Dynamic Portfolio Choice with Parameter Uncertainty

by

Jakša Cvitanić (Social Sciences, Caltech)
Ali Lazrak (Finance Dept., University of British Columbia)
Lionel Martellini (EDHEC, Nice)
Fernando Zapatero (FBE, USC)

OUTLINE

- Motivation and applications: investing in hedge funds; value of analysts recommendations

- The model and the optimal portfolio

- Data calibration to hedge funds and empirical results

- Value of analysts recommendations

:) Disclaimer: We are not responsible for your profits/losses if you follow the method of this paper!!! :)

Treynor and Black (1973)

- Consider α, the deviation from CAPM return

$$\bar{r}_A - r_f = \alpha_A + \beta_A(\bar{r}_M - r_f)$$

- Objective: allocate between market and A

- Denote by σ_{ϵ_A} the firm-specific risk of A and

$$\gamma_A = \frac{\alpha_A/\sigma^2_{\epsilon_A}}{(\bar{r}_M - r_f)/\sigma^2_M}$$

- Solution: invest proportion π_A,

$$\pi_A = \frac{\gamma_A}{1 + (1 - \beta_A)\gamma_A}$$

The Model

- We have three types of securities

- A risk-free security that pays a constant interest rate r

- A market portfolio, whose change in price dS_0 satisfies

$$dS_0/S_0 = \mu_0 dt + \sigma_0 dW_0$$

- Risky securities whose change in price dS_i satisfy

$$dS_i/S_i = \mu_i dt + \sigma_i dW_0 + \sigma_{\varepsilon_i} dW_i$$

"Noise terms" W_i are sources of risk, normally distributed

The Investor

- We assume a risk-averse, non-myopic investor with wealth X_T and maximizing utility

$$\max_{\pi} E\frac{(X_T)^{1-a}}{1-a}$$

- Risk premium:

$$\theta = \sigma^{-1}[\mu - r\mathbf{1}]$$

- Investor knows σ, but is uncertain about μ:

$$\theta \sim \mathcal{N}(\bar{\theta}, \Delta)$$

  So, $\bar{\theta}$ is the initial estimate of risk-premium θ and Δ is its initial variance-covariance matrix.

RELATED LITERATURE

- Black and Litterman (1992), Detemple (1986), Lakner (1995, 1998), Brennan and Xia (2001), Karatzas and Zhao, X. (2001), Rogers (2001), Sekine (2001), Zohar (2001), Stojanovic (2002), Pastor and Stambaugh (1999, 2000), Baks, Metrick and Wachter (2001).

- We have explicit formula in multi-dimensional setting

SOLUTION

- We are looking for $\pi_i(t)$ =percentage (proportion, weight) of investor's wealth invested in stock i at time t

- P orthogonal, D diagonal matrix with elements d_i:

$$\Delta = P'DP$$

- Denote $A^{-1}(t)$ diagonal matrix with elements $A_i^{-1}(t)$:

$$A_i(t) = a-(1-a)\delta_i(t)(T-t), \;\; \delta_i(t) := \frac{d_i}{1+d_i t},$$

  With certainty, $A_i(t) \equiv a$

- Then the **optimal portfolio weights**

$$\pi = (\pi_1, \ldots, \pi_n)$$

are given by

$$\pi(t) = (\sigma')^{-1} P' A^{-1}(t) P \bar{\theta}(t)$$

where $\bar{\theta}(t)$ is the estimate of risk-premium at time t.

Digression on learning/estimation

- Suppose a single asset,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

- The investor knows σ

- The investor is uncertain about μ,

$$\mu \sim \mathcal{N}(\bar{\mu}, \gamma)$$

- The investor sees the dynamics of the stock,

$$\frac{dS_t}{S_t} = \bar{\mu}_t dt + \sigma d\tilde{W}_t$$

- Here, $\tilde{W}$ is the "innovation" noise

$$d\tilde{W}_t = dW_t + \frac{\bar{\mu}_t - \mu}{\sigma} dt$$

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- The "observed" expected return changes,

$$d\bar{\mu}_t = \frac{\gamma_t}{\sigma} d\tilde{W}_t$$

- The volatility decreases in a deterministic way

$$\gamma_t = \frac{\gamma_0 \sigma^2}{\sigma^2 + \gamma_0 t}$$

- It becomes a familiar problem with $\bar{\mu}$ instead of μ

- However, $\bar{\mu}$ is stochastic and for power utility generates hedging components

ALPHA-BETA INTERPRETATION

- Define estimated "alpha" by

$$\overline{\mu}_i = \underbrace{r + \beta_i \left(\overline{\mu}_0 - r\right)}_{\text{normal return}} + \underbrace{\overline{\alpha}_i}_{\text{abnormal return}}$$

- Define "beta" as $\beta_i = \frac{\sigma_i}{\sigma_0}$

- Assume no prior correlation, $\Delta = Id$

- **Optimal portfolio weights**:

$$\pi_i(t) = \frac{\overline{\alpha}_i(t)}{\sigma^2_{\varepsilon_i} A_i(t)}, \quad \pi_0(t) = \frac{\bar{\mu}_0(t) - r}{\sigma^2_0 A_0(t)} - \sum_{i=1}^{n} \beta_i \pi_i(t)$$

where π_0 is money held in the risk-free asset.

- With certainty, $A_i(t) = a$ and we have the risk-adjusted Sharpe ratio

- These are “alpha-driven” holdings (that is, in excess of their participation in the market portfolio)

- Increase with expected abnormal return

- Decrease with specific risk, risk aversion, prior variance and time to maturity

Example: One “hedge fund”

- Funding for the investment in the hedge fund: When $\beta_1 < 1/2$ the majority comes from risk-free security

- Low-beta hedge funds, substitutes of risk-free security, high-beta hedge funds, substitutes of the market portfolio (risky investment)

Correlated Priors

- Assume only two "hedge funds" with correlated priors

- Suppose that the market expected return is known

- Introduce p, positive if positively correlated and negative otherwise

- **Optimal weights** for two "hedge funds":

$$
\begin{aligned}
\pi_1 \;=\; & \frac{1}{\sigma_{\varepsilon_1}}\left(\frac{p^2}{A_1}+\frac{1-p^2}{A_2}\right)\frac{\overline{\alpha}_1}{\sigma_{\varepsilon_1}} \\
& +\frac{1}{\sigma_{\varepsilon_2}}\left(p\sqrt{1-p^2}\left(\frac{1}{A_1}-\frac{1}{A_2}\right)\right)\frac{\overline{\alpha}_2}{\sigma_{\varepsilon_2}}
\end{aligned}
$$

- When they are negatively correlated,
  - Proportions are larger than in the uncorrelated case
  - Higher appraisal ratio $\overline{\alpha}_i/\sigma_{\varepsilon_i}$ of one stock means higher investment in both
  - Diversification result
- When they are positively correlated, holdings in one decreases with the appraisal ratio of the other

Example

- Setting: risk free asset, the market portfolio and the Fama and French SMB and HML portfolios

- We mimic the calibration from Brennan and Xia (2001)

- Our results do not match exactly those in Brennan and Xia (2001)

- Importance of hedging demand, **especially with prior correlation**

a	2	3	4	5
Myopic demand				
Market	1.7793	1.1862	0.8896	0.7117
SMB	1.0399	0.6933	0.5200	0.4160
HML	4.3871	2.9247	2.1935	1.7548
Hedging demand with prior correlation				
Market	-0.4622	-0.3781	-0.3068	-0.2559
SMB	-0.2701	-0.2210	-0.1793	-0.1496
HML	-1.1395	-0.9322	-0.7564	-0.6310
Optimal demand with prior correlation				
Market	1.3171	0.8081	0.5829	0.4558
SMB	0.7698	0.4723	0.3407	0.2664
HML	3.2476	1.9925	1.4371	1.1239
Hedging demand without prior correlation				
Market	-0.6827	-0.5371	-0.4286	-0.3542
SMB	-0.0938	-0.0940	-0.0821	-0.0712
HML	-1.4884	-1.1861	-0.9517	-0.7889
Optimal demand without prior correlation				
Market	1.0966	0.6491	0.4610	0.3575
SMB	0.9461	0.5993	0.4379	0.3448
HML	2.8987	1.7386	1.2419	0.9659

Application to Analysts' Recommendations

- Many papers on analysts' recommendations (AR) New: We have an utility-based framework

Data

- We use IBES data on AR
- Available 11/93 through 12/02
- Then we use CRSP for individual stocks, market and T-Bills
- We use 2,280 stocks with average coverage of more than 4.5 analysts
- Average recommendation is 2.03, corresponding to a "buy"

Implementation

- Translate AR into alphas

- Initial mapping:
  - "Strong buy" corresponds to $\alpha = \frac{2}{\omega}\%$
  - "Buy" corresponds to $\alpha = \frac{1}{\omega}\%$
  - "Holds" corresponds to $\alpha = 0\%$
  - "Sell" corresponds to $\alpha = -\frac{1}{\omega}\%$
  - "Strong sell" corresponds to $\alpha = -\frac{2}{\omega}\%$

- We use a scaling coefficient ω and consider several values for it

- We call this mapping "raw" alphas

- Several refinements:

  - "Centered" alphas (zero average)

  - Changes in alphas

- With respect to the other parameters, we estimate them using two methods:

  - A rolling window of three years

  - The whole sample

  The advantage of the latter is that we use better estimates for the "objective" parameters of the exercise

- With respect to uncertainty over alphas, we assume two components:
  - Standard deviation of AR
  - Average of the dispersion over the whole sample period

- With respect to the correlation, we try two approaches:
  - Ad hoc numbers, with comparison across them
  - Sample correlations

- Problem: analysts upgrade information
  - Is this going to match the formula for Bayesian upgrade?

- We assume analysts observe firm-specific risk and do not observe market risk

- Analysts have information about market risk and upgrade priors in a Bayesian way

- Investors do not observe neither
  - We assume analysts upgrade in a Bayesian way given their information

Accounting for “Expected Utility”

- We want to study the impact of AR on expected utility

- The effect of AR on one path is not enough

- We try two approaches:

  - Bootstrapping

  - We set up portfolios of securities (at random) and we average over a large number of portfolios

- We report the latter

- Assume we start with a level of wealth W_0

- We compute the monetary equivalent ξ such that $W_0 + \xi$ invested using Merton (1971) optimal passive strategy would yield the same expected utility as the portfolio bases on AR

  - When we use bootstrapping, we generate many possible paths and average over them

  - When we compute expected utility averaging over portfolios, we only use one path

Basic Results and Extensions

- Overall we find that AR are not very useful

- Changes in alphas mapping gives higher utility. Correlation among priors improve the usefulness of AR

- In a final exercise, we split each portfolio in two subportfolios, according to the number of AR
  - AR are more useful when there is broader coverage
  - Counterintuitive or conflict of interests?

ω	Centered alphas	Change in alphas	Raw alph
15	-0.02%	0.35%	0.15%
25	0.03%	0.29%	0.29%
35	0.03%	0.23%	0.25%
45	0.03%	0.19%	0.21%
55	0.03%	0.16%	0.18%
65	0.02%	0.13%	0.16%
75	0.02%	0.12%	0.14%

ω	high coverage	low coverage	p-value
15	0.97%	-0.89%	.1400
25	0.58%	-0.32%	.0986
35	0.42%	-0.18%	.0831
45	0.33%	-0.12%	.0753
55	0.27%	-0.09%	.0705
65	0.23%	-0.07%	.0674
75	0.20%	-0.06%	.0651

Utility gains significant for optimal vs naive strategies

A: $x_L = 60\%; x_S = 20\%$.

Horizon			
a	1	3	5
2	8.04%	26.60%	49.15%
5	4.21%	13.22%	23.09%
8	3.95%	12.34%	21.44%

B: $x_L = 40\%; x_S = 10\%$.

Horizon			
a	1	3	5
2	7.84%	25.94%	47.89%
5	3.60%	11.27%	19.60%
8	2.93%	9.06%	15.59%

Conclusions

- Closed form solutions

- Dynamic model of investment in actively managed portfolios

- Application to analysts' recommendations and to hedge funds