REPRESENTATION THEORY XVI

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Organizers:

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ABSTRACTS OF TALKS

Real Groups Section

The local Langlands correspondence and unitary representations of $GL(n)$

Adam Brown, Institute of Science and Technology Austria

Harish-Chandra's Lefschetz principle suggests that representations of real and p-adic split reductive groups are closely related, even though the methods used to study these groups are quite different. The local Langlands correspondence (as formulated by Vogan) indicates that these representation theoretic relationships stem from geometric relationships between real and p-adic Langlands parameters. In this talk we will discuss how the geometric structure of real and p-adic Langlands parameters lead to functorial relationships between representations of real and p-adic groups. I will describe work in progress which applies this functoriality to the study of unitary representations and signatures of invariant hermitian forms for $GL(n)$. The main result expresses signatures of invariant hermitian forms on graded affine Hecke algebra modules in terms of signature characters of Harish-Chandra modules, which are computable via the unitary algorithm for real reductive groups by Adams-van Leeuwen-Trapa-Vogan.

Generalising Vogan's conjecture across Schur-Weyl duality

Kieran Calvert, University of Manchester, UK

We outline Dirac cohomology for Lie algebras and Vogan's conjecture. We then cover some basic material on Schur-Weyl duality and Arakawa-Suzuki functors. Finishing with current efforts and results on generalising Vogan's conjecture to a Schur-Weyl duality setting. This would relate the centre of a Lie algebra with the centre of the relevant tantaliser algebra.

A non-vanishing criterion for Dirac cohomology

Chaoping Dong, Soochow University, China

A formula for the Dirac cohomology of weakly good cohomologically induced modules was obtained by Huang and the speaker in 2015. A remaining question is that whether the Dirac cohomology is zero or not. For linear Lie groups we will provide a criterion. As an application, we will introduce a way to count the number of strings in the Dirac series. We will also mention a few conjectures.

Primitives in the relative Weil algebra

Karmen Grizelj, University of Zagreb

The goal is to define a map of transgression taking primitive symmetric invariants to primitive invariants in the exterior algebra. The result should be compared with the known result in the absolute case called the transgression theorem.

Cubic Dirac operator for $U_q(\mathfrak{sl}_2)$

Andrey Krutov, Institute of Mathematics, Czech Academy of Sciences

Alekseev and Meinrenken developed a noncommutative equivariant de Rham theory for homogeneous manifolds equipped with a transitive action of a Lie group G. This led to the definition of a noncommutative Weil algebra related to the algebra $\mathfrak{g}$ of G. If the Lie algebra $\mathfrak{g}$ admits a nondegenerate invariant bilinear form, then the noncommutative Weil algebra comes equipped with a cubic Dirac element. Since this Dirac element squares to a scalar, it can be seen as an algebraic Dirac operator. Such Dirac operators have numerous applications in geometry and representation theory. In particular, they were used to give a new proof of the celebrated Duflo theorem.

We define the q-deformed noncommutative Weil algebra for $U_q(\mathfrak{sl}_2)$ and the corresponding cubic Dirac element. We calculate the spectrum of the Dirac operator and the corresponding Dirac cohomologies. We discuss a possible q-defomed generalisation of $\mathfrak{g}$-differential algebras.

This is joint work with Pavle Pandžić (Zagreb).

Functoriality via Dirac Cohomology

Jing-Song Huang, CUHKSZ

The Langlands functoriality that defines a lifting of global characters or the spectral transfer for representations of reductive groups is mediated by admissible homomorphisms between the corresponding L-groups. The geometric transfer which is dual to the spectral transfer is closely related to the Dirac cohomology and Dirac index. The aim of this talk is to demonstrate that in many interesting cases the functorial transfer and the transfer factors can be obtained by Dirac cohomology and Dirac index.

On K-types of irreducible representations of $SU(2,2)$

Domagoj Kovačević, University of Zagreb

We analyze the K-types of irreducible representations of $SU(2,2)$ using their highest weight vectors and operators which act on the set of these vectors. The set of K-types resembles the representations of $\mathfrak{sl}(3,\mathbb{C})$. We hope that it will produce irreducible representations of $SU(2,2)$.

Some Comments on the Structure of the Unitary Dual of a Complex Reductive Group

Lucas Mason-Brown, University of Oxford

In his 'Orange Book', David Vogan formulates some general expectations about the structure of the unitary dual of a real reductive group. These expectations can be summarized as follows: every irreducible unitary representation can be constructed from some elementary building blocks (called 'unipotent representations') through some unitarity-preserving operations (unitary induction, cohomological induction, and complementary series). Turning this philosophy into a precise mathematical conjecture turns out to be a subtle and difficult problem. In this talk, I will attempt to do so in the case of spherical representations of a complex group. This talk is partially based on joint work with Ivan Losev.

Hecke module structure on the orbits in double flag varieties

Kyo Nishiyama, AGU, Tokyo, Japan

Let G be a connected reductive algebraic group and K its symmetric subgroup. We consider a double flag variety of finite type $\mathfrak{X} = K/B_K \times G/P$, where B_K is a Borel subgroup of K, and P a parabolic subgroup of G. The orbit space $\mathbb{C}\mathfrak{X}/K$ enjoys a natural Hecke module structure for the Hecke algebra $\mathcal{H} = \mathcal{H}(K, B_K)$ of K. However, it is difficult to find out its explicit Hecke module structure. In this talk, we consider the double flag variety of type AIII, i.e., when $G/K = \mathrm{GL}_n/\mathrm{GL}_p \times \mathrm{GL}_q \ \ (n = p+q)$, and give an explicit action of $\mathcal{H}$ on $\mathbb{C}\mathfrak{X}/K$ in combinatorial way using graphs.

The talk is based on the on-going joint work with Lucas Fresse in Université de Lorraine, IECL (France): `arXiv:2206.10476`.

Dirac inequality for highest weight Harish-Chandra modules

Ana Prlić, University of Zagreb

Let G be a connected simply connected noncompact simple Lie group of Hermitian type. Then G has unitary highest weight representations. The proof of the classification of unitary highest weight representations of G given

by Enright, Howe and Wallach is based on the Dirac inequality of Parthasarathy, Jantzens formula and Howe theory of dual pairs where one group in the pair is compact. In this paper we focus on the Dirac inequality which can be used to prove the classification in a more direct way.

A Soergel bimodule approach to the character theory of real groups

Anna Romanov, UNSW Sydney, Australia

Admissible representations of real reductive groups are a key player in the world of unitary representation theory. The characters of irreducible admissible representations were described by Lusztig-Vogan in the 80s in terms of a geometrically-defined module over the associated Hecke algebra. In this talk, I will. describe a categorification of a block of the LV module using Soergel bimodules. This is joint work with Scott Larson.

Howe duality for exceptional theta correspondences

Gordan Savin, University of Utah

Split exceptional groups have a dual pair where one member is the exceptional group G2. In this setting, I will explain how to prove a Howe duality for spherical representations or real groups. This is a joint work with Hung Yean Loke.

Tilting by Purity

Wolfgang Soergel, Freiburg University

I want to talk about joint work with Matthias Wendt and Rahbar Virk and the underlying formalism of homological algebra. More precisely, the realization functor of [BBD] and Bondarko's weight complex functor can be treated in a common framework and sometimes become quasiinverses to one another. This is what happens typically under the assumption of pointwise purity.

Euklid's plane through Symmetry

Wolfgang Soergel, Freiburg University

I want to discuss an axiomatic description of Euklid's plane as an ordered incidence geometry with parallel axiom and supremum property and with a distinguished group of automorphisms such that for every pair of rays there are precisely two automorphisms transforming one to the other. I will sketch how to deduce in an elementary way that there is only one model for this axiomatic framework.

Generalized BGG resolutions and Grushin complexes

Vladimír Souček, Charles University, Prague

The BGG resolutions on full flag manifolds were constructed first by Bernstein, Gelfand and Gelfand in the dual language of homomorphisms between Verma modules and they were generalized soon by Lepowsky to the case of general flag manifolds. Since their introduction, they were studied, used and applied many times. One of their applications in geometry was described in the paper [1] by Callin, Chang, and Eastwood. They studied the k-th Grushin distributions in the plane, which are distributions generated by two vector fields, which generate by succesive brackets all smooth vector fields but one of them has singularities. They showed that using two simplest BGG resolutions, it is possible to describe explicitely the associated Grushin complexes and to compute their cohomology for $k = 1, 2$.

The talk will describe the first results of the research project with D. Calderbank, M. Eastwood and J. Slovák leading to construction of generalized BGG resolutions on homogeneous spaces $\tilde{G}/P, \tilde{G} = G \ltimes V$, where V is a module over a semisimple Lie group G, and their applications to higher Grushin complexes.

[1] O. Callin, D. C. Chang, M. Eastwood: Integrability conditions for the Grushin and Martinet distribution, Bull. IMAS New Ser. 8(2), 159–168 (2013)

Representations of finite groups and wireless communication

Vít Tuček, Huawei Technologies Sweden

Recent advances in wireless communications renewed interest in so called Grassmannian signaling. In this talk I will explain the motivation and mathematics behind this communication scheme. The main problem is construction of an optimal set of points on a complex Grassmannian akin to the famous Thomson problem. Proving optimality is extremely difficult and for pratical purposes not really neded so we resort to numerical exploration. A particularly advantageous candidates are orbits of a finite groups.

Sharpening the Dirac inequality

David Vogan, MIT

Suppose G is a real reductive Lie group with maximal compact subgroup K, and μ is an irreducible representation of K. The set

$$\Pi_u(G,\mu) = \text{irr unitary reps of lowest } K\text{-type } \mu \tag{1}$$

is a compact rational polyhedron. This means that it admits a finite description (for example by writing down all the rational coordinates of vertices of facets). One of the goals of the `atlas` software is to write a computer program that will compute the rational polyhedron $\Pi_u(G,\mu)$.

A necessary ingredient in such a program is an *a priori* bound on the answer: for example, a computable positive number $R(\mu)$ so that the polyhedron is known to be contained in the ball of radius $R(\mu)$. Such a number $R(\mu)$ is provided by Parthasarathy's Dirac inequality, proved fifty years ago, and since then exploited constantly in unitary representation theory.

I will recall what the Dirac inequality looks like, and how Parthasarathy proved it. I will then conjecture a related collection of inequalities, which would give far stronger bounds on $\Pi_u(G,\mu)$.

How to compute the unitary dual

David Vogan, MIT

There are two fundamental methods to construct unitary representations of a real reductive group $G(\mathbb{R})$. The oldest is *real parabolic induction*. One begins with a non-central split torus $A(\mathbb{R})$ inside $G(\mathbb{R})$, and forms its centralizer $M(\mathbb{R})$. *Real parabolic induction* takes an irreducible unitary representation π_M of $M(\mathbb{R})$ to a finite sum Π of irreducible unitary representations of $G(\mathbb{R})$; if the imaginary part of the infinitesimal character of π is suitably regular, then Π is irreducible, and one gets in this way *all* irreducible unitary representations of $G(\mathbb{R})$ with *non-real* infinitesimal character.

Slightly less familiar is Zuckerman's *cohomological induction*. One begins with a non-central *compact* torus $T(\mathbb{R})$ inside $G(\mathbb{R})$, and forms its centralizer $L(\mathbb{R})$. A nice domain for cohomological induction is irreducible unitary representations π_L of $L(\mathbb{R})$, having suitably regular real infinitesimal character. On this domain, cohomological induction of π_L is an irreducible unitary representation Π of G; and one gets in this way *all* irreducible unitary representations of $G(\mathbb{R})$ with having suitably regular *real* infinitesimal character.

The *fundamental parallelepiped* FPP consists of all real weights γ such that

$$0 \le \langle \gamma, \alpha^\vee \rangle \le 1 \qquad (\text{all } \alpha^\vee \text{ simple coroot}).$$

Conjecture. Each irr unitary of $G(\mathbb{R})$ with infinitesimal character *not* in the the FPP is obtained by real or cohomological parabolic induction.

I will explain why the determination of irreducible unitary representations with infinitesmal character in the FPP is a finite calculation. My other talk can be thought of as discussing a way to try to prove the Conjecture.

Revisiting a conjecture of Salamanca-Riba and Vogan

Kayue Daniel Wong, Chinese University of Hong Kong, Shenzhen, China

One major unsolved problem in real reductive Lie groups is the classification of the unitary dual. In their 1998 Annals paper, Salamanca-Riba and Vogan proposed that one can reduce the classification problem to Hermitian representations π with unitarily small lowest K-types. Their reduction relies on a(n unproved) non-unitarity conjecture on the infinitesimal character of π. In this talk, we propose a sharper non-unitarity conjecture, which immediately implies the conjecture of Salamanca-Riba and Vogan. We will sketch a proof of the refined conjecture for complex classical groups. One expects that similar techniques can be applied to other groups.

Theta correspondence and unitary representations

Chengbo Zhu, National University of Singapore

Abstract: The theory of theta correspondence, initiated by R. Howe, provides a powerful method of constructing irreducible admissible representations of classical groups. In this lecture, I will discuss some of the key applications of theta correspondence towards unitary representation theory, including unitary highest weight modules, small unitary representations, and special unipotent representations.

Vertex Algebra Section

VOAs arising from 4-dimensional superconformal field theories

Tomoyuki Arakawa, RIMS, Kyoto

This talk is about the 4D/2D duality discovered by Beem et al. rather recently. It associates a VOA to any 4-dimensional superconformal field theory, which is conjecturally a complete invariant of the 4-dimensional theory. The VOAs in this manner need not be rational nor lisse, but are still expected to have some interesting properties, and may be regarded as chiralization of various symplectic singularities such as $\mathbb{C}^{2n}//S_n$.

Rationality settings for $\mathbb{C}$-graded vertex algebras, and graded pseudo-traces of indecomposable modules for some irrational C_1-cofinite vertex operator algebras

Katrina Barron, University of Notre Dame, Notre Dame, Indiana USA

We present recent joint work with K. Batistelli, F. Orosz Hunziker, V. Pedic Tomic, and Y. Yamskulna on questions of rationality for $\mathbb{C}$-graded vertex algebras and applications to the Weyl algebra under conformal flow. In addition, results on the computation of graded pseudo-traces for all indecomposable modules for the Heisenberg vertex operator algebra and for all indecomposable modules induced from the level zero Zhu algebra for the universal Virasoro vertex operator algebra will be presented.

Combinatorial bases of modules of affine Lie algebras

Marijana Butorac, Faculty of Mathematics, University of Rijeka

In this talk I will present some results concerning the construction of combinatorial bases of standard modules of affine Lie algebras. The construction relies on the combinatorial bases of Feigin–Stoyanovsky principal subspaces of standard modules. In the case of untwisted affine Lie algebras from the construction of combinatorial bases of principal subspaces followed the construction of combinatorial bases of the corresponding parafermionic spaces and the proof of the character formulae of A. Kuniba, T. Nakanishi and J. Suzuki.
This talk is based on the joint works with Slaven Kožić and Mirko Primc.

Module tensor categories and the Landau-Ginzburg/conformal field theory correspondence

Ana Ros Camacho, Cardiff University

The Landau-Ginzburg/conformal field theory correspondence is a physics result from the late 80s and early 90s predicting some relation between categories of representations of vertex operator algebras and categories of matrix factorizations. At present we lack an explicit mathematical statement for this result, yet we have examples available. The only example of a tensor equivalence in this context was proven back in 2014 by Davydov-Runkel-RC, for representations of the $N = 2$ unitary minimal model with central charge $3(1 - 2/d)$ (where d integer bigger than 2) and matrix factorizations of the potential $x^d - y^d$. This equivalence was proven back in the day only for d odd, and in this talk we explain how to generalize this result for any d, realising these categories as module tensor categories enriched over $\mathbb{Z}_d$-graded vector spaces. Joint work with T. Wasserman (University of Oxford).

Rationality of exceptional subregular W-algebras associated with $\mathfrak{sp}_4$

Justine Fasquel, University of Melbourne

The classification of rational vertex algebras and W-algebras is a wide open problem. Rationality was recently proved for a large family of W-algebras called *exceptional*. However, the description of their simple modules remains

mysterious. In this talk, we give an explicit description of simple modules of exceptional subregular W-algebra associated with $\mathfrak{sp}_4$.

In addition, we present some applications of the description of spectral flow twists: we show that the W-algebra at certain levels is isomorphic to another unitary vertex algebra. If times permits, we will also explore the consequence of our results to the principal W-superalgebras associated with $\mathfrak{osp}_{2|4}$, which is the dual side of the Feigin-Semikhatov duality stated by Creutzig-Genra-Nakatsuka for orthosymplectic type.

Strongly tame modules for affine vertex algebras

Vyacheslav Futorny, University of São Paulo and SUSTech

We will discuss the Gelfand-Tsetlin tableaux realization of Zhu's algebras for admissible representations of simple affine vertex algebras in type A.

Feigin-Semikhatov duality

Naoki Genra (IPMU)

We talk about Feigin-Semikhatov duality, which is dualities between subregular W-algebras and principal W-superalgebras, super analogs of FeiginFrenkel duality and generalizations of Kazama-Suzuki cosets correspondences. The Feigin-Semikhatov duality gives rise to correspondences of fusion rules of dual W-(super)algebras in rational cases, and the induced coset functors imply equivalences between blocks of the module categories in general levels. This is joint work with Thomas Creutzig, Shigenori Nakatsuka and Ryo Sato.

Noncommutative PVAs and Courant-Dorfman algebras

Reimundo Heluani (IMPA)

We introduce the notion of double Courant-Dorfman algebra and prove a one-to-one correspondence between these and double Poisson vertex algebras, introduced by De Sole-Kac-Valeri, that are freely generated in degrees 0 and 1.

Induction and Wakimoto functors

Libor Křižka, Universidade de Saõ Paulo

The usual induction functor $\mathbb{M}_{\kappa,\mathfrak{g}}$ goes from the category $\mathcal{M}(\mathfrak{g})$ of $\mathfrak{g}$-modules to the category $\mathcal{E}_+(\widehat{\mathfrak{g}}_\kappa)$ of positive energy $\widehat{\mathfrak{g}}_\kappa$-modules. In the talk, we will describe a geometric counterpart of this functor called the Wakimoto functor $\mathbb{W}_{\kappa,\mathfrak{g}}$.

Hecke symmetries and quantum vertex algebras

Slaven Kožić, University of Zagreb

In this talk, we consider two generalizations of the Yangian of type A associated with a certain class of Hecke symmetries, which were recently introduced by Gurevich and Saponov. We discuss the underlying quantum vertex algebraic framework, where a certain action of these Yangian-like algebras plays the role of annihilation operators. In particular, we give a vertex-operator theoretic interpretation of the quantum determinants for the aforementioned algebras and establish a connection with the theory of ϕ-coordinated modules.

Unitary forms for holomorphic VOAs of central charge 24

Ching Hung Lam, Institute of Mathematics, Academia Sinica, Taiwan

We show that all holomorphic VOAs of central charge 24 with non-trivial weight one spaces are unitary. The main idea of the proof is to use the orbifold constructions of these VOAs from Niemeier lattice VOAs and some information about their automorphism groups.

On the Classification of Holomorphic Vertex Operator Superalgebras

Sven Möller, Universität Hamburg

I will discuss the classification of holomorphic vertex operator superalgebras of central charge between 0 and 24 using the 2-neighbourhood method. This is joint work with Gerald Höhn.

Associated varieties and finite extension of vertex algebras

Anne Moreau, Orsay

Motivated by recent works joint with Tomoyuki Arakawa and Jethro Van Ekeren on collapsing levels, we conjectured that if W is a finite extension of a vertex subalgebra V, then the natural morphism between the corresponding associated varieties is dominant.

In the case where W is a simple W-algebra and V is its simple affine vertex algebra, the conjecture is deeply related with the singularities of nilpotent Slodowy slices.

In this talk, I will explain some results toward the conjecture and interesting examples. This is a work in progress joint with Jethro Van Ekeren.

Duality of hook-type W-superalgebras via convolution operations

Shigenori Nakatsuka, University of Alberta

Hook type W-superalgebras are W-superalgebras whose affine cosets appear at junctions of supersymmetric interfaces in $N = 4$ Super Yang Mills gauge theory. Their affine cosets enjoy a Feigin-Frenkel type duality as proven by Creutzig and Linshaw. I will explain how this duality is enhanced to a reconstruction theorem for the W-superalgebra themselves via convolution operation with "shifted" chiral differential operators. If time permits, I will talk about its representation theoretic applications and module characters. The talk is based on my joint work with Thomas Creutzig, Andrew Linshaw and Ryo Sato.

On collapsing levels (II)

Paolo Papi, Sapienza Università di Roma

We prove a general result saying that under mild hypothesis an embedding of an affine vertex algebra into an affine W–algebra is conformal if and only if their central charges coincide. This result extends previous results obtained in the case of minimal affine W-algebras. We also discuss sufficient conditions showing that certain conformal levels are collapsing and applications to W-algebras of hook or rectangular type for $sl(n+m)$ and $sl(m|n)$. Joint work with D.Adamović and Pierluigi Möseneder Frairja.

Permutation orbifolds of Virasoro vertex algebras and $\mathcal{W}$-algebras

Michael Penn, Randolph College

There has been significant recent attention to the study of permuration orbifolds of vertex operator algebras. In this talk we will consider the S_3 orbifold algebra of the tensor product of the three copies of the universal Virasoro vertex algebra. For generic values of the central charge we construct a minimal set of strong generators. We also investigate the simple orbifold for special cases of $c = 1/2$ and $c = -22/5$, leading to a new unitary $\mathcal{W}$-type algebra and connections to two affine $\mathcal{W}$-algebras associated to G_2.

A higher-rank logarithmic Kazhdan–Lusztig correspondence

David Ridout, University of Melbourne

Kazhdan–Lusztig correspondences are equivalences between categories of modules over a quantum group and a vertex operator algebra (VOA). We say that they are logarithmic when the categories are nonsemisimple. Here, I'll report on recent work verifying some nontrivial consistency checks on a conjectural logarithmic example associated with $\mathfrak{sl}_3$. Assuming this conjecture leads to many interesting (and hitherto unimaginable) conjectures for certain VOA categories.

Weight-one elements of vertex operator algebras and automorphisms of categories of generalized twisted modules

Christopher Sadowski, Ursinus College

Given a weight-one elements u of a vertex operator algebra V, we construct an automorphism of the category of generalized g-twisted modules for automorphisms of g fixing u. We apply these results to the case that V is an affine vertex algebra to obtain explicit results on these automorphisms of categories. In particular, we give explicit constructions of certain generalized twisted modules from generalized twisted modules associated to diagram automorphisms of finite-dimensional simple Lie algebras and generalized (untwisted) modules.

This talk is based on a joint work with Yi-Zhi Huang.

Reflective forms on orthogonal groups and their expansions at 1-dimensional cusps

Nils Scheithauer, TU Darmstadt

We classify reflective automorphic forms of singular weight on regular lattices of even rank and determine their expansions at 1-dimensional cusps. We show that these expansions correspond exactly to the holomorphic vertex operator algebras of central charge 24. This is joint work with Thomas Driscoll-Spittler and Janik Wilhelm.

Combinatorial relations among relations for level 2 standard $C_n^{(1)}$-modules

Tomislav Šikić, University of Zagreb

In the first part of the talk, it will be will be presented the construction of combinatorial bases of basic modules for affine symplectic Lie algebras $C_n^{(1)}$ (Journal of Mathematical Physics [PŠ 2016]). This construction is some kind generalization of A. Meurman and M. Primc's results (Memoirs of AMS [MP 1999]).

The rest of the talk will be devoted to the construction of combinatorial bases of standard $C_n^{(1)}$-modules. Special accent of this talk will be devoted to the combinatorial parametrization of leading terms of defining relations for all standard modules for affine Lie algebra of type $C_n^{(1)}$. This parametrization is the base of a conjecture on the standard modules $L(k\Lambda_0)$ where $n \geq 2$ and $k \geq 2$ (The Ramanujan Journal [PŠ 2016]). The numerical evidence for some characteristic examples which supports our conjecture will be given. At the end of talk, will be presented result for level 2 standard $C_n^{(1)}$-modules. This talk is based on joint work with Mirko Primc.

The representation theory of affine $sl(4)$ at level $-5/2$ and application to the higher ranks

Ivana Vukorepa University of Zagreb

In the first part of the talk we study the representation theory of non-admissible simple affine vertex algebra $L_{-5/2}(sl(4))$. This case is of particular interest since it appears in conformal embeddings of affine vertex algebras. We determine an explicit formula for the singular vector which generates the maximal ideal in the universal affine vertex algebra $V^{-5/2}(sl(4))$ and we classify irreducible $L_{-5/2}(sl(4))$–modules in the category $\mathcal{O}$. We determine the fusion rules between irreducible modules in the category of ordinary modules $KL_{-5/2}$ and prove that $KL_{-5/2}$ is a semi-simple, rigid braided tensor category.

In the second part, we discuss how these results can be extended to the category $KL_k(sl(2n))$, $k = -(2n+1)/2$, for any $n \geq 3$ using tensor category approach. This is joint work with D. Adamović, O. Perše and T. Creutzig.

An exceptional mirror extension and constructions of $c = 24$ holomorphic VOAs

Hiroshi Yamauchi (Tokyo Woman's Christian University)

We will consider a mirror extension corresponding to an exceptional conformal embedding $A_{1,28} \subset G_{2,1}$ and classify its extensions to $c = 24$ holomorphic VOAs. This is a joint work with C.H. Lam and T. Creutzig.

On $\mathbb{N}$-graded vertex algebras associated with cyclic non-Lie Leibniz algebras

Gaywalee Yamskulna, Illinois State University

Much work on vertex algebras has been concentrated on the vertex algebras that are of CFT-type. A natural question that one might ask is following: "what will happen to the algebraic structure of the $\mathbb{N}$-graded vertex algebras V when V are no longer of CFT-type?"

In this talk, I will discuss a study of a family of $\mathbb{N}$-graded vertex algebras $V = \oplus_{n=0}^{\infty} V_n$ such that $\dim V_0 \geq 2$ by using cyclic Leibniz algebras and exploring relationships between this family of vertex algebras and a vertex operator algebra associated with a rank one Heisenberg algebra.

This talk is based on joint work with C. Barnes, E. Martin, G. Seelinger, and J. Service.

The classification of simple (semi)relaxed admissible affine $sl(3)$ modules and their modular properties at level $-3/2$

Simon Wood, Cardiff

I will present joint work with Kazuya Kawasetsu and David Ridout on admissible affine $sl(3)$ weight modules. The main emphasis will be on the classification of simple modules in the category of weight modules and their use in evaluating the standard module formalism at level $-3/2$.

Number Theory Section

Rational points on modular curves $X_0(N)^+$ and $X_0(N)^*$

Nikola Adžaga, University of Zagreb

In this talk we discuss how to provably determine all rational points on curves $X_0^+(p)$ of genus g up to 6 (for prime p). Let r be the rank of the Jacobian of the curve (over the rationals). As these curves satisfy $r = g$, we use Quadratic Chabauty. We also determine all rational points on hyperelliptic curves $X_0^*(N)$ where we used other methods as well: quotients, Mordell-Weil Sieve and variations of Chabauty's method. Since the points on these curves parametrize elliptic curves with additional structure, we also classify rational points on all $X_0^+(p)$ and on hyperelliptic $X_0^*(N)$ for N squarefree. We also discuss future work and applications.

This talk presents several works written in co-authorship with V. Arul, L. Beneish, M. Chen, S. Chidambaram, T. Keller, O. Padurariu and B. Wen.

On some special Diophantine quintuples in $\mathbb{Z}[\sqrt{D}]$

Zrinka Franušić, University of Zagreb

In 2018, Gibbs conducted a search for Diophantine quintuples in $\mathbb{Z}[\sqrt{D}]$ for square free D with $|D| < 50$ and found 160 examples for positive D and no Diophantine quintuples were found for negative D. This motivated us to try to find formulas for Diophantine quintuples at least in some special cases. Precisely, we considered Diophantine quintuples $\mathbb{Z}[\sqrt{D}]$ containing two pairs of conjugates which include two regular Diophantine quadruples and were able to find some polynomial families of such sets. This is a joint work with A. Dujella and V. Petričević.

Torsion growth of rational elliptic curves over number fields of degree pq

Tomislav Gužvić, University of Zagreb

Define $\Phi_{\mathbb{Q}}(d)$ to be the set of possible isomorphism classes of groups $E(K)_{tors}$, where K runs through all number fields K of degree d, and E runs through all elliptic curves defined over $\mathbb{Q}$. We wish to determine the sets $\Phi_{\mathbb{Q}}(pq)$, for all prime numbers p and q. It is not hard to show that $\Phi_{\mathbb{Q}}(p) \cup \Phi_{\mathbb{Q}}(q)$ is contained in $\Phi_{\mathbb{Q}}(pq)$ and one can conjecture that the equality holds. It turns out that this is true for all but finitely many choices of p and q. We explicitly describe when such anomalies occur and classify the sets $\Phi_{\mathbb{Q}}(pq)$.

Some classes of generalized number systems over rings of integers of imaginary quadratic number fields

Borka Jadrijević, University of Split

In 2018, Pethő and Thuswaldner introduced a general notion of number systems defined over orders of number fields. This generalizes the well-known number systems and canonical number systems in number fields to relative extensions and allows for the definition of "classes" of digit sets by the use of lattice tilings.

Let $\mathbb{K}$ be a number field of degree k and let $\mathcal{O}$ be an order in $\mathbb{K}$. A *generalized number system* (GNS) over $\mathcal{O}$ is a pair $(p, \mathcal{D})$ where $p \in \mathcal{O}[x]$ is monic and $\mathcal{D} \subset \mathcal{O}$ is a complete residue system modulo $p(0)$ containing 0. The polynomial p is called basis of this GNS, $\mathcal{D}$ is called its set of digits. If each $q \in \mathcal{O}[x]$ admits a representation of the form $q \equiv \sum_{j=0}^{l-1} d_j x^j \pmod p$, with $l \in \mathbb{N}$ and $d_0, ..., d_{l-1} \in \mathcal{D}$ then the GNS $(p, \mathcal{D})$ is said to have the *finiteness property*. To a given fundamental domain $\mathcal{F}$ of the action of $\mathbb{Z}^k$ on $\mathbb{R}^k$ we associate a class $\mathcal{G}_{\mathcal{F}} = \{(p, \mathcal{D}_{\mathcal{F}}) : p \in \mathcal{O}[x]\}$ of GNS whose digit sets $\mathcal{D}_{\mathcal{F}}$ are defined in terms of $\mathcal{F}$ in a natural way. The main objective is the investigation of the finiteness property for these number systems. Pethő and Thuswaldner have proved some general results on the finiteness property of GNS in $\mathcal{G}_{\mathcal{F}}$. Some of these results depend on mild conditions on the topology of $\mathcal{F}$.

In recent joint work with Kristina Miletić, we investigate of the finiteness property of the classes $\mathcal{G}_{\mathcal{F}}^{(n)} = \bigcup_d \{(p, \mathcal{D}_{\mathcal{F}}) : p \in \mathcal{O}_d[x],\ \deg p = n\}$ of GNS, where $\mathcal{O}_d$ is a ring of integers of imaginary quadratic number field $\mathbb{K}_d = \mathbb{Q}(\sqrt{-d})$, associated with three fixed fundamental domains $\mathcal{F}$ of the action of $\mathbb{Z}^2$ on $\mathbb{R}^2$. Using the new theory of Pethő and Thuswaldner, we are able to determine all GNS $(p, \mathcal{D}_{\mathcal{F}})$ with finiteness property in the class $\mathcal{G}_{\mathcal{F}}^{(1)}$.

We also discuss some results obtained for the class $\mathcal{G}_{\mathcal{F}}^{(2)}$ and the possible application of our results to *étale number systems*, i.e. number systems over orders of étale $\mathbb{K}$–algebras introduced by Győry, Pethő and Thuswaldner in 2019.

Bernstein-Zelevinsky classification of irreducible representations of covering groups of GL_n over local non-archimedean fields

Erez Lapid, Weizmann Institute of Science

Distributions and interactions in various settings

Ivan Mirković, University of Masschusetts, Amherst

I will attempt to formally re-sketch in a "uniform" language the setting for the moderate symbol in number theory and vertex operators in the theory of vertex algebras. The intended language is a quasiphysical notion of interactions.

Circle method and counting transversals in group multiplication tables

Rudi Mrazović, University of Zagreb

A transversal in an $n \times n$ latin square is a collection of n entries not repeating any row, column, or symbol. Hall and Paige conjectured in 1955 that the multiplication table of a finite group G contains a transversal if and only if G satisfies a straightforward necessary condition. This was proved in 2009 by Wilcox, Evans, and Bray using the classification of finite simple groups and extensive computer algebra. I will discuss joint work with Sean Eberhard and Freddie Manners in which we approach the problem using the circle method from analytic number theory that enables us to asymptotically count transversals.

Models of $X_0(N)$ and beyond via modular forms and some applications

Goran Muić, University of Zagreb

We consider congruence subgroups $\Gamma_0(N)$, $N \geq 1$, and the corresponding curves $X_0(N)$. We construct various realizations (representations) of them in projective plane via modular forms of various weights. We discuss generalizations on wider class of discrete groups of $SL_2(\mathbb{R})$. We also discuss applications on computing certain Galois groups. This is joint work with I. Kodrnja.

Higher moments of elliptic curves

Bartosz Naskrecki, Adam Mickiewicz University, Poznań

In this talk we will discuss some new developments in higher moment sums of 1-parametric families of elliptic curves. These sums have connections to modular forms and algebraic curves. I will sketch a result about the second moment of cubic curves leading to a connection with intermediate Jacobians in threefolds. Next we will discuss proofs of modularity of certain rigid Calabi-Yau threefolds which uses directly higher moments, universal families of elliptic curves and Deligne's results, avoiding completely the standard approach via Faltings-Serre method. This is joint work with Matija Kazalicki.

Gonality of the modular curve $X_0(N)$

Petar Orlić, University of Zagreb

We determine the $\mathbb{Q}$-gonalities of the modular curve $X_0(N)$ for all $N < 135$ and some larger values of N. We also determine the $\mathbb{C}$-gonality for many of these curves. As a consequence of these results, we determine all modular

curves $X_0(N)$ that are tetragonal over $\mathbb{Q}$ and show that $X_0(109)$ is the only pentagonal curve over $\mathbb{Q}$. We also find the first known instances of pentagonal curves $X_0(N)$ over $\mathbb{C}$. This is a joint work with Filip Najman.

Asymptotics, strange identities and the Habiro ring

Robert Osburn, University College Dublin

In 2001, Zagier determined the asymptotic behavior for numbers which appear in the $1-q$ expansion of a particular element in the Habiro ring. In this talk, we discuss a recent generalization whereby asymptotics are obtained for coefficients in various expansions for any Habiro element which satisfies a general type of strange identity. As an application, we obtain asymptotics and positivity statements for numbers which arise from the Kontsevich-Zagier series associated to the colored Jones polynomial for a family of torus knots.

Schneider's p-adic continued fractions

Tomislav Pejković, University of Zagreb

We study Schneider's version of p-adic continued fractions. We are interested in the finiteness of rational number expansion, the quality of approximation by convergents, the irrationality exponent of a number with a given continued fraction expansion, and the convergence of Schneider's continued fractions in the field of real numbers. The main requirement for all of these problems is a good estimate of growth for the sequences of numerators and denominators of convergents.

$D(n)$-quintuples with square elements

Vinko Petričević, University of Zagreb

For an integer n, a set of m distinct nonzero integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \leq i < j \leq m$, is called a $D(n)$-m-tuple. We have shown that there are infinitely many essentially different $D(n)$-quintuples with square elements.

In this presentation, I will show computational background of this search.

References:

- A. Dujella, M. Kazalicki and V. Petričević, *D(n)-quintuples with square elements*, Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Math. RACSAM **115** (2021), Article 172, (10pp)

Diffusion process as a computational engine: integer factorisation algorithm

Lejla Smajlović, University of Sarajavo

We discuss how to use a diffusion process on a suitably constructed graph as a computational engine. More precisely, we describe the construction of the graph and implementation of the "diffusion-based" algorithm for factorisation of a given integer N which is not a prime or a prime power. Along the way, the definition of a "diffusion step" which is analogous to "quantum step" is introduced, and it is shown, using analysis of short exponential sums, that the factorization algorithm uses at most $O((\log N)^2)$ diffusion steps to terminate.

This is a joint work with Carlos A. Cadavid, Paulina Hoyos, Jay Jorgenson and Juan D. Vélez

On unitarizability of representations of classical p-adic groups

Marko Tadić, University of Zagreb

In the talk we will discuss some questions related to the problem of classifying unitary duals of classical p-adic groups.

Using deep learning for heuristic detection of elliptic curves having high rank

Domagoj Vlah, University of Zagreb

We investigate a novel possibility for heuristic computation of the rank of elliptic curves over $\mathbb{Q}$. We develop a deep learning model for classification of a given set of elliptic curves by rank. The model is based on convolutional neural network (CNN) architecture. It takes as an input the conductor of an elliptic curve together with the sequence of a_p-s in a fixed range. For training we use either curves from LMFDB or custom generated curves having conductors up to 10^{30} and ranks up to 10. For the baseline we develop and train simple neural network classifiers, involving as an input only computed values of different Mestre-Nagao-like sums and conductors thus mimicking the usual heuristic approach. We compare our CNN model performance to the Mestre-Nagao-like sums baseline.

CNN approach greatly outperforms baseline models for LMFDB dataset. For our custom generated dataset CNN and baseline models have similar performance. Interestingly, contrary to using Mestre-Nagao-like sums, CNN model achieved near perfect classification of the LMFDB dataset when using a_p-s in the highest considered range of $p < 10^5$.

We think that this new method could probably help in finding record breaking curves of high rank. This is joint work with Matija Kazalicki.

Isogenies over low degree number fields of elliptic curves with rational j-invariant

Borna Vukorepa, University of Zagreb

For a number field K, it is well known that elliptic curves E/K with a cyclic K-rational n-isogeny are parametrized (up to an isomorphism over $\overline{K}$) by the K-rational points on the modular curve $X_0(n)$. Bruin and Najman, Ozman and Siksek, and Box described all the quadratic points on the modular curves of genus $2 \leq g(X_0(n)) \leq 5$.

That information tells us something about the elliptic curves and isogenies defined over quadratic fields. It is therefore reasonable to ask ourselves a slightly different question: if E has a rational j-invariant, what are the possible degrees of a cyclic isogeny of E defined over a quadratic field or even over a higher degree number field?

This question is closely connected to the question of possible images of the mod n Galois representation of an elliptic curve E defined over $\mathbb{Q}$. It was proved by Najman that if E is a non-CM elliptic curve with a rational j-invariant with an isogeny of prime degree p, then $p \in \{2, 3, 5, 7, 11, 13, 17, 37\}$ as long as the degree of the field of definition of the isogeny is at most 7.

In recent work, we answered the same question for quadratic number fields and for all possible cyclic isogeny degrees (not only prime degrees). Motivated by that work, we analyze how those methods extend to the case of number fields of slightly higher degree (up to 7).

New results on the non-vanishing of Poincaré series

Sonja Žunar, University of Zagreb

Poincaré series are a famous simple tool for constructing modular forms. But in the case when the constructed modular form is cuspidal, the question whether it vanishes identically is not simple at all. Most existing approaches to this question are based on estimates of Fourier coefficients of the cusp forms concerned. In this talk, we will discuss a different idea, based on Muić's integral non-vanishing criterion for Poincaré series on unimodular locally compact Hausdorff groups. We will present new applications of this idea to cuspidal vector-valued modular forms and Siegel modular forms.