Aggregation and Diversification of Risks in the Solvency II Framework

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Croatian Quants Day
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Overview

- Way to Solvency II: QIS3 et al
- Fallacy: Multi-Level Aggregation
- Realizable Group Diversification Effects

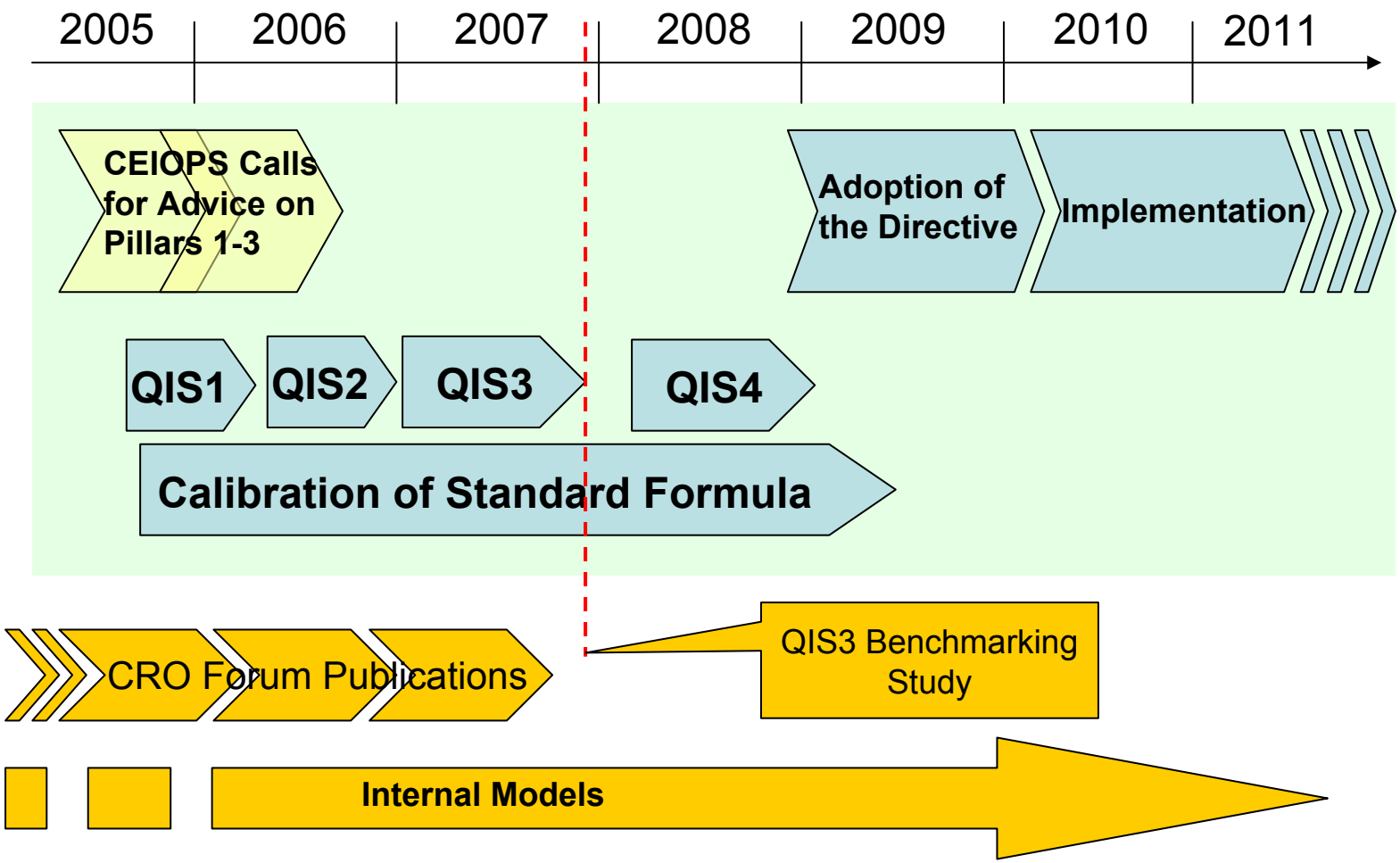
Solvency II

New European framework for regulating the solvency of insurers

Aiming at ensuring adequate policyholder protection

URL: http://ec.europa.eu/internal_market/insurance/solvency/index_en.htm

Way to Solvency II



CRO Forum QIS3 Benchmarking Study

Major differences between QIS3 and internal models:

1. **Diversification between risks within solo entities**
2. **Diversification across solo entities (geographic diversification)**

Available on www.croforum.org

CRO Forum ≠ CRO Forum

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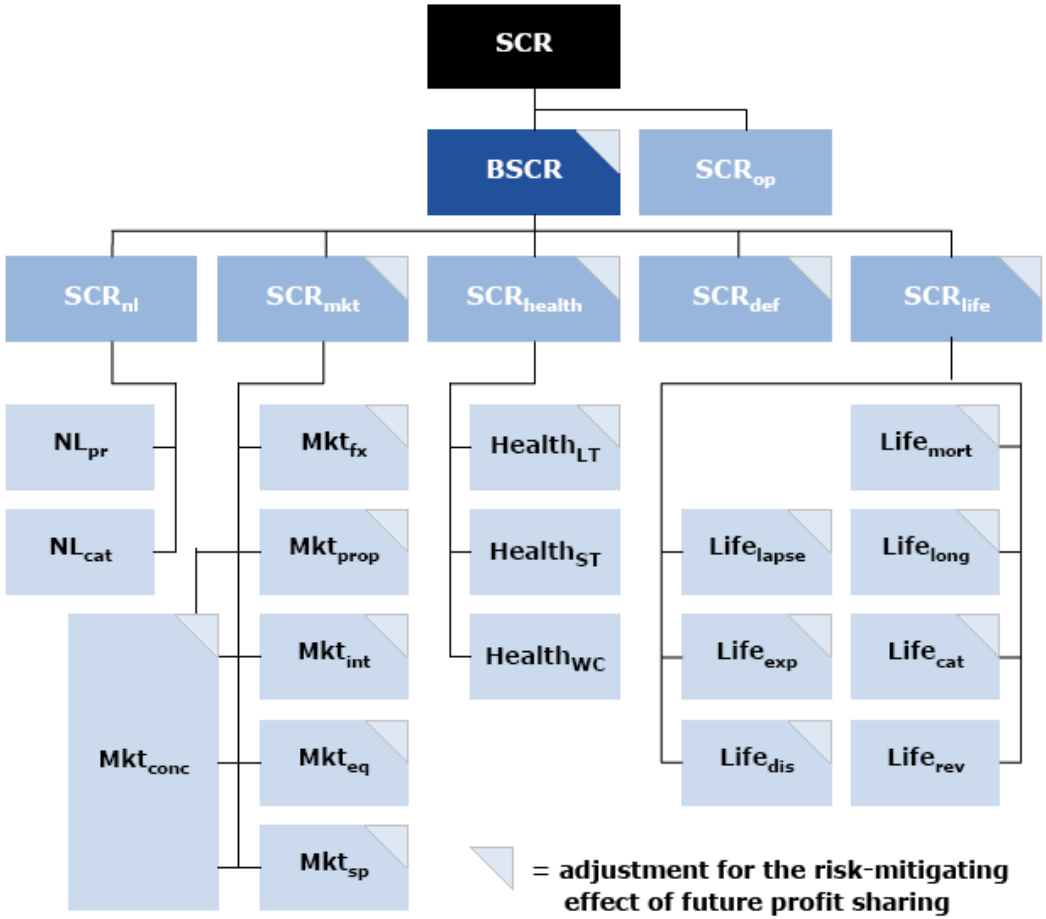
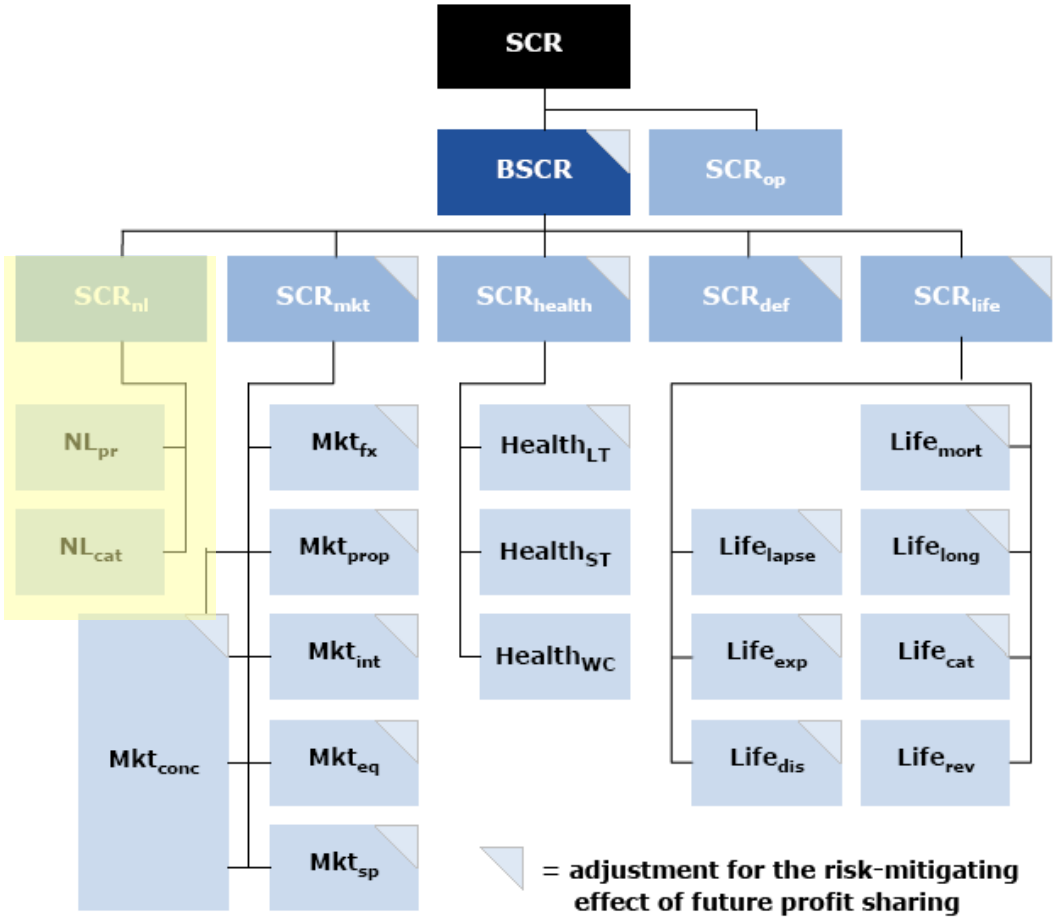
QIS4 Standard Model



(CEIOPS: QIS4 Technical Specifications. December 2007)

Conceptual Fallacy: Multi-Level Aggregation

Level 2: Between Risk Types, Within Risk Class



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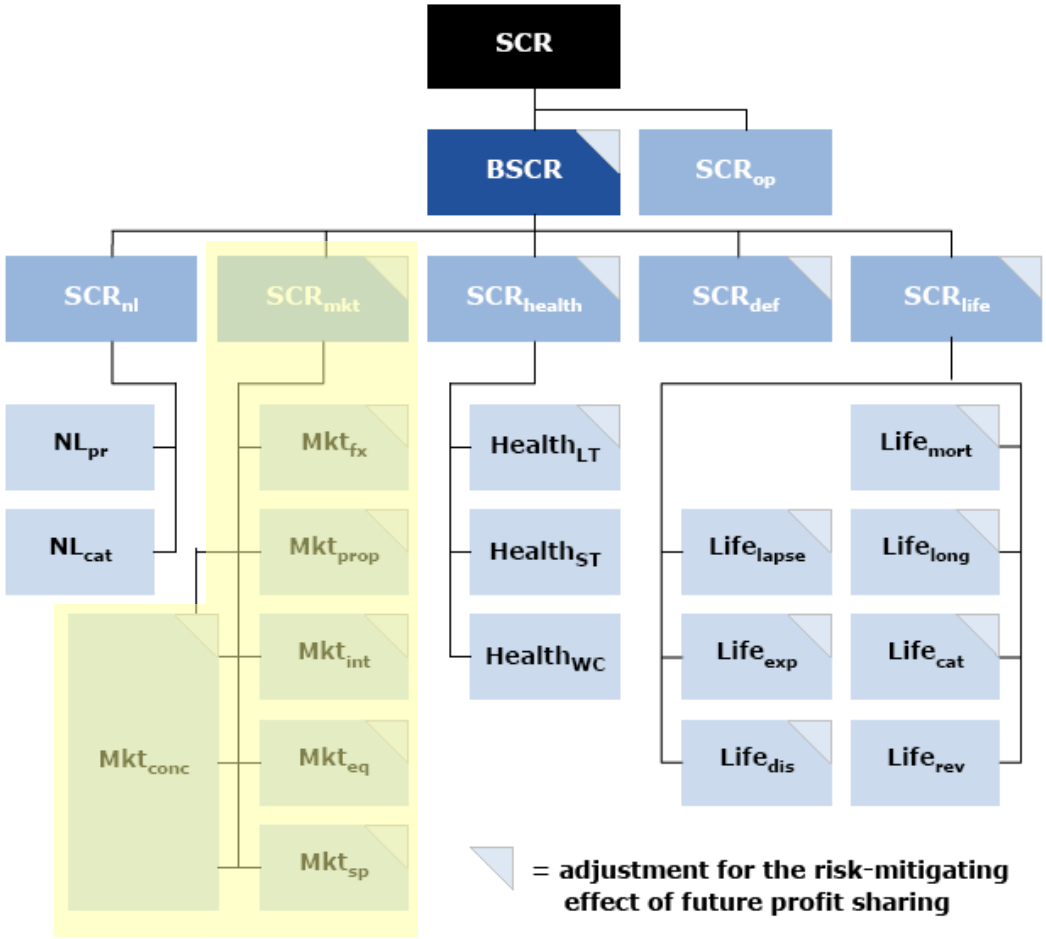

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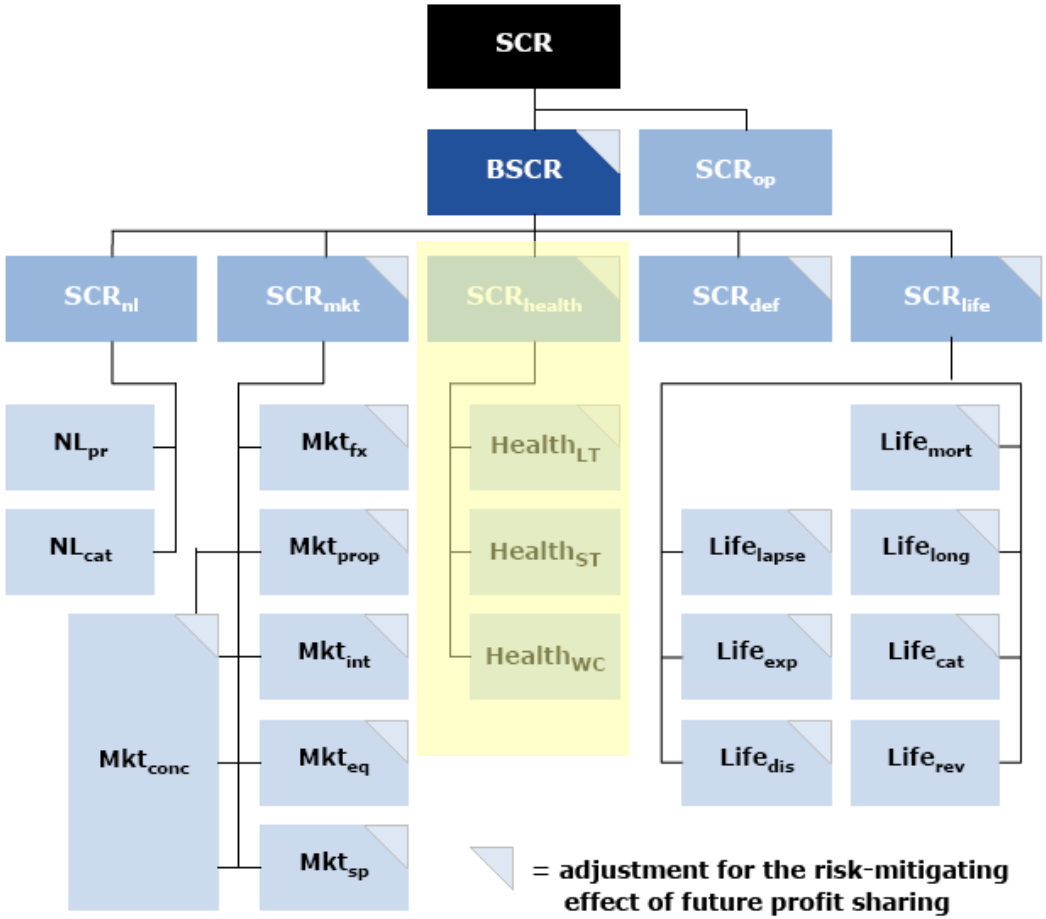

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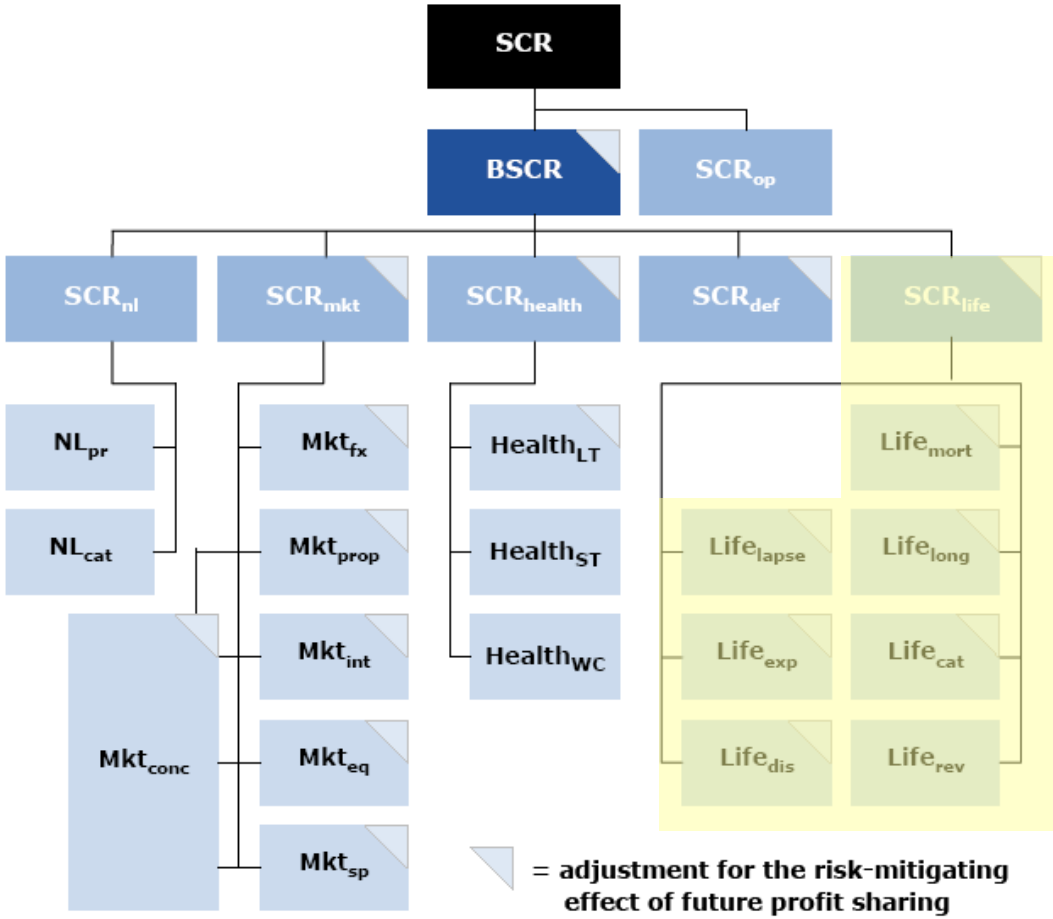

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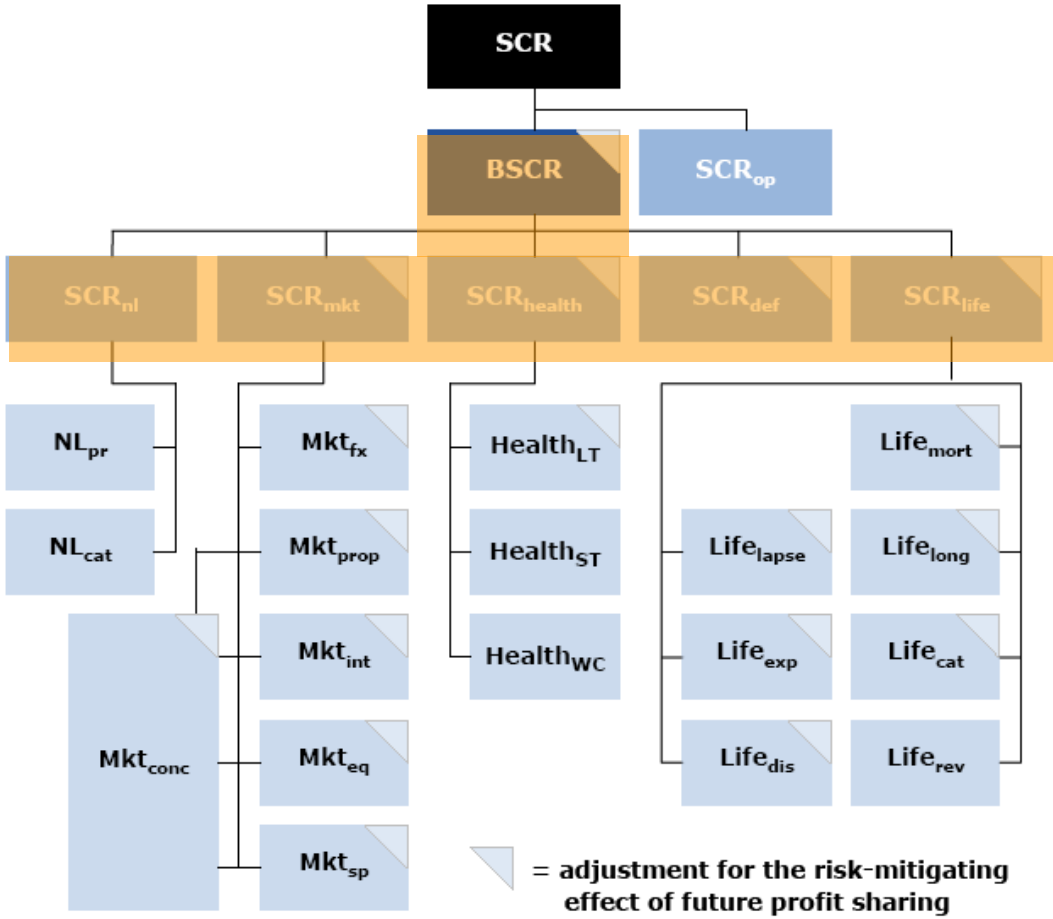
Conceptual Fallacy: Multi-Level Aggregation

Level 1: Between Risk Classes



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Conceptual Fallacy: Multi-Level Aggregation

Level 1: Correlation matrix for risk classes

CorrSCR=	SCR_{mkt}	SCR_{def}	SCR_{life}	SCR_{health}	SCR_{nl}
SCR_{mkt}	1				
SCR_{def}	0.25	1			
SCR_{life}	0.25	0.25	1		
SCR_{health}	0.25	0.25	0.25	1	
SCR_{nl}	0.25	0.5	0	0,25	1

Level 2: Correlation matrices for risk types

Market

CorrMkt	Mkt_{int}	Mkt_{eq}	Mkt_{prop}	Mkt_{sp}	Mkt_{conc}	Mkt_{fx}
Mkt_{int}	1					
Mkt_{eq}	0	1				
Mkt_{prop}	0.5	0.75	1			
Mkt_{sp}	0.25	0.25	0.25	1		
Mkt_{conc}	0	0	0	0	1	
Mkt_{fx}	0.25	0.25	0.25	0.25	0	1

Life

CorrLife =	$Life_{mort}$	$Life_{lon g}$	$Life_{dis}$	$Life_{lapse}$	$Life_{exp}$	$Life_{rev}$	$Life_{CAT}$
$Life_{mort}$	1						
$Life_{long}$	0	1					
$Life_{dis}$	0.5	0	1				
$Life_{lapse}$	0	0.25	0	1			
$Life_{exp}$	0.25	0.25	0.5	0.5	1		
$Life_{rev}$	0	0.25	0	0	0.25	1	
$Life_{CAT}$	0	0	0	0	0	0	1

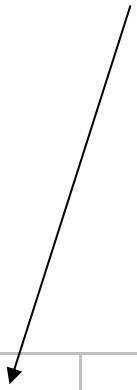
Non-Life

CorrNL=	NL_{pr}	NL_{CAT}
NL_{pr}	1	
NL_{CAT}	0	1

(CEIOPS: QIS4 Technical Specifications. December 2007)

Conceptual Fallacy: Multi-Level Aggregation

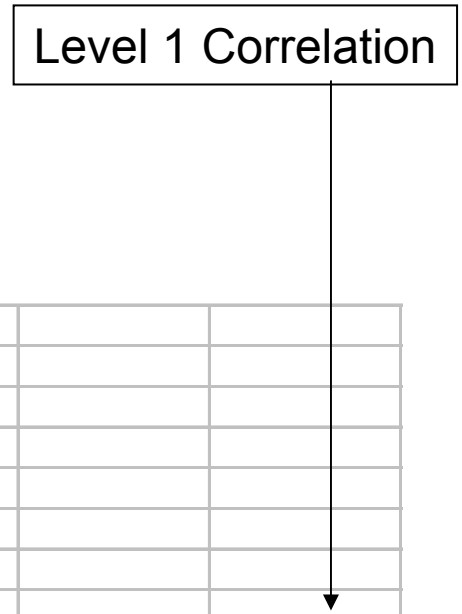
Internal models use correlation matrix across all risk types



	int	eq	pr	CAT			
int	1	0,8	0,2	0,1			
eq	0,8	1	0,1	0,3			
pr	0,2	0,1	1	0			
CAT	0,1	0,3	0	1			
SCR							
Level 2	7	22	40	10			
Top				54			

Conceptual Fallacy: Multi-Level Aggregation

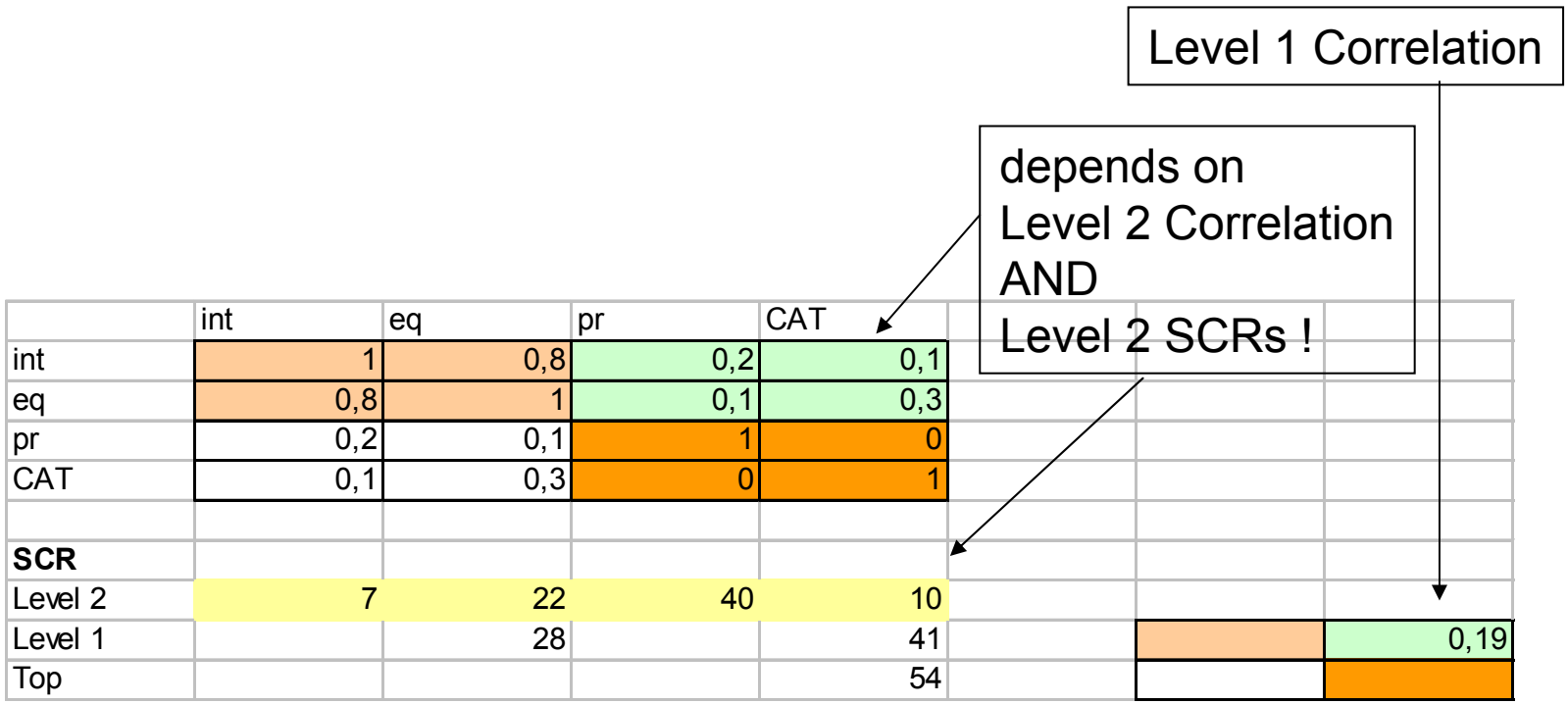
Implied Level 1 Correlation



	int	eq	pr	CAT			
int	1	0,8	0,2	0,1			
eq	0,8	1	0,1	0,3			
pr	0,2	0,1	1	0			
CAT	0,1	0,3	0	1			
SCR							
Level 2	7	22	40	10			
Level 1		28		41			0,19
Top				54			

Conceptual Fallacy: Multi-Level Aggregation

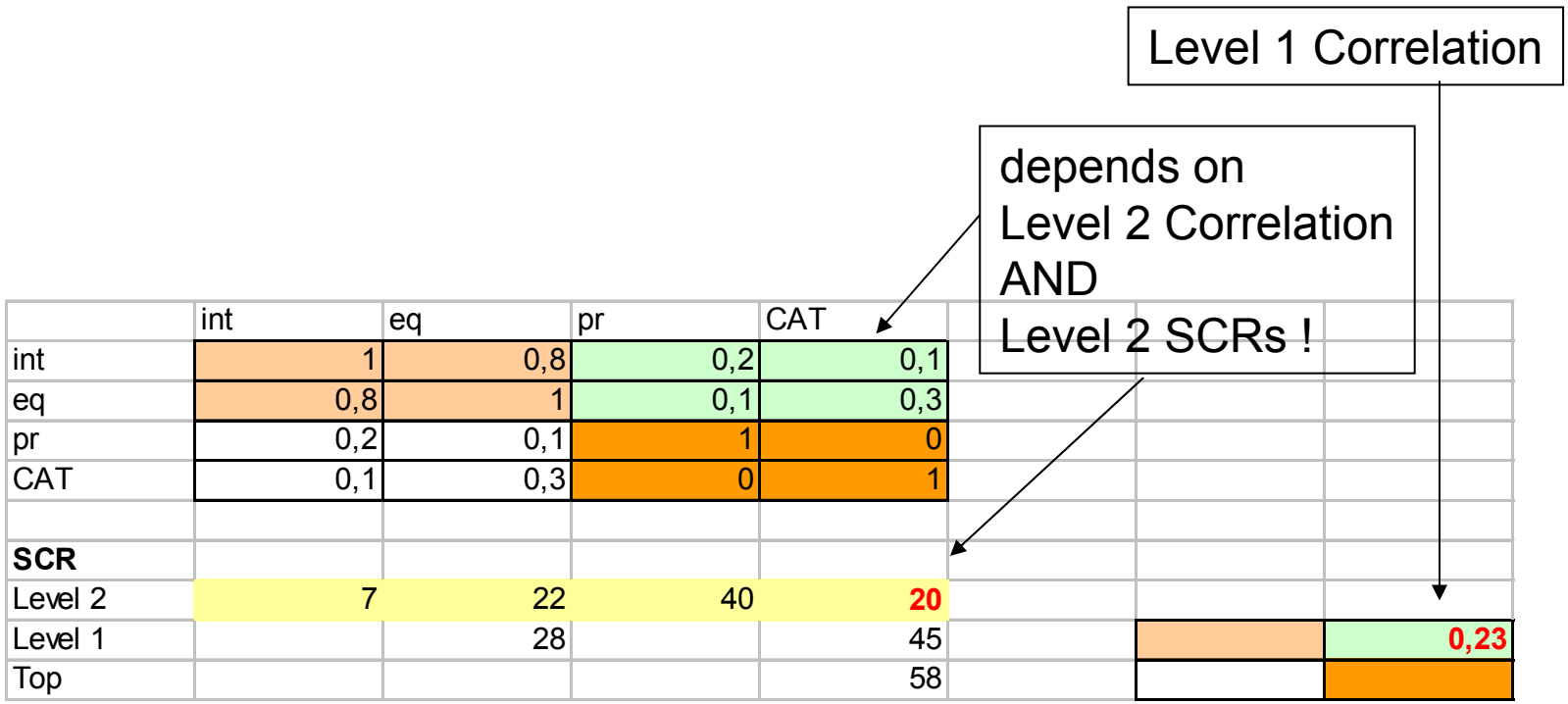
Implied Level 1 Correlation



	int	eq	pr	CAT			
int	1	0,8	0,2	0,1			
eq	0,8	1	0,1	0,3			
pr	0,2	0,1	1	0			
CAT	0,1	0,3	0	1			
SCR							
Level 2	7	22	40	10			
Level 1		28		41			0,19
Top				54			

Conceptual Fallacy: Multi-Level Aggregation

Change Level 2 input changes Level 1 correlation



	int	eq	pr	CAT
int	1	0,8	0,2	0,1
eq	0,8	1	0,1	0,3
pr	0,2	0,1	1	0
CAT	0,1	0,3	0	1
SCR				
Level 2	7	22	40	**20**
Level 1		28		45
Top				58

	0,23

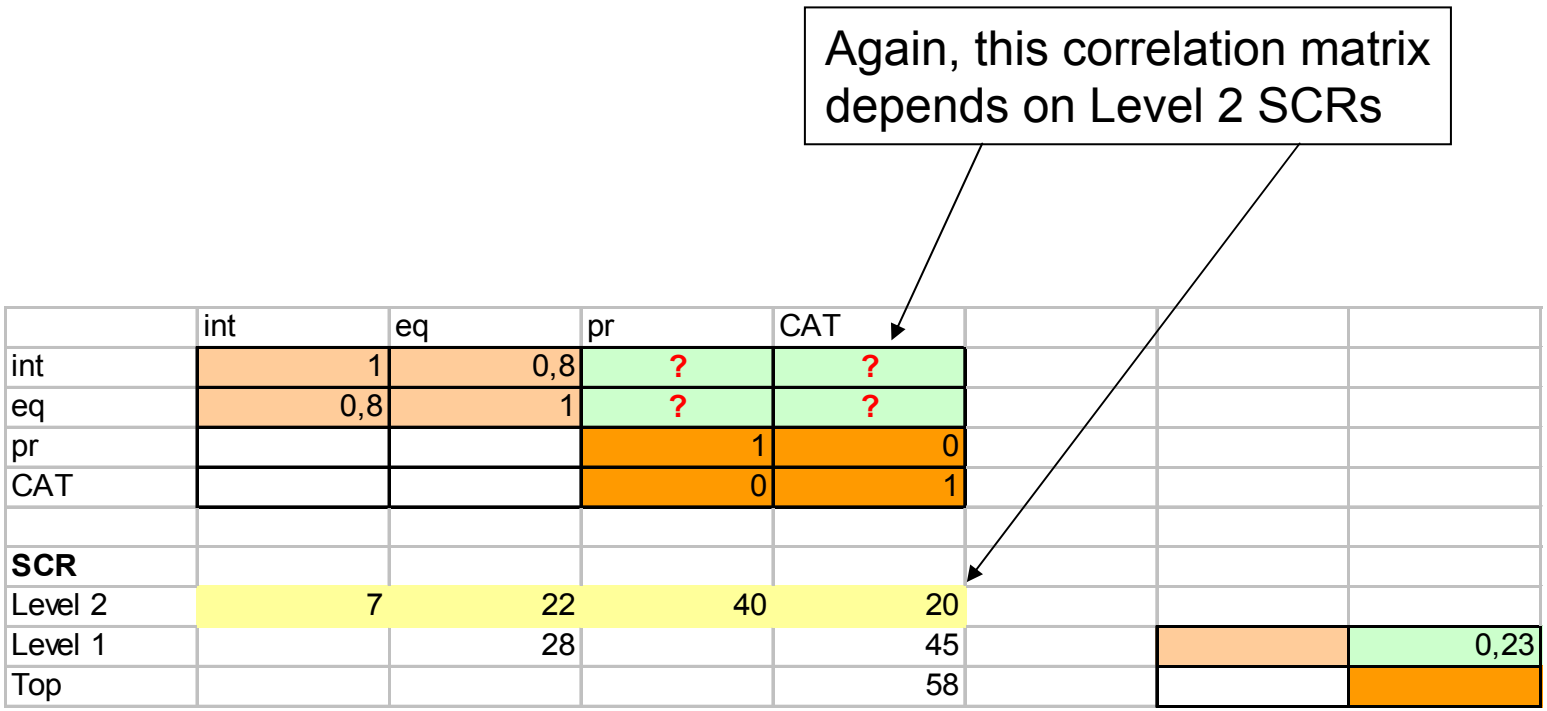
Conceptual Fallacy: Multi-Level Aggregation

How to infer from QIS3 parameters on full correlation matrix?

	int	eq	pr	CAT			
int	1	0,8	?	?			
eq	0,8	1	?	?			
pr			1	0			
CAT			0	1			
SCR							
Level 2	7	22	40	20			
Level 1		28		45			0,23
Top				58			

Conceptual Fallacy: Multi-Level Aggregation

How to infer from QIS3 parameters on full correlation matrix?



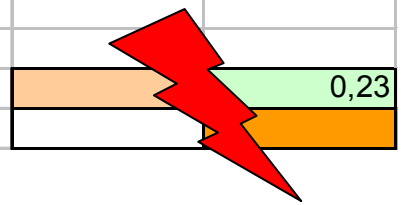
	int	eq	pr	CAT			
int	1	0,8	?	?			
eq	0,8	1	?	?			
pr			1	0			
CAT			0	1			
SCR							
Level 2	7	22	40	20			
Level 1		28		45			0,23
Top				58			

Conceptual Fallacy: Multi-Level Aggregation

Messages:

1. Direct comparison of standard and implied internal Level 1 correlation – and thus **DIVERSIFICATION** – is meaningless
2. Best solution: standard model provides Level 2 correlation

	int	eq	pr	CAT
int	1	0,8	0,2	0,1
eq	0,8	1	0,1	0,3
pr	0,2	0,1	1	0
CAT	0,1	0,3	0	1
SCR				
Level 2	7	22	40	20
Level 1		28		45
Top				58

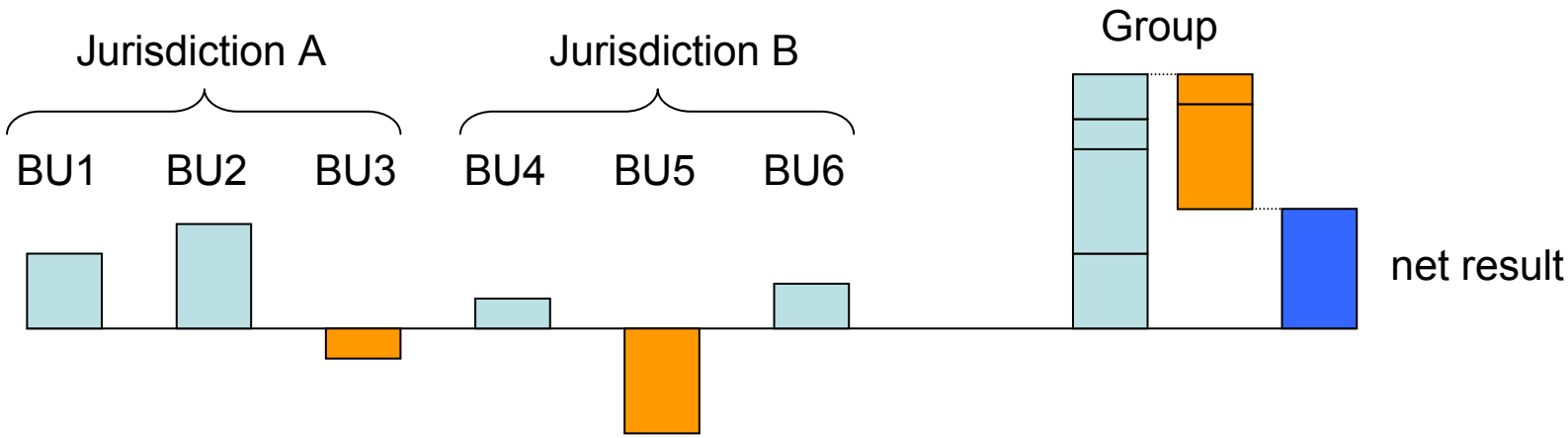


Overview

- Way to Solvency II: QIS3 et al
- Fallacy: Multi-Level Aggregation
- Realizable Group Diversification Effects

Group Diversification

Diversification effects require full fungibility of capital!



Regulatory constraints: regulators may prevent capital to be transferred between jurisdictions

Management constraints: companies' managements may refuse to provide necessary capital injections

→ **need of standardization for capital & risk transfers!**

Realizable Group Diversification Effects

- Difficult to quantify fungibility constraints via standard model

→ Approach via Capital and Risk Transfer (CRT) Instruments

- Developed as joint project with Munich Re to determine haircut on acknowledged diversification

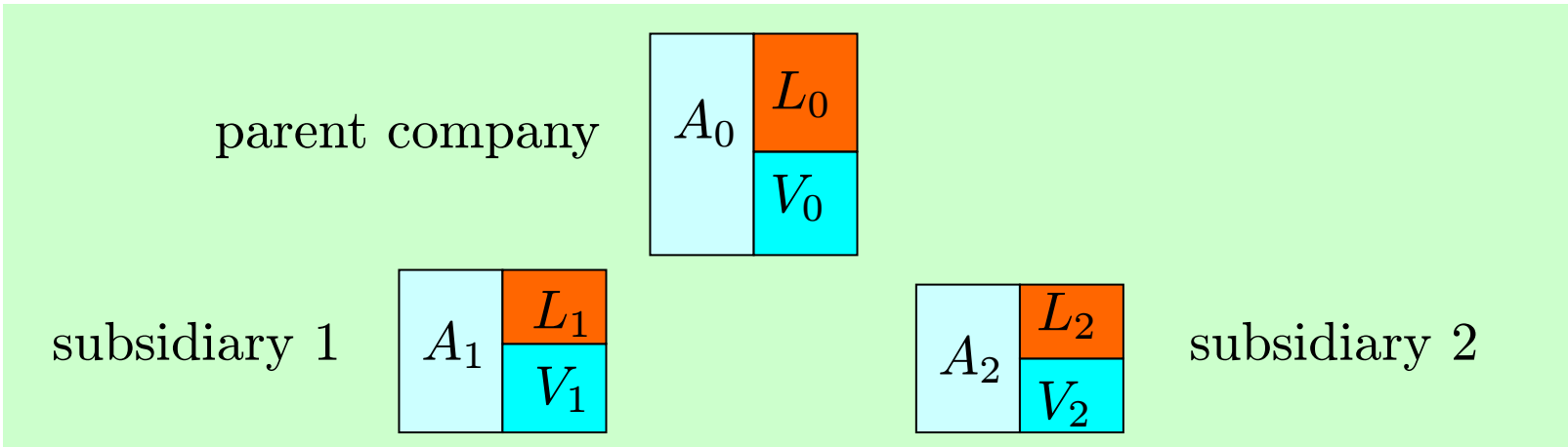
  (Filipovic—Kunz: "Realizable Group Diversification Effects")

- Also recommended by CRO Forum

  ("CRO Forum Response to Standard&Poor's": www.croforum.org)

Group Structure

Insurance group: parent company ($i = 0$), 2 subsidiaries ($i = 1, 2$)



Current available capital of entity i: $c_i = a_i - \ell_i$

Terminal value of entity i's asset-liability portfolio: $V_i = A_i - L_i$

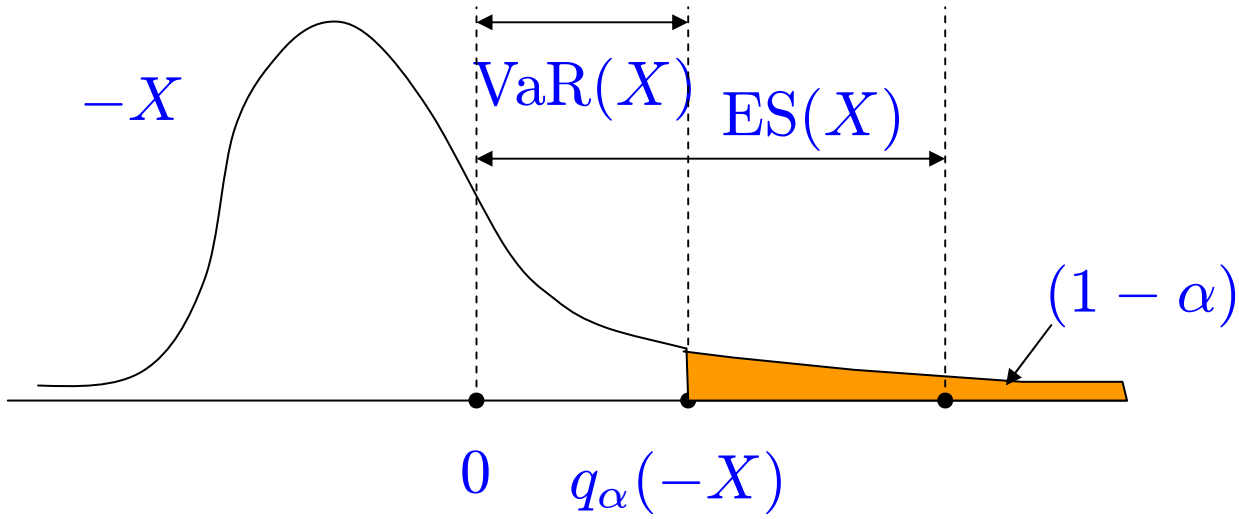
Value-at-Risk vs. Expected Shortfall

Let X be the change of portfolio value, α some confidence level

$$\mathrm{VaR}(X) := q_\alpha(-X) \quad \text{α-quantile of loss } -X$$

$$\mathrm{ES}(X) := \mathrm{VaR}(X) + \frac{1}{1-\alpha}\mathrm{E}\left[(-X - \mathrm{VaR}(X))^+\right]$$

Note: VaR is not convex (danger for aggregation!)



Required Capital

Three views on aggregation: stand alone (no diversification), considated (full diversification), CRT (realizable diversification)

Compare VaR and ES based results. Mathematical Fact:

$$\mathrm{VaR}_{99.5\%}(Z) = \mathrm{ES}_{98.7\%}(Z)$$

for normally distributed risks Z.

Write ρ as placeholder for $\mathrm{VaR}_{99.5\%}$ and $\mathrm{ES}_{98.7\%}$ in what follows

Stand Alone View (No Diversification)

Stand alone required capital for entity i:

$$k_i^{stal} = \rho[V_i - c_i]$$

Resulting group required capital is

$$k^{stal} = k_0^{stal} + k_1^{stal} + k_2^{stal}$$

sum of solo requirements

Consolidated view (full diversification)

Assumes one group balance sheet under full fungibility of capital.

Group required capital k^{cons} is

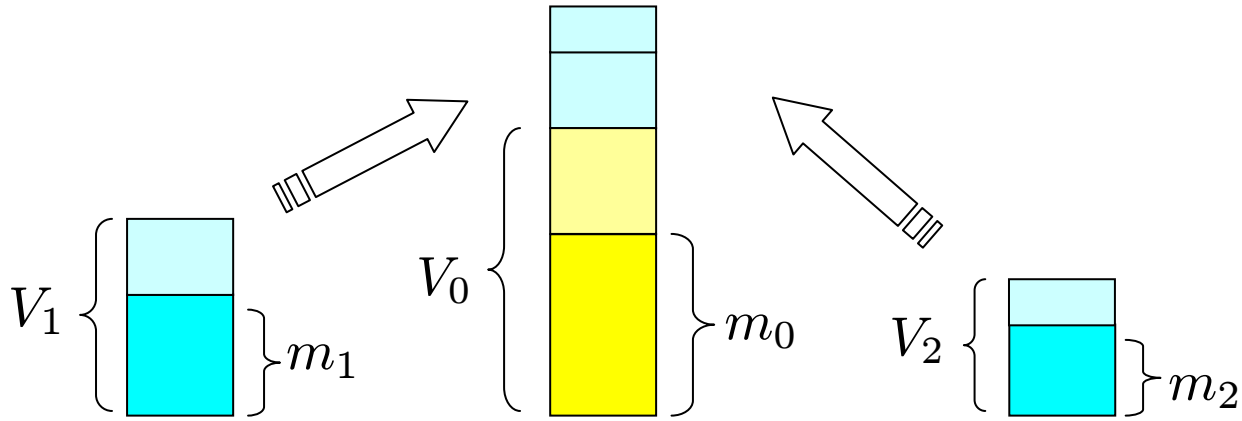
$$k^{cons} = \rho\left[(V_0 - c_0) + (V_1 - c_1) + (V_2 - c_2)\right].$$

The resulting **relative diversification effect (RDE)** becomes

$$RDE^{cons} = \frac{k^{stal} - k^{cons}}{k^{stal}}.$$

CRT view (realizable diversification)

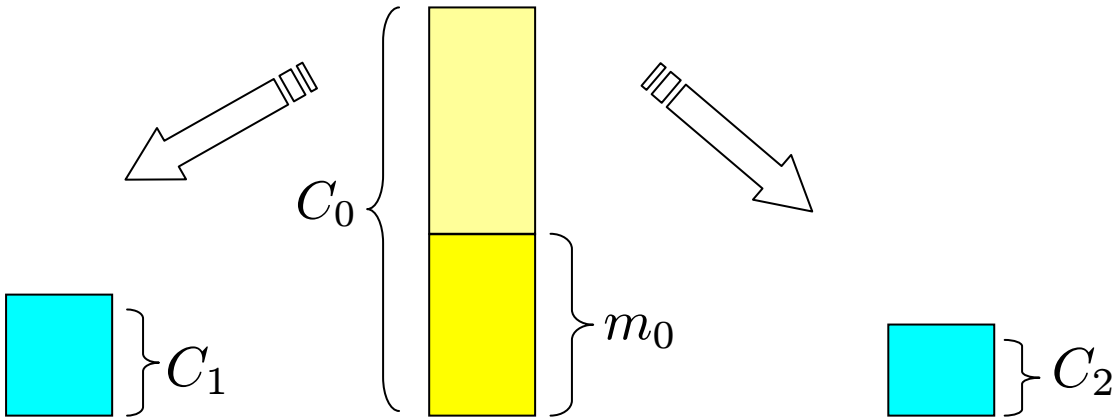
Capital is fungible above **tied capital level** m_i



$\rightarrow$ a priori distribution of total surplus across entities:

$$C_0 = V_0 + (V_1 - m_1)^+ + (V_2 - m_2)^+$$
$$C_i = \min\{V_i, m_i\}, \quad i = 1, 2.$$

Parent company in turn provides guarantees G_1, G_2



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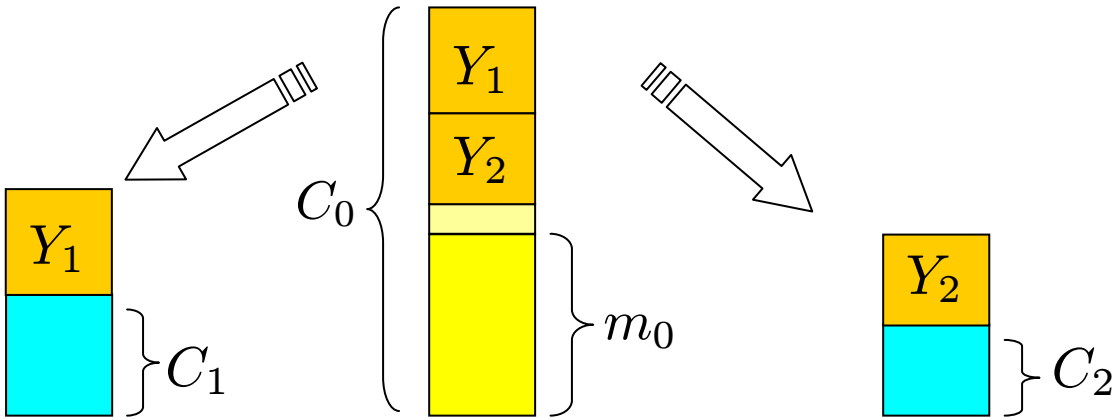


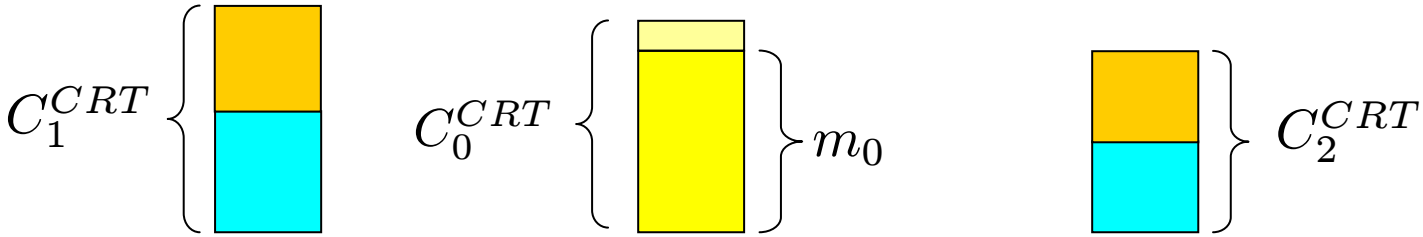
Default option on guarantees: Actual cash flow for guarantee G_i

$$Y_i = \begin{cases} G_i & \text{if } (C_0 - m_0)^+ \geq G_1 + G_2, \\ \frac{G_i}{G_1+G_2} \times (C_0 - m_0)^+ & \text{otherwise.} \end{cases}$$

Default probability is $\mathbb{P}\left[(C_0 - m_0)^+ < G_1 + G_2\right]$

CRT yields realizable distribution of available capital:

$$C_0^{CRT} = C_0 - Y_1 - Y_2,$$
$$C_i^{CRT} = C_i + Y_i, \quad i = 1, 2.$$



Resulting group capital requirement is the sum

$$k^{CRT} = k_0^{CRT} + k_1^{CRT} + k_2^{CRT}$$

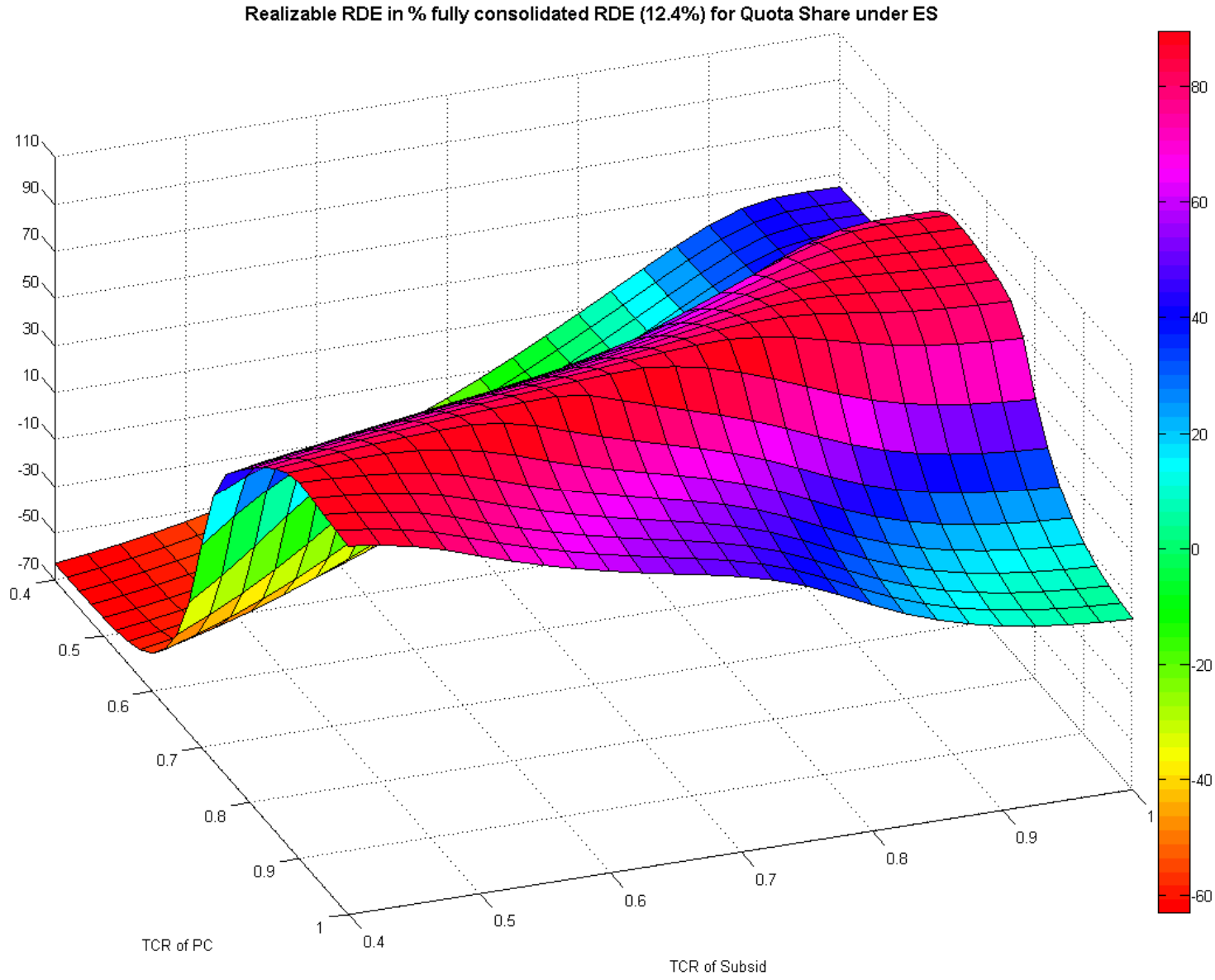
of the stand alone requirements $k_i^{CRT} = \rho[C_i^{CRT} - c_i]$.

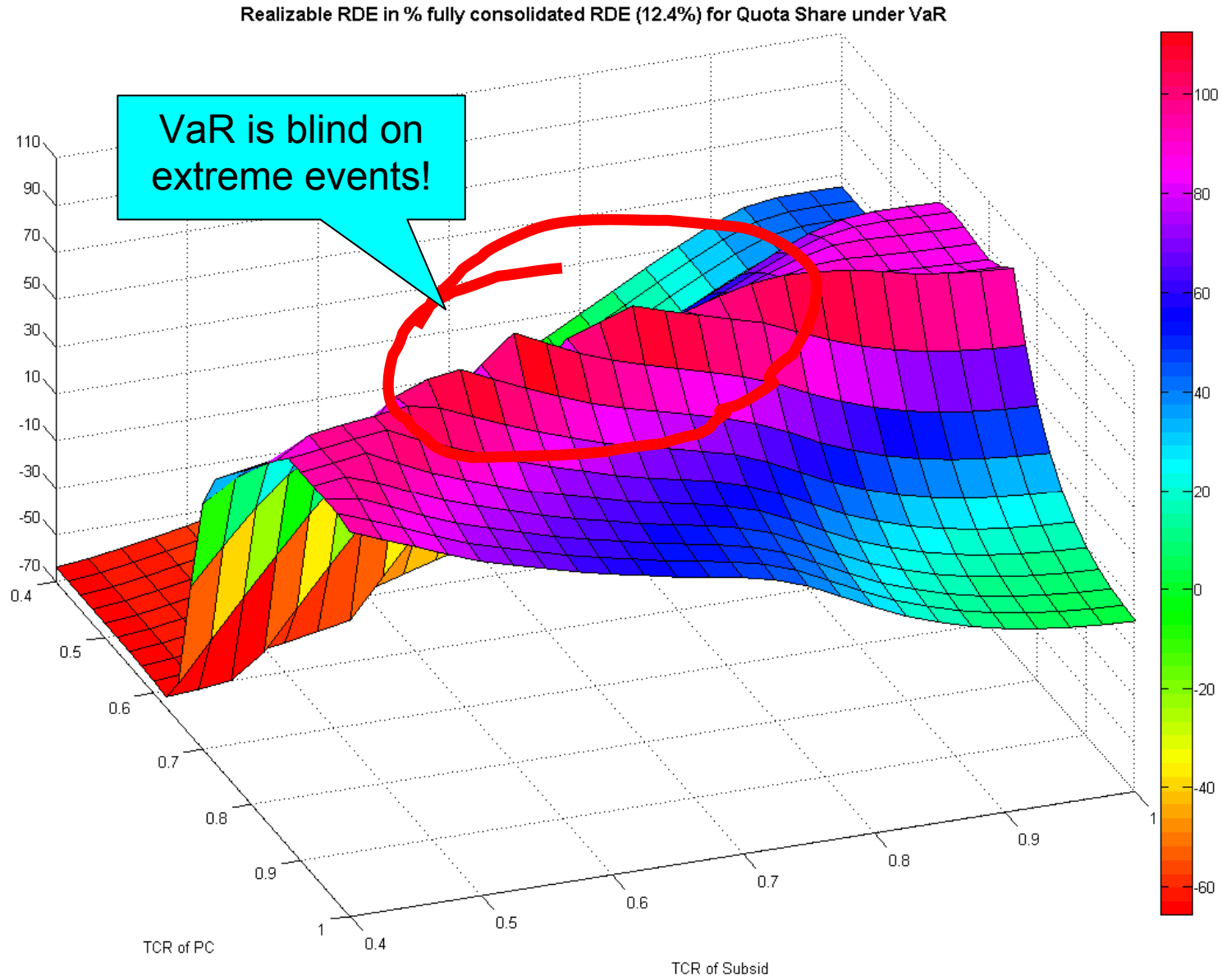
Example

Quota share guarantees with cash flow

$$G_i = 40\% \times L_i$$

Multi-dimensional normal terminal asset and liability values.


Realizable RDE in % fully consolidated RDE (12.4%) for Quota Share under ES
110
90
70
50
30
10
-10
-30
-50
-70
0.4
0.5
0.6
0.7
0.8
0.9
1
TCR of PC
0.4
0.5
0.6
0.7
0.8
0.9
1
TCR of Subsid
80
60
40
20
0
-20
-40
-60


Realizable RDE in % fully consolidated RDE (12.4%) for Quota Share under VaR
VaR is blind on extreme events!
110
90
70
50
30
10
-10
-30
-50
-70
100
80
60
40
20
0
-20
-40
-60
0.4
0.5
0.6
0.7
0.8
0.9
1
TCR of PC
TCR of Subsid

Conclusion

- Diversification is major issue for Solvency II
- Conceptual improvements for risk aggregation needed
- Diversification realizable via capital and risk transfer instruments (beware of VaR!)
- **Thank you for your attention!**