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ON INVERSE LIMITS OF COMPACT SPACES

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In this paper*) we are concerned with inverse systems $\{X_\alpha, \pi_{\beta\alpha}\}$ of Hausdorff compact spaces X_α; the systems are taken over arbitrary directed sets $M = \{\alpha\}$. X will always denote the inverse limit of the system and $\pi_\alpha : X \to X_\alpha$ will be the corresponding natural projections.[1]

We first introduce a Hausdorff paracompact space X^* associated to every inverse system and consisting of all the spaces X_α of the system (taken as disjoint sets) and of the limit X. The topology of X^* is such that the subset X is actually the limit (in the sense of the directed set M) of subsets X_α. Several properties of X^* are given. This generalizes a procedure given by H. Freudenthal ([6], p. 153) in the case of inverse sequences of metrizable compacta.

Next we consider the mapping spaces $\langle X, R\rangle$ of all mappings of a Hausdorff compact X into an ANR and we consider the singular homology group $H_q(\langle X, R\rangle; G)$ (with coefficients in an arbitrary Abelian group G) as a contravariant functor of X. Using the properties of X^* we show that $H_q(\langle X, R\rangle; G)$ is continuous with respect to inverse limits (for Hausdorff compacta). This generalizes a previous result of the author ([9], Theorem 13, p. 200) and settles a question raised in the same paper ([9], p. 202).

1. *The Space X^**

Let

$$X^* = (\bigcup_\alpha X_\alpha) \cup X, \ \alpha \in M, \tag{1}$$

where all X_α and their limit X are considered as being disjoint sets. If U_α is an open set of X_α, let $U_\alpha^* \subset X^*$ be the set defined by

$$U_\alpha^* = \bigcup_{\alpha \leq \beta} (\pi_{\beta\alpha}^{-1} U_\alpha) \ \cup (\pi_\alpha^{-1} U_\alpha). \tag{2}$$

Let $\mathcal{U}$ be the family of subsets of X^* consisting of all open sets $U_\alpha \subset X_\alpha$, $\alpha \in M$, and of all sets U_α^*, $\alpha \in M$. Since the sets $\pi_\alpha^{-1} U_\alpha$

*) This paper has been written while the author held a membership at The Institute for Advanced Study in Princeton.

[1]) Basic definitions and facts concerning inverse systems and their limits can be found in [5] and [8].

form a basis of open sets for X, it follows that $\mathcal{U}$ is a covering of X^*. Moreover, the intersection of any two members of $\mathcal{U}$ is the union of some members of $\mathcal{U}$. It suffices to prove this statement for the sets $U_\alpha{}^*$ and $U_{\alpha'}{}^*$, $\alpha, \alpha' \in M$. Let $x \in U_\alpha{}^* \cap U_{\alpha'}{}^*$, if $x \in X_\beta$, for a $\beta \in M$, then $\alpha \leqq \beta$, $\alpha' \leqq \beta$, and x belongs to the set $(\pi_{\beta\alpha}{}^{-1} U_\alpha) \cap (\pi_{\beta\alpha'}{}^{-1} U_{\alpha'})$, which is open in X_β and thus belongs to $\mathcal{U}$. On the other hand, if $x \in X$, then x belongs to the set $(\pi_\alpha{}^{-1} U_\alpha) \cap (\pi_{\alpha'}{}^{-1} U_{\alpha'})$ which is open in X. Therefore, there is a $\beta \in M$ and an open set $U_\beta \subset X_\beta$ such that $x \in \pi_\beta{}^{-1} U_\beta \subset (\pi_\alpha{}^{-1} U_\alpha) \cap (\pi_{\alpha'}{}^{-1} U_{\alpha'})$. One can also achieve that $U_\beta \subset U_\alpha \cap U_{\alpha'}$, so that $U_\beta{}^* \subset U_\alpha{}^* \cap U_{\alpha'}{}^*$.

We now define the topology of X^* by taking the family $\mathcal{U}$ for a basis of all open sets. The properties that we established above show that $\mathcal{U}$ can be given such a role. Notice that X_α and X inherit from X^* their natural topologies as the relative topologies. X^* is clearly a Hausdorff space if all X_α are Hausdorff spaces; this enables us to use in X^* nets and their limits (see [7], Chapter 2).

Theorem 1. *Let $\{X_\alpha, \pi_{\beta\alpha}\}$, $\alpha \in M$, be an inverse system of nonempty Hausdorff compacta (over a directed set M). Choose for every $\alpha \in M$ an arbitrary point $x_\alpha \in X_\alpha \subset X^*$. Then $\{x_\alpha\}$, $\alpha \in M$, is a net in X^* which has at least one cluster point $x \in X \subset X^*$.*

Proof. Let M_α denote the set of all $\beta \in M$ with $\alpha \leqq \beta$. Then $\{\pi_{\beta\alpha} x_\beta\}$, $\beta \in M_\alpha$, is a net of X_α. Let $A \subset X_\alpha$ be the set of all cluster points of this net. A is non-empty, because X_α is compact. Furthermore, A is closed in X_α. Thus the sets $B_\beta = \pi_{\beta\alpha}{}^{-1}(A) \subset X_\beta$, $\beta \in M_\alpha$, and $B = \pi_\alpha{}^{-1}(A) \subset X$ are also closed. We shall now prove the following proposition:

(i) The set $B \subset X$ is not empty.

Take any $a \in A$ (A is not empty) and any open set $U_\alpha \subset X_\alpha$ containing a. Since a is a cluster point of the net $\{\pi_{\beta\alpha} x_\beta\}$, $\beta \in M_\alpha$, for every $\beta \in M_\alpha$ there is a $\gamma \geqq \beta$ such that $\pi_{\gamma\alpha} x_\gamma \in U_\alpha$. On the other hand, $\pi_{\gamma\alpha} x_\gamma = \pi_{\beta\alpha}(\pi_{\gamma\beta} x_\gamma) \in \pi_{\beta\alpha}(X_\beta)$ so that $(\pi_{\beta\alpha}(X_\beta)) \cap U_\alpha \neq 0$. Consequently, a is a cluster point of $\pi_{\beta\alpha}(X_\beta)$ and thus $a \in \pi_{\beta\alpha}(X_\beta)$, for all $\beta \in M_\alpha$ ($\pi_{\beta\alpha}(X_\beta)$ is compact). This proves that the sets $B_\beta = \pi_{\beta\alpha}{}^{-1}(A)$ are non-empty compact spaces. Since obviously $\pi_{\beta'\beta}(B_{\beta'}) \subset B_\beta$, for $\beta \leqq \beta'$, the sets B_β form an inverse system. The inverse limit of this system is contained in $B = \pi_\alpha{}^{-1}(A) \subset X$ and is non-empty (see Theorem 3.6, p. 217 of [5]), proving the assertion (i).

Now assume that $\{x_\alpha\}$, $\alpha \in M$, has no cluster points in X. Then for every $x \in X$ one can find an opet set $U_\alpha{}^*$ (given by (2)) and an $\alpha(x) \in M$ such that $U_\alpha{}^*$ contains no points of $\{x_\beta\}$, $\beta \in M_{\alpha(x)}$ and $x \in U_\alpha{}^*$. Since X is compact, there is a finite collection of sets $U_{\alpha(1)}{}^*, \ldots, U_{\alpha(n)}{}^*$ covering X and disjoint with $\{x_\beta\}$, $\beta \in M_\gamma$, where γ is a suitable element of M, $\gamma \geqq \alpha(1), \ldots, \alpha(n)$. Consider now the net $\{\pi_{\beta\gamma} x_\beta\}$, $\beta \in M_\gamma$, and the open set $U_\gamma = \pi_{\gamma\alpha(1)}{}^{-1}(U_{\alpha(1)}) \cup \ldots \cup \pi_{\gamma\alpha(n)}{}^{-1}(U_{\alpha(n)})$ of X_γ. Clearly,

$$\pi_\gamma{}^{-1}(U_\gamma) \supset X. \tag{4}$$

On the other hand, it is readily seen that $U_\gamma{}^*$ is contained in the union of the sets $U_{\alpha(1)}{}^*, \ldots, U_{\alpha(n)}{}^*$ and therefore contains no points of $\{x_\beta\}$, $\beta \leqslant M$. Consequently, $\{\pi_{\beta\gamma} x_\beta\}$, $\beta \leqslant M_\gamma$, is a net entirely contained in the closed set $X_\gamma \setminus U_\gamma$. Hence, the set A of its cluster points belongs also to $X_\gamma \setminus U_\gamma$. According to (i) the set $B = \pi_\gamma^{-1}(A) \subset \subset X$ is not empty and is contained in $\pi_\gamma^{-1}(U_\gamma)$ by (4). Therefore, $(A \cap U_\gamma) \supset \pi_\gamma B \neq 0$, which is a contradiction to $A \subset X_\gamma \setminus U_\gamma$.

Theorem 2. *Let $\{X_\alpha, \pi_{\beta\alpha}\}$, $\alpha \leqslant M$, be an inverse system of (non-empty) Hausdorff compacta and let U be an open set in X^* such that $X \subset U$. Then there is a $\gamma \leqslant M$ such that $X_\beta \subset U$, for all $\beta \geqq \gamma$.*

Proof. Since the sets (2) form a basis for open sets around points of X and since X is compact, it is easy to find an open set V of X^* such that $X \subset V \subset U$ and that

$$V = U_{\alpha(1)}^* \cup \ldots \cup U_{\alpha(n)}^*. \tag{5}$$

In order to prove Theorem 2, it sufficies to find a $\gamma \leqslant M$, $\gamma \geq \alpha(1), \ldots, \alpha(n)$, such that

$$X_\gamma \subset V, \tag{6}$$

because (6) will then imply

$$X_\beta \subset V \subset U, \text{ for all } \beta \geq \gamma. \tag{7}$$

Suppose now that no $\gamma \leqslant M$ satisfies (6). Then one could find a point $x_\gamma \leqslant X_\gamma \setminus V$ for every $\gamma \leqslant M$. $\{x_\gamma\}, \gamma \leqslant M$, would be a net in X^*, satisfying the conditions of Theorem 1 and contained entirely in $X^* \setminus V$. Hence, this net could not have cluster points in $X \subset V$, which contradicts Theorem 1.

Theorem 3. *If $\{X_\alpha, \pi_{\beta\alpha}\}$ is an inverse system of (non-empty) Hausdorff compacta, then the space X^* is Hausdorff and paracompact.*

Proof. Let $\{V_\mu\}$ be an open covering of X^*. Since X is compact, there is a finite subcollection, consisting of sets $V_{\mu(1)}, \ldots, V_{\mu(n)}$, which covers X. If V denotes the union of this subcollection, then there is an $\alpha \leqslant M$ such that all X_β, $\beta \leqslant M_\alpha$, are contained in V (Theorem 2). Notice that the set

$$X_\alpha^* = (\bigcup_{\beta \geq \alpha} X_\beta) \cup X \tag{8}$$

is an open subset of X^*, because it is of type (2) (with $U_\alpha = X_\alpha$).

Now consider the following collection $\mathcal{V}$ of open sets of X^*: take first the open sets $(X_\alpha^*) \cap V_{\mu(1)}, \ldots, (X_\alpha^*) \cap V_{\mu(n)}$ for members of $\mathcal{V}$. Furthermore, for every $\beta \leqslant M \setminus M_\alpha$, consider the open covering $\{X_\beta \cap V_\mu\}$ of X_β and take elements of a finite subcovering as new elements of $\mathcal{V}$ (recall that X_β is compact and open in X^*). The family $\mathcal{V}$ of open sets of X^*, which we just defined, is clearly a star-finite covering of X^* which refines the covering $\{V_\mu\}$. $\mathcal{V}$ is a fortiori a locally finite refinement of $\{V_\mu\}$.

2. *Mappings of X into ANR-s*

In this section we are concerned with absolute neighborhood retracts R for metric spaces (abbreviated as ANR). Recall that ANR-s can be characterized as neighborhood retracts of convex subsets C of Banach spaces (see [4], p. 363). We shall also use the following theorem due to R. Arens (Theorem 4.1, p. 18 of [3]; see also [2]):

Let C *be a convex subset of a Banach space. Every mapping* f *of a closed subset of a Hausdorff paracompact space into* C *admits an extension* f_* *to the whole space (the values of* f_* *are in* C).

The following theorem generalizes a lemma by M. Abe ([1], 2.2, p. 188) and Theorem 11.9, p. 287 of [5].

Theorem 4. *Let* $\{X_\alpha, \pi_{\beta\alpha}\}$, $\alpha \in M$, *be an inverse system of Hausdorff compacta and let* $f: X \to R$ *be a mapping of their limit into an ANR. Then there is an* $\alpha \in M$ *such that for every* $\beta \in M_\alpha$ *one can define a map* $f_\beta : X_\beta \to R$ *with the property that* $f_\beta \pi_\beta$ *is homotopic to* f *and* $f_\beta \pi_{\gamma\beta}$ *is homotopic to* f_γ, *for all* $\gamma \geq \beta > \alpha$.

Proof. Consider R as a neighborhood retract of a convex set C of a Banach space. Let V be a neighborhood of retraction of R in C and let $\Theta : V \to R$ be a retraction. Consider f as a mapping of X into C. Since X is a closed subset of X^* and X^* is Hausdorff and paracompact (Theorem 3), we can apply the theorem of Arens and obtain a mapping $f_* : X^* \to C$ extending f.

Choose now, for every $x \in X$, a convex open set $V(x)$ of C such that $f(x) \in V(x) \subset V$ and choose an open set $U_{\alpha(x)}^*$ of type (2) such that $x \in U_{\alpha(x)}^* \subset f_*^{-1}(V(x))$. Notice that $X \cap U_{\alpha(x)}^* = \pi_{\alpha(x)}^{-1} U_{\alpha(x)}$, so that for $\beta \in M_{\alpha(x)}$, we get

$$\pi_\beta(X \cap U_{\alpha(x)}^*) \subset \pi_{\beta\alpha(x)}^{-1}(U_{\alpha(x)}) \subset U_{\alpha(x)}^* \subset f_*^{-1}(V(x)). \tag{9}$$

Thus, for $\beta \in M_{\alpha(x)}$,

$$f_* \pi_\beta(X \cap U_{\alpha(x)}^*) \subset V(x). \tag{10}$$

The collection $\{U_{\alpha(x)}^*\}, x \in X$, is an open covering of X and we can choose a finite subcovering consisting of sets $U_{\alpha(1)}^*, \ldots, U_{\alpha(n)}^*$, where $\alpha(i) = \alpha(x_i)$, $x_i \in X$. If we denote the convex set $V(x_i)$ by V_i, then (10) goes over into

$$f_* \pi_\beta(X \cap U_{\alpha(i)}^*) \subset V_i, \; i = 1, \ldots, n, \tag{11}$$

and is valid for all β larger than $\alpha(1), \ldots, \alpha(n)$.

Now define a homotopy in C, connecting f and $f_*\pi_\beta$, $\beta \geqq \alpha$, by joining points $f(x)$ and $f_*\pi_\beta(x)$ by a line segment, obviously lying in C. We want to show that this segment lies actually in the retraction neighborhood V. Given any $x \in X$, there is an $i \in \{1, \ldots, n\}$ such that $x \in U_{\alpha(i)}^* \subset f_*^{-1}(V_i)$. Thus, $f(x) = f_*(x) \in V_i$. On the other hand, (11) shows that $f_*\pi_\beta(x) \in V_i$. Since V_i is convex and is lying in V, it follows that the segment joining $f(x)$ and $f_*\pi_\beta(x)$ is contained in V_i and thus in V too. In other words, for $\beta \geqq \alpha(1), \ldots, \alpha(n)$, we have a homotopy in V connecting $f(x)$ and $f_* \pi_\beta(x)$. Choose now an $\alpha \geqq \alpha(1), \ldots, \alpha(n)$ such that all X_β, $\beta \in M_\alpha$, lie in $U_{\alpha(1)}^* \cup \ldots \cup U_{\alpha(n)}^* \subset f_*^{-1}(V)$; this is possible due to Theorem 2. Now define

$$f_\beta = \Theta f_* | X_\beta, \ \beta \in M_\alpha. \tag{12}$$

We have obtained already a homotopy, connecting f and $f_* \pi_\beta$ in V, for all $\beta \in M_\alpha$. Composing this homotopy with the retraction Θ, we now get a homotopy connecting f and $\Theta f_* \pi_\beta = f_\beta \pi_\beta$ in R. A similar argument shows that $f_\beta \pi_{\gamma\beta}$ and f_γ are homotopic in R, for all $\gamma \geqq \beta \geqq \alpha$.

Theorem 5. *Let $\{X_\alpha, \pi_{\beta\alpha}\}$ and $\{Y_\alpha, \sigma_{\beta\alpha}\}$, $\alpha \in M$, be two inverse systems of Hausdorff compacta and let $X_\alpha \subset Y_\alpha$, $\sigma_{\beta\alpha} | X_\beta = \pi_{\beta\alpha}$; let $X \subset Y$ be the corresponding limits. Let R be an ANR and let, for a fixed $\alpha \in M$, $f_\alpha : X_\alpha \to R$ be a given mapping such that $f_\alpha \pi_\alpha : X \to R$ is extendible to Y. Then there is a $\beta \in M_\alpha$ such that $f_\alpha \pi_{\beta\alpha} : X_\beta \to R$ is extendible to Y_β.*

This theorem generalizes Lemma 8, p. 199 of [9]. Disposing of Theorem 2 and other properties of the spaces X^* and Y^* it is easy to carry on the necessary modifications in the proof given in [9] in order to obtain a proof of Theorem 5. Notice in particular that the space X_α^*, defined in (8), is a closed subset of the corresponding space Y_α^*. Furthermore, let $\pi_\alpha^* : X_\alpha^* \to X_\alpha$ be a mapping coinciding with $\pi_{\beta\alpha}$ on X_β, $\beta \geqq \alpha$, and coinciding with π_α on X. The fact that the sets (2) are open in X^* insures the continuity of π_α^*.

3. *Continuity Theorem for Homology of Function Spaces*

Let X be a Hausdorff compact and Y a metrizable space. We denote by $\langle X, Y\rangle$ the space of all continuous mappings $f : X \to Y$; $\langle X, Y\rangle$ is given the compact-open topology (e. g., see [7], p. 221). If X' is another Hausdorff compact and $g : X' \to X$ is a mapping, then the transformation $G : \langle X, Y\rangle \to \langle X', Y\rangle$ defined by

$$G(f) = f g, \tag{13}$$

i. e. by composing f and g. If C' is a closed subset of X', then C' and $g(C')$ are compact. If U is an open set of Y, then

$$G^{-1}\{f' \mid f' \in \langle X', Y\rangle, \ f'(C) \subset U\} = \{f \mid f \in \langle X, Y\rangle, \ f g(C') \subset U\}. \tag{14}$$

This shows that G is continuous.

Now consider an inverse system of Hausdorff compact spaces $\{X_\alpha, \pi_{\beta\alpha}\}$, $\alpha \in M$, and a metrizable space Y. Let $\Pi_{\alpha\beta} : \langle X_\alpha, Y\rangle \to \langle X_\beta, Y\rangle$ be the induced mappings. Let $H_q(\langle X_\alpha, Y\rangle, G)$ denote the q-th singular homology group of $\langle X_\alpha, Y\rangle$ with coefficients in the group G and let $\Pi_{\alpha\beta*}$ be the homomorphism induced by $\Pi_{\alpha\beta}$. Then $\{H_q(\langle X_\alpha, Y\rangle, G), \Pi_{\alpha\beta*}\}$, $\alpha \in M$, is a direct system of groups. Furthermore, if X is the limit of X_α, then the mappings $\pi_\alpha : X \to X_\alpha$

induce mappings $\Pi_\alpha : \langle X_\alpha, Y\rangle \to \langle X, Y\rangle$ and we have homomorphisms $\Pi_{\alpha *} : X_q(\langle X_\alpha, Y\rangle, G) \to H_q(\langle X, Y\rangle, G)$, which induce a natural homomorphism π of the direct limit of $H_q(\langle X_\alpha, Y\rangle, G)$ into $H_q(\langle X, Y\rangle, G)$.

Theorem 6. *Let* $\{X_\alpha, \pi_{\beta\alpha}\}$, $\alpha \in M$, *be an inverse system of Hausdorff compacta and let* R *be an absolute neighborhood retract. Then* π *establishes a natural isomorphism between the direct limit of* $H_q(\langle X_\alpha, R\rangle, G)$ *and the group* $H_q(\langle X, R\rangle, G)$, *where* X *is the inverse limit of* $\{X_\alpha, \pi_{\beta\alpha}\}$ *and* G *is any group of coefficients (the homology is taken in the sense of singular theory).*

The proof is carried on first by interpreting singular homology of the mapping spaces $\langle X, R\rangle$ as X-homology of R, in the sense of [9] (see I. 4, p. 190). Obvious modifications of the arguments on p. 200—202 of [9] give a proof of Theorem 6. Notice that the Lemma of Abe and Lemma 8 of [9] have to be replaced by the above Theorems 4 and 5.

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O INVERZNIM LIMESIMA KOMPAKTNIH PROSTORA

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Sadržaj

U ovom članku se promatraju inverzni[1] sistemi $\{X_\alpha, \pi_{\beta\alpha}\}$ Hausdorffovih kompaktnih prostora X_α i to nad proizvoljnim usmjerenim skupovima $M = \{\alpha\}$. Relacijom (1) se uvodi u razmatranje skup sastavljen od svih članova sistema (koje smatramo disjunktnima)

[1]) Osnovne definicije i svojstva inverznih sistema izloženi su na pr. u [5] i [8].

i od graničnog skupa X. U skup X^* se uvodi topologija time, što se definira jedna baza otvorenih skupova $\mathfrak{U}$ na ovaj način. $\mathfrak{U}$ se sastoji iz svih skupova $U_\alpha \subset X_\alpha$, koji su otvoreni u X_α, $\alpha \in M$, te iz svih skupova oblika (2); pri tome je $\pi_\alpha : X \rightarrow X_\alpha$ prirodno preslikavanje, koje pripada promatranom sistemu.

Pokazuje se više svojstava prostora X^*. Napose se pokazuje da svaki otvoreni skup U iz X^*, koji sadrži X, sadrži i sve X_β, počevši od nekog dovoljno velikog $\alpha \in M$ (Theorem 2.). Kao posljedica dobiva se da je X^* Hausdorffov i parakompaktan. Ove činjenice omogućuju da se primijeni jedan teorem R. Arensa o proširivanju neprekidnih preslikavanja, koja su definirana na nekom zatvorenom dijelu nekog parakompaktnog prostora, a vrijednosti im leže u nekom konveksnom dijelu nekog Banachovog prostora. Služeći se tim teoremom dokazuje se na primjer ovo (Theorem 4):

Neka je $\{X_\alpha, \pi_{\beta\alpha}\}$ jedan inverzni sistem Hausdorffovih kompakata, neka je R jedan apsolutni okolinski retrakt (za metričke prostore) i neka je dano neprekidno preslikavanje $f : X \rightarrow R$. Tada postoji $\alpha \in M$ sa svojstvom da je, za svaki $\beta \geqq \alpha$, moguće definirati jedno neprekidno preslikavanje $f_\beta : X_\beta \rightarrow R$ i to na takav način, da je preslikavanje $f_\beta \pi_\beta$ homotopno sa f, dok je $f_\beta \pi_{\gamma\beta}$ homotopno sa f_γ, za sve $\gamma \geqq \beta \geqq \alpha$.

Služeći se ovim i još jednim sličnim rezultatom (Theorem 5) dokazuje se glavni rezultat radnje:

Neka je R *jedan apsolutni okolinski retrakt a* $\langle X_\alpha, R\rangle$ *i* $\langle X, R\rangle$ *neka su prostori svih neprekidnih preslikavanja od* X_α *u* R, *odnosno od* X *u* R. *Neka je* G *neka Abelova grupa, a* $H_q(Y, G)$ *neka označuje* q-*dimenzionalnu singularnu grupu homologije prostora* Y *s koeficijentima u* G. *Tada inverznom sistemu* $\{X_\alpha, \pi_{\beta\alpha}\}$ *pripada direktni sistem grupa* $\{H_q(\langle X_\alpha, R\rangle, G)\}$. *Direktni limes ovog sistema je grupa izomorfna grupi* $H_q(\langle X, R\rangle, G)$.

Ovaj teorem, dakle, pokazuje da je funktor homologije funkcionalnog prostora $\langle X_\alpha, R\rangle$ neprekidan s obzirom na prijelaz varijable X_α na inverznu granicu. Time je dobiveno poopćenje jednog teorema iz autorove disertacije (vidi Theorem 13, str. 200 u [9]) i riješen je Problem 1, koji se tamo navodi ([9], str. 202).

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