

# REPRESENTATION THEORY XIV

Dubrovnik, Croatia, June 21-27, 2015

## ABSTRACTS OF TALKS

# Explicit realization of certain affine and superconformal vertex algebras

Dražen Adamović, University of Zagreb, Croatia

We shall present our recent results on realization of certain admissible affine vertex algebras and study their relations with vertex algebras appearing in LCFT.

## Admissible representations and minimal models of $W$ -algebras

Tomoyuki Arakawa, RIMS, Kyoto University, Japan

Admissible representations are (conjecturally all) modular representations of affine Kac-Moody algebras. In my talk I will prove two important conjectures on admissible representations, that is, the Adamović-Milas conjecture and the Feigin-Frenkel conjecture, and apply them to prove the Kac-Wakimoto conjecture on the modular invariance of modules over  $W$ -algebras.

## Twisted logarithmic modules of vertex algebras

Bojko Bakalov, North Carolina State University

I will introduce a notion of a twisted module of a vertex algebra relative to any (not necessarily semisimple) automorphism, generalizing that considered previously by Y.-Z. Huang. Two features of such twisted modules are that they involve the logarithm of the formal variable and the action of the Virasoro operator  $L_0$  on them is not semisimple. I will derive a Borcherds identity and commutator formula for twisted modules. Examples for affine and Heisenberg vertex algebras will be presented.

## Universal $K$ -matrices via quantum symmetric pairs

Martina Balagović, Newcastle University, UK

The construction of the universal  $R$ -matrix for quantum groups produces solutions of the Yang-Baxter equation on tensor products of representations of that quantum group. This gives an action of the braid group of type A, endowing the category of finite dimensional  $U_q(\mathfrak{g})$  representations with a structure of a braided tensor category.

I will explain how the theory of quantum symmetric pairs allows an analogous construction of a universal  $K$ -matrix, which produces solutions of the reflection equation on tensor products of representations of that quantum group. This gives a representation of the braid group of type B, endowing the category of finite dimensional  $U_q(\mathfrak{g})$  representations with a structure of a braided tensor category with a cylinder twist.

This is joint work with Stefan Kolb.

## Unipotent representations and the Theta-correspondence

Dan Barbasch, Cornell University, Ithaca, USA

Unipotent representations for real groups are the building blocks of the unitary dual and residual spectrum. For classical groups large classes of such representations are related by the Theta correspondence. In this talk I will review some older known, and some new results about this relationship.

## On twisted constructions for vertex operator superalgebras

Katrina Barron, University of Notre Dame, USA

We will discuss the known twisted constructions for vertex operator superalgebras and implications for the classification of twisted modules in terms of other twisted and non-twisted modules.

## The irreducible modules for the Lie algebra $Der(C_q) \times C_q$

Punita Batra, HRI, Allahabad, India

Let  $C_q$  be the quantum torus associated with the matrix  $q$ , where all the diagonal entries of the matrix  $q$  are 1 and all other entries are roots of unity. I will define the Lie Algebra  $DerC_q \times C_q$  and classify its irreducible modules with finite dimensional weight spaces.

## Invariant measures on nilpotent orbits associated with holomorphic discrete series

Mladen Božičević, University of Zagreb, Croatia

Let  $G_{\mathbb{R}}$  be a linear semisimple Lie group and  $K_{\mathbb{R}}$  a maximal compact subgroup of  $G_{\mathbb{R}}$ . Suppose that the symmetric space  $G_{\mathbb{R}}/K_{\mathbb{R}}$  has a Hermitian structure. Let  $\pi$  be a holomorphic discrete series representation of  $G_{\mathbb{R}}$ , and let  $\mathcal{O}$  be a nilpotent orbit contained in the wavefront set of  $\pi$ . We compute the Liouville measure on  $\mathcal{O}$  in terms of the Liouville measures on elliptic coadjoint orbits defined by the holomorphic discrete series.

## Combinatorial bases of principal subspaces for affine Lie algebras of type $B_l^{(1)}$ and $C_l^{(1)}$

Marijana Butorac, University of Rijeka, Croatia

Extending our work on principal subspaces of standard module  $L_{B_2^{(1)}}(k\Lambda_0)$  and generalized Verma module  $N_{B_2^{(1)}}(k\Lambda_0)$  at level  $k \geq 1$  for affine Lie algebra of type  $B_2^{(1)}$ , we consider principal subspaces of affine Lie algebras of type  $B_l^{(1)}$  and  $C_l^{(1)}$ . By using the theory of vertex operator algebras, we find combinatorial bases of principal subspaces in terms of quasi-particle monomials.

## Biparabolic Lie subalgebras of $\mathfrak{gl}(n)$ with commutative maximal reductive stabilizer.

Michel Duflo, University Paris VII, France

Let  $\mathfrak{g}$  be a biparabolic Lie subalgebra of  $\mathfrak{gl}(n)$ . A "Maximal Reductive Stabilizer" is a subalgebra of  $\mathfrak{g}$  which is the stabilizer of some linear form on  $\mathfrak{g}$ , is reductive, and is maximal for these properties. MRSs do exist, and they are pairwise conjugate. I present results on the number of biparabolic subalgebras of  $\mathfrak{gl}(n)$  with commutative MRS. This is a joint work with Rupert Yu.

## Symplectic fermions and a quasi-Hopf algebra structure on the small quantum group

Azat Gainutdinov, Fachbereich Mathematik, Universität Hamburg

In a joint work with Ingo Runkel, we consider the (finite-dimensional) small quantum group  $\overline{U}_q\mathfrak{sl}(2)$  at  $q = \sqrt{-1}$  and show that  $\overline{U}_q\mathfrak{sl}(2)$  does not allow for an R-matrix, even though  $U \otimes V \cong V \otimes U$  holds for all finite-dimensional representations  $U, V$  of  $\overline{U}_q\mathfrak{sl}(2)$ . We then give an explicit coassociator  $\Phi$  and a new R-matrix  $R$  such that  $\overline{U}_q\mathfrak{sl}(2)$  becomes a quasi-triangular quasi-Hopf algebra.

Our construction is motivated by the two-dimensional logarithmic conformal field theory of symplectic fermions, where a braided monoidal category,  $\mathcal{SF}$ , has been computed from the factorisation and monodromy properties of conformal blocks, and we prove that the category  $\mathbf{Rep}(\overline{U}_q\mathfrak{sl}(2), \Phi, R)$  is braided monoidally equivalent to  $\mathcal{SF}$ . We also observe that our new R-matrix defines an  $SL(2, \mathbb{Z})$ -action on the centre of the quasi-Hopf algebra  $\overline{U}_q\mathfrak{sl}(2)$ , equivalent to the one given by Feigin–Gainutdinov–Semikhatov–Tipunin (2006), which agrees with the  $SL(2, \mathbb{Z})$ -action via modular transformations on the space of torus conformal blocks.

## Theta correspondence of tempered representations and Langlands parameters

Wee Teck Gan, National University of Singapore

I will discuss joint work with Hiraku Atobe, which gives a precise determination of the theta lifts of tempered representations for unitary and symplectic-orthogonal dual pairs over p-adic fields, in terms of Langlands parameters. This is achieved by exploiting the (now established) Gross-Prasad conjecture.

## Symplectic reflection algebras and eigenspace representations associated with finite reflection groups

Jing-Song Huang, The Hong Kong University of Science and Technology

We study the space of eigenfunctions of the differential operators with constant coefficients that are invariant under finite linear groups, in particular finite reflection groups, in the framework of representation theory of symplectic reflection algebras, in particular the representation theory of the rational Cherednik algebras. The Dirac cohomology and Lie algebra cohomology for semisimple Lie groups have natural analogues in this setting.

## W-algebras with non-admissible levels and the Deligne exceptional series

Kazuya Kawasetsu, The University of Tokyo, Japan

In this talk, structure of certain simple  $\mathcal{W}$ -algebras associated with the Deligne exceptional Lie algebras and non-admissible levels are described as the simple current extensions of certain vertex operator algebras. As an application, the  $C_2$ -cofiniteness and  $\mathbb{Z}_2$ -rationality of the algebras are proved. Since  $\mathcal{W}$ -algebras has been conjectured to be  $C_2$ -cofinite and rational only if the level  $k$  is admissible, they are new example of  $C_2$ -cofinite rational  $\mathcal{W}$ -algebras.

## On unitary representations of disconnected real reductive group.

Domagoj Kovačević, University of Zagreb, Croatia

The main goal of this paper is the analysis of the unitary dual of disconnected real reductive group when the unitary dual of the component of the unity is known.

## Multiplicities of $K$ -types in $U(\mathfrak{su}(n, 1))$ and $U(\mathfrak{so}(n, 1))$

Hrvoje Kraljević, University of Zagreb, Croatia

Let  $\mathfrak{g}$  be a real simple Lie algebra,  $G$  its adjoint group,  $K$  a maximal compact subgroup of  $G$  and  $\mathfrak{k} \subseteq \mathfrak{g}$  the Lie algebra of  $K$ . Furthermore, let  $U(\mathfrak{k}) \subseteq U(\mathfrak{g})$  be universal enveloping algebras of the complexifications  $\mathfrak{k}^{\mathbb{C}}$  and  $\mathfrak{g}^{\mathbb{C}}$  and  $\mathfrak{Z}(\mathfrak{k})$  and  $\mathfrak{Z}(\mathfrak{g})$  their centers. Let  $U(\mathfrak{g})^K$  be the unital subalgebra of all  $K$ -invariants in  $U(\mathfrak{g})$ . The multiplication defines an algebra homomorphism  $\mathfrak{Z}(\mathfrak{g}) \otimes \mathfrak{Z}(\mathfrak{k}) \rightarrow U(\mathfrak{g})^K$ . Knop has shown that in the case of noncompact  $G$  this homomorphism is injective and its image is the center of  $U(\mathfrak{g})^K$ . Furthermore, the algebra  $U(\mathfrak{g})^K$  is commutative, i.e. equal to  $\mathfrak{Z}(\mathfrak{g})\mathfrak{Z}(\mathfrak{k})$  if and only if  $\mathfrak{g}$  is either  $\mathfrak{su}(n, 1)$  or  $\mathfrak{so}(n, 1)$  and in these two cases  $U(\mathfrak{g})$  is free as a  $U(\mathfrak{g})^K$ -module. Then there exists a  $K$ -submodule  $H$  of  $U(\mathfrak{g})$  such that the linear map  $U(\mathfrak{g})^K \otimes H \rightarrow U(\mathfrak{g})$ , defined by multiplication is an isomorphism of vector spaces. I have proved that the multiplicity of any irreducible finite dimensional representation  $\delta$  of  $K$  in  $H$  is less than or equal  $\dim(\delta)$ . My conjecture is that it is always equal and I shall give some evidence for this conjecture. The consequence is that for any finite dimensional  $K$ -module  $E$  the  $U(\mathfrak{g})^K$ -module  $(U(\mathfrak{g}) \otimes E)^K$  is free of rank  $\dim E$ .

## New results and applications related to Kirillov-Reshetikhin modules and Macdonald polynomials

Cristian Lenart, State University of New York at Albany, USA

In a series of papers with S. Naito, D. Sagaki, A. Schilling, and M. Shimozono, I developed a uniform combinatorial model for (tensor products of one-column) Kirillov-Reshetikhin (KR) modules of affine Lie algebras; we also showed that their graded characters coincide with the specialization of symmetric Macdonald polynomials at  $t = 0$ . I will first present our latest work, on the extension of the above results corresponding to the non-symmetric Macdonald polynomials. I will then present a brief survey of related applications, which involve: various computations, properties of KR crystals, local and global Weyl modules, a categorification of Macdonald polynomials, and the quantum  $K$ -theory of flag varieties.

## Local theta correspondence between supercuspidal representations

Hung Yean Loke, National University of Singapore

There is a construction and parametrization of supercuspidal representations of  $p$ -adic groups via compact inductions due to Howe, Yu, Kim, Hakim-Murnaghan and others. In this talk, I will describe the local theta correspondences between supercuspidal representations of Type I reductive dual pairs in terms of Yu and Kim's induction parameters. These parameters are related by the moment maps. This is a joint work with Jiajun Ma.

## Realizations of Lie algebras and differential calculus on noncommutative spaces

Tea Martinić, University of Split, Croatia

In the first part of the talk we construct an extension of an  $n$ -dimensional Lie algebra  $\mathfrak{g}$  by an abelian family of generators and define an action of the extended algebra  $H$  on the subalgebra  $U(\mathfrak{g})$ . We then consider realization of  $H$  by formal power series with coefficients in the Weyl algebra  $A_n$ . The generators of  $U(\mathfrak{g})$  are realized as deformations of commutative coordinates in  $A_n$ . The realization of the algebra  $H$  provides a proper framework for studying left-right dual realizations of  $\mathfrak{g}$  and associated star-products on the algebra of symmetric coordinates in  $A_n$ . The problem is illustrated with kappa-deformed space. Finally we will consider differential calculus on noncommutative spaces. We will construct Lie super algebra by extending arbitrary finite dimensional Lie algebra  $\mathfrak{g}$  with one-forms and we will construct extension of such Lie super algebra analog as in first part.

## The determinant formula for the Neveu-Schwarz and Ramond Algebras

Arne Meurman, Lund University Sweden

We present an old result, joint work with A. Rocha-Caridi, on the highest weight representations of the Neveu-Schwarz and Ramond Lie superalgebras, including in particular the determinant formula for the Shapovalov form on highest weight modules.

## Quantum modular forms and characters of modules of vertex algebras

Antun Milas, University at Albany, USA

As defined by Zagier, a "quantum modular form" is a complex function defined on the set of rational numbers that exhibits usual modular transformations, up to a "nice" continuous or analytic function. Zagier also provided several motivating examples coming from quantum invariants of knots and 3-manifolds. In this talk, we show that quantum modular forms are also relevant to representation theory of certain irrational vertex algebras. This talk is partially based on a joint work K.Bringmann.

## A geometric formula for u-homology of representations

Dragan Miličić, University of Utah, USA

We shall describe a simple geometric formula for Lie algebra homology of a representation of a complex semisimple Lie algebra with respect to the nilpotent radical of a parabolic subalgebra. One can view this formula as a generalization of Kostant's generalized Borel-Weil-Bott theorem. This is a part of a joint project with Wilfried Schmid.

## Zastava spaces via local spaces over curves

Ivan Mirković, University of Massachusetts at Amherst, USA

We will consider zastava spaces as a point of view on loop Grassmannians ("affine Grassmannians"), a construction relevant for automorphic representation theory (Hecke operators in positive characteristic), vertex algebras (geometric constructions) and physics ('t Hooft operators).

On the other hand, the notion of a space  $Z$  local over a given scheme  $X$  is one of formalizations of the idea of locality in quantum field theory. It is a slight generalization of the fundamental structure of a factorization space developed by Beilinson and Drinfeld. The weakening of the requirements formalizes some well known examples of almost factorization spaces". The change of emphases leads to new constructions.

Our examples of local spaces will be of a "cohomological" nature. They include a reconstruction of the loop Grassmannians (affine Grassmannians) and zastava spaces, and their generalizations corresponding to quadratic forms on based lattices. The commutative examples suggest to me a "more geometric" homology theory in algebraic geometry.

# Minimal nilpotent orbit closures and lisse $W$ -algebras

Anne Moreau, Universite de Poitiers, France

Motivated by a recent work of Kawasetsu, we consider a lifting of Joseph ideals for minimal nilpotent orbit closures to the setting of affine Kac-Moody algebras and find new examples of affine vertex algebras whose associated varieties is a minimal nilpotent orbit closure. As an application, we obtain a new family of lisse  $W$ -algebras. This is a joint work with Tomoyuki Arakawa.

## Modular forms, 3rd order ordinary differential equations and affine vertex operator algebras

Kiyokazu Nagatomo, Osaka University, JAPAN

On this occasion I talk on so-called modular invariance properties of rational  $C_2$ -cofinite vertex operator algebras, particularly, affine vertex operator algebras. The affine vertex operator algebras (affine VOA for short) whose number of simple modules are not exceeding 20 are studied. One can determine the dimension of the space of characters of each affine VOA under this condition. We completely classify affine VOA's whose dimensions of space of characters are at most 5. Further we show that any basis of the space of characters forms a fundamental system of solutions of a special kind of 3rd order linear differential equations with a regular singularity at  $q (= e^{2\pi i\tau}) = 0$ . These linear differential equations are called *modular linear differential equations* (of weight  $k$ ) whose space of solutions are invariant under the slash action  $|\_k\gamma$  for each  $\gamma \in SL_2(\mathbb{Z})$ . We may expect any space of characters of affine VOA's form a fundamental system of solutions, but it is not true. In fact, we have a counterexample, i.e., the first non-trivial example is the affine VOA associated to complex simple Lie algebra of type  $E_8$  with level 1. However, this is an exceptional example. There is another example that is the affine VOA associated to complex simple Lie algebra of type  $A_2$  with level 3 whose number of simple modules is 10 and the dimension of the space of characters is 6, but this space does not form a fundamental system of solutions of a modular linear differential equations. Finally we mention one of advantages of our approach. Since a basis of space of characters forms a fundamental system of solutions of linear differential equations of regular singular type, by using the Frobenius method, we can recursively determine coefficients of the powers of  $q$ , which are dimensions of eigenspaces of the graded operator  $L_0$ .

## Orbit embedding for enhanced Lie algebra

Kyo Nishiyama, Aoyama Gakuin University, Tokyo, Japan

Let  $G$  be an algebraic group acting on a variety  $X$ . Let  $H$  be a subgroup of  $G$  and assume that a subvariety  $Y$  of  $X$  is stable under the action of  $H$ . Then we have a natural map  $p$  from the orbit space  $Y/H$  to  $X/G$ . In general, this is not surjective nor injective. In this talk, we give certain sufficient condition which implies the injectivity of  $p$  (a generalization of Ohta's result). As an application, we discuss the adjoint orbit embedding for enhanced Lie algebra, which contains, as a special case, the enhanced nilpotent cone of Achar-Henderson and the exotic nilpotent cone of Syu Kato.

Reference: arXiv:1410.2336 (Commentarii Mathematici Univ. St. Pauli, 63(2014), p. 223-232.)

## Singular BGG resolutions via Penrose transform

Pavle Pandžić, University of Zagreb, Croatia

We use Penrose transform to relate parabolic categories  $\mathcal{O}$  for different parabolic subalgebras of a fixed simple complex Lie algebra  $\mathfrak{g}$  of type  $A$ . In particular, using an appropriate twistor space, we can construct analogues of BGG resolutions at singular infinitesimal characters.

This is joint work with Vladimir Souček.

## Representations of some affine vertex algebras at negative integer levels

Ozren Perše, University of Zagreb, Croatia

In this talk we present a construction of singular vectors in universal affine vertex operator algebras associated to  $D_\ell^{(1)}$  of levels  $n - \ell + 1$ , for  $n \in \mathbb{Z}_{>0}$ . For  $n = 1$ , we study the representation theory of the quotient vertex operator algebra modulo the ideal generated by that singular vector. In the case  $\ell = 4$ , we show that the adjoint module

is the unique irreducible ordinary module for simple vertex operator algebra  $L_{D_4}(-2, 0)$ . We also show that the maximal ideal in associated universal affine vertex algebra is generated by three singular vectors. When  $\ell$  is odd, we obtain the classification of irreducible ordinary modules for associated simple vertex operator algebra. We use these results to obtain the branching rules for certain conformal embeddings. Part of the talk is based on joint work with Dražen Adamović.

## Quasi-particles in the principal picture of $\widehat{\mathfrak{sl}}_2$ and Rogers-Ramanujan-type identities

Mirko Primc, Department of Mathematics, University of Zagreb, Croatia

In this talk, I will present a joint work with Slaven Kozic. We define quasi-particles in the principal picture of  $\widehat{\mathfrak{sl}}_2$  and construct quasi-particle monomial bases of standard  $\widehat{\mathfrak{sl}}_2$ -modules. We show that their principally specialized characters are given as products of sum sides of the corresponding analytic Rogers-Ramanujan-type identities with the character of the Fock space for the principal Heisenberg subalgebra.

## Translation of Dolbeault representations on reductive homogeneous spaces

Nicolas Prudhon, University of Lorraine - Metz, France

We will explain how to adapt techniques used in the study of the Dirac operator on homogeneous reductive spaces to the Dolbeault operator on elliptic coadjoint orbits in order to prove that cohomologically induced representations have an infinitesimal character, that cohomological induction and Zuckerman translation functor commute and give a geometric interpretation of the Zuckerman translation functor in this context.

## Free field realization and weight representation theory of the twisted Heisenberg-Virasoro algebra

Gordan Radobolja, University of Split, Croatia

In this talk we discuss weight representations of the twisted Heisenberg-Virasoro Lie algebra at level zero  $\mathcal{H}$ . By using free field realization of the Heisenberg-Virasoro vertex algebra  $L^{\mathcal{H}}(c_L, c_{L,I})$  we construct highest weight  $\mathcal{H}$ -modules and present explicit formulas for singular vectors in Verma modules over  $\mathcal{H}$ . These formulas are crucial for proving irreducibility of certain tensor product modules. As a result, we get the fusion rules for an interesting subcategory of irreducible highest weight  $\mathcal{H}$ -modules.

In the end, we give a nontrivial homomorphism between the universal vertex-algebra  $L^{W(2,2)}(c_W, c_L)$  associated to Lie algebra  $W(2, 2)$  and  $L^{\mathcal{H}}(c_L, c_{L,I})$ . As a consequence, highest weight  $\mathcal{H}$ -modules become  $W(2, 2)$ -modules. This talk is based on a recent joint work with Dražen Adamović.

## Arthur packets for quasi-split classical real groups

David Renard, École polytechnique, Palaiseau, France

In a recent joint work with Nicolas Arancibia and Colette Moeglin, we prove that the packets of representations of quasi-split classical real groups defined by J. Arthur via twisted endoscopy to  $GL(N)$  coincide with packets previously defined in a more elementary way by Adams and Johnson. In the talk we will mostly explain basic facts about Arthur packets and their properties.

## Vertex-algebraic structure of principal subspaces of $D_4^{(3)}$ -modules

Christopher Sadowski, Ursinus College

Principal subspaces of standard modules for untwisted affine Lie algebras were introduced by Feigin and Stoyanovsky, and have been further studied by many other authors. In this talk, we introduce and discuss principal subspaces of certain standard  $D_4$  modules for the twisted affine Lie algebra  $D_4^{(3)}$ . We extend earlier results and vertex-algebraic techniques of Calinescu, Lepowsky, and Milas on principal subspaces of standard  $A_2^{(2)}$ -modules.

This is joint work with Michael Penn.

## Kantor-Koecher-Tits construction from Galois cohomology perspective

Gordan Savin, University of Utah, USA

In the KKT construction, starting with a Jordan algebra, one constructs a Lie algebra with a short  $\mathbb{Z}$ -grading. A difficulty is in the proofs. We present an approach based on Galois descent. (From a work with Toshi Kobayashi.)

## Quantum walled Brauer spin chains

Aleksei Semikhatov, Moscow, Russia

A spin chain defined by fundamental representations of a suitable finite-dimensional (braided) Hopf algebra  $H$  provides a discretized counterpart of a “continuum” model (presumably, logarithmic conformal field theory) centralized by  $H$ . I will discuss a particular setting where the continuum version is based on a logarithmic extension of the affine  $sl(2)$  algebra; the spin chain symmetries are then expressed in terms of the quantized walled Brauer algebra. For this algebra, I will present a family of commuting Jucys–Murphy elements and diagonalize them (which amounts to the construction of seminormal representations) for generic values of the parameters. I will also propose a Baxterization of the algebra, yielding commutative families of elements (“conservation laws”).

# Graded Representation categories and motives

Wolfgang Soergel, University of Freiburg, Germany

I want to explain a characterization of the usual graded version of category  $\mathcal{O}$  by its compatibility with the action of the center. I then want to discuss motivic constructions for graded versions of categories of representations.

## Leading terms of relations for standard modules of affine Lie algebras $C_n^{(1)}$

Tomislav Šikić, University of Zagreb

In this talk we give a combinatorial parametrization of leading terms of defining relations for level  $k$  standard modules for affine Lie algebra of type  $C_n^{(1)}$ . Using this parametrization we conjecture colored Rogers-Ramanujan type combinatorial identities for  $n \geq 2$  and  $k \geq 2$ ; the identity in the case  $n = k = 1$  is equivalent to one of Capparelli's identities. This talk is based on joint work with Mirko Primc.

## A Hopf algebroid related to the automorphisms of a Lie algebra

Zoran Škoda, Zagreb

Given a finite dimensional Lie algebra  $\mathfrak{g}$  over a field of characteristic zero, we observe a partly degenerate Hopf pairing between the universal enveloping algebra  $U(\mathfrak{g})$  and the algebra  $Fun(Aut(\mathfrak{g}))$  of regular functions on the affine algebraic group of automorphisms of  $\mathfrak{g}$ . This Hopf pairing induces on  $Fun(Aut(\mathfrak{g}))$  the structure of an algebra in the category of Yetter-Drinfeld modules over  $U(\mathfrak{g})$ . As a consequence, the semidirect (smash) product  $U(\mathfrak{g}) \# Fun(Aut(\mathfrak{g}))$  is a Hopf algebroid over the noncommutative base  $U(\mathfrak{g})$  (the notions of smash product, Yetter-Drinfeld modules and Hopf algebroids will be explained in the talk). This is a part of a joint work with my student M. Stojic, which, unlike her thesis work in progress, does not require formal completions. On the other hand, our Hopf algebroid can be explicitly embedded into a larger (and formally completed) Hopf algebroid studied with S. Meljanac and M. Stojic, which has a structure of a formally completed Heisenberg double. If time permits, I will finish with comments on the wider context within deformation quantization.

## Coherent sheaves on nilpotent cones

David Vogan, Massachusetts Institute of Technology, Cambridge, USA

Suppose  $G$  is a complex reductive algebraic group, and  $\mathcal{N} \subset \mathfrak{g}^*$  is the nilpotent cone. A conjecture of Lusztig, proved by Bezrukavnikov, says that there is a natural bijection

$$\text{irr. } G\text{-equivariant vector bundles on } G\text{-orbits on } \mathcal{N} \longleftrightarrow \text{dom. weights for } G.$$

(The coherent sheaves in the title arise because the left side is more or less obviously a basis for the Grothendieck group of  $G$ -equivariant coherent sheaves on  $\mathcal{N}$ .)

In the case of  $SL(2)$ , the dominant weights are non-negative integers, and the bijection is

$$\begin{aligned} 0 &\longleftrightarrow \text{trivial bundle on the regular orbit} \\ 1 &\longleftrightarrow \text{nontrivial bundle on the regular orbit} \\ p &\longleftrightarrow (p-1)\text{-dimensional representation of } G \text{ at } 0 \quad (p \geq 2). \end{aligned}$$

This bijection was computed explicitly in the case of  $GL(n)$  by Achar in his 2001 thesis; it has not been computed completely for any other infinite series of groups.

I'll explain a definition and description of this bijection in terms of finite-dimensional representation theory (due to Achar and Bezrukavnikov); applications to infinite-dimensional representations that would follow from computing it; and possible generalizations to real and  $p$ -adic reductive groups.

## Capelli identity and Radon transform

Genkai Zhang, Chalmers University, Gothenburg, Sweden

We construct a class of Cayley type differential operators on Grassmannian manifolds and find their Harish-Chandra homomorphisms. We prove that that Radon transform can be inverted using the Cayley operators. (Joint work with S. Sahi.)

## Irreducible Whittaker modules

Kaiming Zhao, Wilfrid Laurier University, Waterloo, Canada

I will recall some results on irreducible Whittaker modules over various Lie algebras, including finite-dimensional simple Lie algebras, the Virasoro algebra, the Heisenberg-Virasoro algebra and affine Kac-Moody algebras. Then I will show you some recent development on this topic. At last I might give some possible further studies.

## Graph complexes and their cohomology

Marko Živković, University of Zürich, Switzerland

I will introduce the notion of Graph vector spaces, Lie algebra structure on them and a differential  $d$  that turns it to a chain complex. Among a number of different classes of Graph complexes we will consider only one of the simplest examples,  $GC_0$ . There can be some small differences in definitions, whether we allow tadpoles, 1- or 2-valent vertices, but it turns out to be almost irrelevant for the cohomology because it can be shown that these graph complexes are essentially quasi-isomorphic. An example of such a proof will be explained, that largely uses spectral sequences.

Though very simple to define, the cohomology of Graph complexes is very hard to compute and largely unknown at present. Computer has been used to compute the low degree classes. I will explain two techniques lately developed to gain some more information on the cohomology. The first are computer calculations of Euler characteristics, what can be done up to much higher degree than a direct calculation of cohomological classes. The second is introducing an extra differential  $\nabla$  that can be used to show that classes of standard cohomology occur in pairs.