{R})$. We will see that there are exactly 11 such forms. They have expansions at 0- and 1-dimensional cusps and these cusps can also be classified. The 1-dimensional cusps are parametrised by root systems and the root systems we find are exactly the 70 affine structures in Schellekens' list. We describe the relationship between the 1-dimensional cusps and holomorphic vertex operator algebras of central charge 24 to explain this observation. This talk is based on joint work with Thomas Driscoll-Spittler and Nils Scheithauer.

Duality structures on tensor categories coming from vertex operator algebras

Simon Wood, Cardiff University

The categories of admissible modules over rational vertex operator algebras famously give modular tensor categories and one of the most important features of such categories is the fact that each module has a rigid dual. However, there are known families of vertex operator algebras (such as the p,q triplet models) for which such duals have been proven not to exist. In this talk I will discuss a weaker notion of duality, called Grothendieck-Verdier (GV) duality that appears to be the natural notion of duality for vertex operator algebras in general and some of its implications.

Recent developments of tensor categories of vertex operator algebras

Jinwei Yang, Shanghai Jiao Tong University, Shanghai

In this survey talk, we will discuss the applicability of vertex tensor category theory of Huang-Lepowsky-Zhang, and report our recent progress on the construction of vertex tensor categories as well as the proofs of the rigidity. This talk is based on a series of joint work with T. Creutzig, S. Kanade and R. McRae.

On irreducibility of modules of Whittaker type

Nina Yu, Xiamen University

In [Adamović-Lam-Pedić-Yu; 2019], we generalized Dong-Mason's theorem on irreducibilty modules for cyclic orbifold vertex algebra to the entire category weak modules and applied these results to Whittaker modules. In this talk, I will present further generalizations of these results for nonabelian orbifolds of vertex operator superalgebras. I will also talk about applications of these results to examples and give irreducibility of modules of Whittaker type for orbifolds of vertex (super)algebras.

Number Theory Section

Asymptotics of the number of $D(q)$-pairs and $D(q)$-triples via L-functions

Nikola Adžaga, University of Zagreb

Let q be an integer. A $D(q)$-*m-tuple* is a set of m distinct positive integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + q$ is a perfect square for all $1 \leqslant i < j \leqslant m$. We count $D(q)$-pairs and triples with both elements up to N by counting integer solutions $x \in [1, b]$ of congruences $x^2 \equiv q \pmod{b}$ with $b \leqslant N$. We show that for prime q, the number of such $D(q)$-pairs and $D(q)$-triples grows linearly with N. Up to a factor of 2, the gradient of this linear function is the quotient of the value of the L-function of an appropriate Dirichlet character (usually a Kronecker symbol) and of $\zeta(2)$.

Modular Galois representations: twists and Diophantine applications

Samuele Anni, Aix-Marseille Université

Let ℓ be a prime number. To any mod ℓ modular form, which is an eigenform for all Hecke operators, it is associated a 2-dimensional residual representation of the absolute Galois group of the rationals. A residual modular Galois representation can arise as twist of a representation of lower conductor. In this talk, I will address this problem and describe applications towards solving Diophantine equations.

On the modularity of reducible Galois representations

Nicolas Billerey, Université Clermont Auvergne

I will discuss analogues in the reducible case of classical questions on the modularity of 2-dimensional residual Galois representations such as (weak and strong forms of) Serre's modularity conjecture and the level raising problem.

Counting D_4 quartic extensions of a number field ordered by discriminant

Alina Bucur, University of California, San Diego

A guiding question in number theory, specifically in arithmetic statistics, is counting number fields of fixed degree and Galois group as their discriminants grow to infinity. We will discuss the history of this question and take a closer look at the story in the case of quartic fields. In joint work with Florea, Serrano Lopez, and Varma, we extend and make explicit the counts of extensions of an arbitrary number field that was done over the rationals by Cohen, Diaz y Diaz, and Olivier.

The Zelevinsky classification of unramified representations of the metaplectic group

Igor Ciganović, University of Zagreb

We show that irreducible unramified representations of the metaplectic group, the unique non-trivial, two-fold central extension of the p-adic symplectic group with $p \neq 2$, are fully parabolically induced from unramified characters of general linear groups and a genuine irreducible negative unramified representation of a smaller metaplectic group. This is a joint work with Neven Grbac.

Effective Sato-Tate conjecture for abelian varieties and applications

Francesc Fité, University of Barcelona

I will present an effective version of the Sato–Tate conjecture for an abelian variety A defined over a number field with connected Sato–Tate group that is derived from the generalized Riemann hypothesis. The "effectivity" refers to the obtaintion of an upper bound on the error term in the count predicted by the Sato-Tate measure that only depends on certain invariants of the Lie algebra of the Sato-Tate group of A. I will discuss three applications of this conditional result: an interval variant of Linnik's problem for an abelian variety, a sign variant of Linnik's problem for a pair of abelian varieties, and the determination (up to multiplication by a nonzero constant) of the asymptotic number of primes whose Frobenius trace attains the integral part of the Hasse–Weil bound when A is an elliptic curve with complex multiplication. This is a joint work with Alina Bucur and Kiran Kedlaya.

Fermat-type equations via computation of elliptic curves with prescribed trace of Frobenius

Nuno Freitas, Instituto de Ciencias Matemáticas, Madrid

When applying the modular method to solve Diophantine equations the main difficulty one usually faces is at the final contradiction step; this can be due to the presence of solutions but also due to the inability to compute the required spaces of classical or Hilbert newforms. This spaces of newforms are usually filled with forms that we know in advance are not obstructions to the method but with current algorithms we are still required to compute them, making the modular method impractical very easily. In this talk we will discuss how to use the multi-Frey technique together with recent algorithms for computing elliptic curves over number fields due to Matschke to overcome some computational limitations, allowing us to tackle new cases of Fermat-type equations of the form $x^r + y^r = dz^p$.

Parametrizations of isogeny-torsion graphs of elliptic curves over $\mathbb{Q}$

Enrique González Jiménez, Universidad Autónoma de Madrid

The isogeny graph of a $\mathbb{Q}$-isogeny class of elliptic curves defined over $\mathbb{Q}$ consists in a vertex for each elliptic curve in the isogeny class and an edge for each rational isogeny of prime degree between elliptic curves in the isogeny class, with the degree recorded as a label on the edge. The isogeny graphs of elliptic curves over $\mathbb{Q}$ first appeared in the so-called Antwerp tables [1]. Although the first proof (in press) seems to be due to Chiloyan and Lozano-Robledo [2, §6]. In the first part of this talk we will show parametrizations of these isogeny graphs. Moreover, Chiloyan and Lozano-Robledo [2] define isogeny-torsion graph to be an isogeny graph where, in addition, each vertex is labeled with the abstract group structure of the torsion subgroup of the corresponding elliptic curve. They classify all the possible isogeny-torsion graphs that occur for $\mathbb{Q}$-isogeny classes of elliptic curves defined over $\mathbb{Q}$. In the last part of this talk we will show parametrizations of these isogeny-torsion graphs.

1. B.J. Birch, W. Kuyk (Eds.), *Modular Functions of One Variable IV*. Lecture Notes in Mathematics, vol. 476, Springer-Verlag, 1975.
2. G. Chiloyan, Á. Lozano-Robledo, *A classification of isogeny-torsion graphs of $\mathbb{Q}$-isogeny classes of elliptic curves.* Trans. London Math. Soc. **8** (2021), 1–34.
3. E. González–Jiménez, *Parametrizations of isogeny-torsion graphs of elliptic curves over $\mathbb{Q}$*. Preprint.

Characterization of quadratic $\varepsilon-$CNS polynomials and determination of all $\varepsilon-$CNS bases in quadratic number fields

Borka Jadrijević, University of Split

In this talk, we give the characterization of the quadratic $\varepsilon-$canonical number system ($\varepsilon-$CNS) polynomials for all values $\varepsilon \in [0,1)$. Our characterization provides a unified view of the well-known characterizations of the classical quadratic CNS polynomials ($\varepsilon = 0$) and quadratic SCNS polynomials ($\varepsilon = 1/2$). This result is a consequence of our new characterization results of $\varepsilon-$shift radix systems ($\varepsilon-$SRS) in the two-dimensional case and their relation to quadratic $\varepsilon-$CNS polynomials. Using the characterization of quadratic $\varepsilon-$CNS polynomials, we also obtain a complete description of the $\varepsilon-$CNS basis in the ring of integers of quadratic number fields. This description fits into the general framework of generalized number systems (GNS) introduced by A. Pethő and J. Thuswaldner.

This is joint work with Kristina Miletić.

Quadratic twists of genus one curves and Diophantine quintuples

Matija Kazalicki, University of Zagreb

Motivated by the theory of Diophantine m-tuples, we study rational points on quadratic twists $H^d : dy^2 = (x^2 + 6x-18)(-x^2+2x+2)$, where $|d|$ is a prime. If we denote by $S(X) = \{d \in \mathbb{Z} : H^d(\mathbb{Q}) \neq \emptyset, |d| \text{ is a prime and } |d| < X\}$, then, by assuming some standard conjectures about the ranks of elliptic curves in the family of quadratic twists, we prove that as $X \to \infty$

$$\frac{43}{256} + o(1) \leq \frac{\#S(X)}{2\pi(X)} \leq \frac{46}{256} + o(1).$$

Computing quadratic points on modular curves $X_0(N)$

Timo Keller, Groningen University

We improve on existing methods to compute quadratic points on modular curves and apply them to successfully find all the quadratic points on all modular curves $X_0(N)$ of genus up to 8, and genus up to 10 with N prime, for which they were previously unknown.

This is joint work with Nikola Adzaga, Philippe Michaud-Jacobs, Filip Najman, Ekin Ozman, and Borna Vukorepa.

A simple dichotomy in Serre's uniformity question

Davide Lombardo, University of Pisa

Let E be an elliptic curve defined over the field of rational numbers and suppose that E does not have (potential) complex multiplication. For every prime p, the action of the absolute Galois group of $\mathbb{Q}$ on the p-torsion points of E gives rise to a representation $\rho_{E,p} : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{Aut}(E[p]) \cong \mathrm{GL}_2(\mathbb{F}_p)$. Serre's celebrated open image theorem shows that this representation is surjective for all p greater than some bound $p_0 = p_0(E)$, depending in principle on E. Serre has asked whether there is a universal p_0 that works for all (non-CM) elliptic curves over $\mathbb{Q}$. Building on results by many authors, including most recently Le Fourn and Lemos, we prove that for $p > 37$ there are at most two possibilities (up to conjugacy) for the image of $\rho_{E,p}$: the whole group $\mathrm{GL}_2(\mathbb{F}_p)$ and the normaliser of a so-called non-split Cartan subgroup.

This is joint work with Lorenzo Furio.

Sieving for quadratic points on bielliptic curves

Philippe Michaud-Jacobs, Warwick University

Computing rational and low-degree points on curves is a major area of research in number theory. In recent years, there has been much interest in computing low-degree points on modular curves, and in particular in computing

quadratic points on the curves $X_0(N)$. Such results yield insight into the arithmetic of elliptic curves and have direct applications in solving Diophantine equations using the modular method. In this talk, I will introduce a Mordell–Weil sieve, and we will see how it can be used to study quadratic points on certain bielliptic modular curves $X_0(N)$. I will also discuss how these computations lead to examples of twisted modular curves that violate the Hasse principle.

The modular approach for solving $x^r + y^r = z^p$ over totally real number fields

Diana Mocanu, Warwick University

Since Wiles' famous proof of Fermat's Last Theorem, number theorists extensively studied Diophantine equations using the modular approach. We will briefly describe a variation of this method using (partial results about) modularity of elliptic curves over totally real fields, image of inertia comparison, and the study of certain S-unit equations.

Then, we will describe how to attack the equation $x^r + y^r = z^p$ (fixed r, varying p) using the above method. If time permits, we will sketch how to attack the very similar family of equations $x^p + y^p = z^2$ and $x^p + y^p = z^3$ (varying p).

Counterexamples to the Hasse principle in quadratic twist families of genus 1 hyperelliptic curves

Adam Morgan, Glasgow University

Let C be the genus 1 hyperelliptic curve $y^2 = f(x)$ for $f(x)$ an irreducible quartic polynomial with rational coefficients. I will discuss joint work in progress with Alex Bartel in which we study the number of squarefree integers d for which the quadratic twist $dy^2 = f(x)$ violates the Hasse principle. Specifically, I will describe heuristics leading to a conjecture for the proportion of everywhere locally soluble twists which have no rational point, and describe progress towards this conjecture when $f(x)$ has Galois group the Klein four-group.

A random model for the Paley graph

Rudi Mrazović, University of Zagreb

For a prime p we define the Paley graph to be the graph with the set of vertices $\mathbf{Z}/p\mathbf{Z}$, and with edges connecting vertices whose sum is a quadratic residue. Paley graphs are notoriously difficult to study, particularly finding bounds for their clique numbers. A well known result of Graham and Ringrose shows that the clique number of the Paley graph is at least $c \log p \log\log\log p$ for some constant $c > 0$ (even $c \log p \log\log p$, under the GRH) for infinitely many primes p – a behaviour not detected by the random Cayley graph which is hence deficient as a random model for the Paley graph. In this talk we present a new probabilistic model which incorporates some multiplicative structure and as a result captures the Graham-Ringrose phenomenon.

On Higher Order Weierstrass Points on $X_0(N)$ and beyond

Goran Muić, University of Zagreb

Let Γ be the Fuchsian group of the first kind. For an even integer $m \geq 4$, we describe the space $H^{m/2}(\mathfrak{R}_\Gamma)$ of $m/2$–holomorphic differentials in terms of a subspace $S_m^H(\Gamma)$ of the space of (holomorphic) cuspidal modular forms $S_m(\Gamma)$. This generalizes classical isomorphism $S_2(\Gamma) \simeq H^1(\mathfrak{R}_\Gamma)$. We describe the properties of the space $S_m^H(\Gamma)$. As an application, we describe the algorithm implemented in SAGE for testing if a cusp at ∞ for non–hyperelliptic $X_0(N)$ is a $\frac{m}{2}$–Weierstrass point. This is a joint work with Damir Mikoč.

Counting S_4 and S_5 extensions satisfying the Hasse norm principle

Rachel Newton, King's College London

Let L/K be an extension of number fields. The norm map $N_{L/K} : L^* \to K^*$ extends to a norm map from the ideles of L to those of K. The Hasse norm principle is said to hold for L/K if, for elements of K^*, being in the image of the idelic norm map is equivalent to being the norm of an element of L^*. The frequency of failure of the Hasse norm principle in families of abelian extensions is fairly well understood, thanks to previous work of Christopher Frei, Daniel Loughran and myself, as well as recent work of Peter Koymans and Nick Rome. In this talk, I will focus on the non-abelian setting and discuss joint work with Ila Varma on the statistics of the Hasse norm principle in field extensions with normal closure having Galois group S_4 or S_5.

Modular curves $X_0(N)$ with infinitely many quartic points

Petar Orlić, University of Zagreb

We determine all modular curves $X_0(N)$ with infinitely many quartic points. This is a joint work with Maarten Derickx.

On solutions of the curve $x^4 + dy^2 = z^p$

Ariel Pacetti, University of Aveiro

Let d be a square-free integer. In this talk we will explain how to use the modular method to study integral solutions of the equation $x^4 + dy^2 = z^p$ for d fixed and varying p. If time allows, we will explain how this approach (together with a result on endomophisms of modular abelian varieties related by a congruence) imply the following result.

Theorem: If d is a prime number congruent to 3 modulo 8 and such that the class group of $\mathbb{Q}(\sqrt{-d})$ is not divisible by 3 then there are no non-trivial primitive solutions of the equation $x^4 + dy^2 = z^p$ for p large enough.

Similar arguments prove analogue statements for the curve $x^2 + dy^6 = z^p$.

The primes of bad reduction of the modular star quotient $X_0(N)^*$

Oana Padurariu, Boston University

Let $X_0(N)^*$ be the quotient of the modular curve $X_0(N)$ by the full group of Atkin-Lehner involutions. We know that a prime p is a prime of bad reduction for $X_0(N)$ if and only if $p \mid N$. In joint work with John Voight, we prove that the primes of bad reduction of $X_0(N)^*$ are the same as the primes of bad reduction of $X_0(N)$, outside a finite, explicitly computable set of values for the level N.

Searching for elliptic curves with high rank in the PARI/GP software package

Vinko Petričević, University of Osijek

In the new version of PARI/GP software package, they added the command ellrank, which in some situations can calculate the rank of the elliptic curve very fast. Using this and connecting PARI/GP and C++, we were able to find record-breaking curves for some torsion groups.

Curves with prescribed rational points

Katerina Santicola, University of Warwick

Given a nonsingular curve $C/\mathbb{Q}$ with genus ≥ 2, we know by Falting's Theorem that $C(\mathbb{Q})$ is finite. Determining $C(\mathbb{Q})$ is a difficult problem, as we have no effective results on computing this set. Here we ask the reverse question: given a finite set of rational points S, does there exist a nonsingular curve such that $C(\mathbb{Q}) = S$? We answer this question in the affirmative by constructing a separable polynomial $f_S(x) \in \mathbb{Q}[x]$ of degree d such that the curve $C : y^d = f_S(x)$ has the desired property. In this talk I will go over which ingredients are necessary to construct such a polynomial (one of them being Falting's theorem itself!).

Torsion points on QM abelian surfaces

Ari Shnidman, Hebrew University of Jerusalem

I'll present two results on rational torsion subgroups of geometrically simple abelian surfaces A over $\mathbb{Q}$ with quaternionic multiplication. First, we show that if the endomorphism ring is maximal, then $|A(\mathbb{Q})_{tors}|$ is at most 18 and this is sharp. Second, for the family of Pryms $y^3 = x^4 + ax^2 + b$, we give a complete Mazur-type classification of the surfaces with non-trivial torsion. This is joint work with Laga-Schembri-Voight and Laga.

Conjectural asymptotics for prime orders of points on elliptic curves over number fields

Michael Stoll, Bayreuth University

Define, for a positive integer d, $S(d)$ to be the set of all primes p that occur as the order of a point $P \in E(K)$ on an elliptic curve defined over a number field K of degree d. We discuss how some plausible conjectures on the sparsity of newforms with certain properties would allow us to deduce a fairly precise result on the asymptotic behavior of $\max S(d)$ as d tends to infinity.

This is joint work with Maarten Derickx.