The autocallable conundrum

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Who am I? *(not Umberto ☺)*

- Alberto Cherubini: alberto@eqfltd.com
- EQ Finance Ltd: www.eqfltd.com
  - Consulting and training on risk, derivatives, trading
  - Derivatives software products *(hedge funds, vanilla and exotics trading and structuring, risk management)*
- University of Bologna: equity derivatives course
- Formerly:
  - Head of Trading for Exotics Equity Derivatives, Citi (London)
  - Vol trader, flow and prop, Salomon Bros, Citi and briefly ABN Amro
  - Quant at Salomon Bro's and Citi
  - Physicist at JET Joint Undertaking – Nuclear Fusion largest lab
  - Software development
  - PhD in Physics, etc

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Plan of the talk

- Introduction and revision
  - Financial derivatives
  - Structured products
  - Many conundrums
- Case study *"Autocall: too good to be true!"*
  - Review of the autocallable family
  - Building an exotic payoff
  - The conundrum: How can the coupons be so high?
  - Five steps to the autocall
    - Step 1: the call
    - Step 2: the reverse convertible
    - Step 3: digital coupons
    - Step 4: the KI barrier
    - Step 5: the Worst-Of feature
  - Additional risks generated
- Summary and Conclusion

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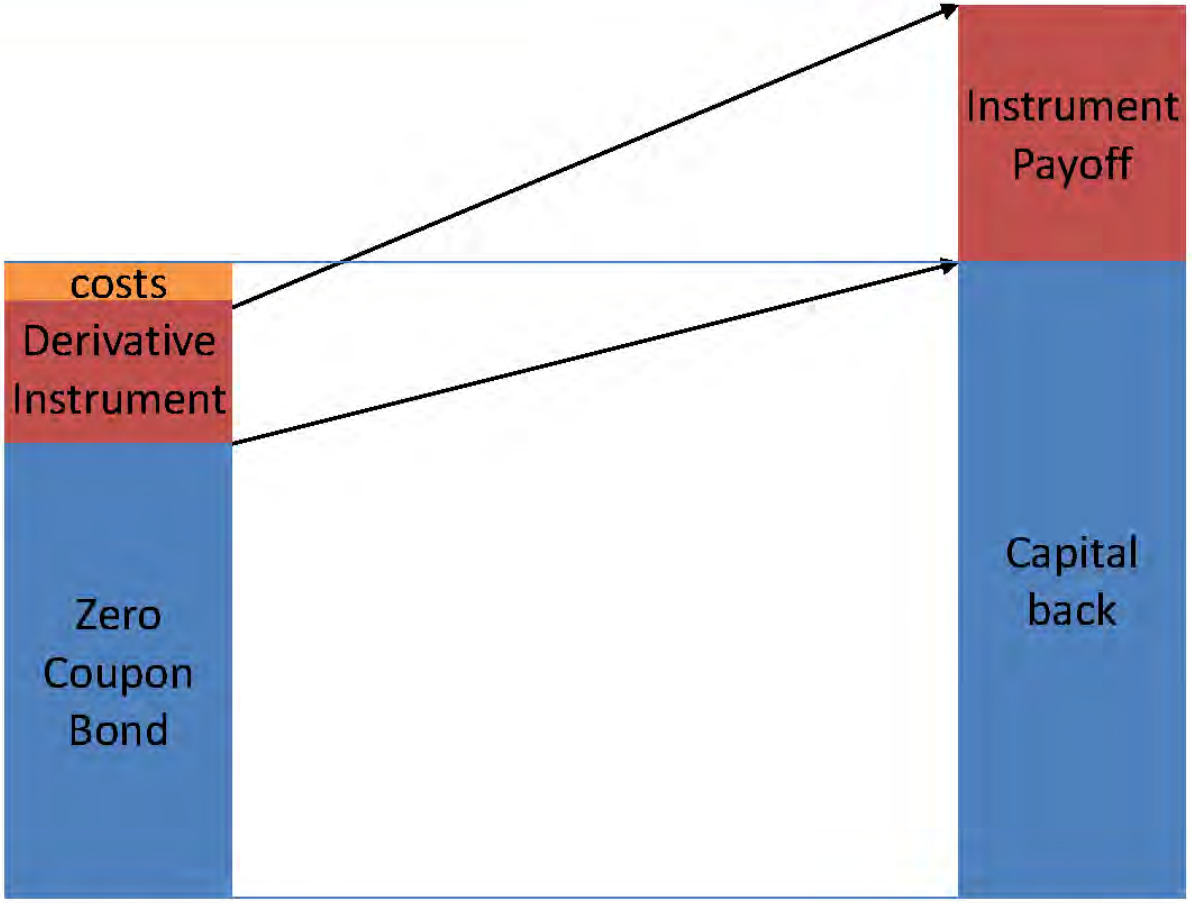
Financial derivatives

- Types:
  - Vanillas: calls, puts
  - Exotics: the others
  - Structured products *(with defined payoffs)*
- Diffusive framework:
  - Dynamic delta hedging
  - Gamma vs. Theta

Structured Products

- Main types:
  1. “Guaranteed” capital plus participation
  2. “Reverse” convertible

Basic SP type 1



"Too good to be true"

- Introduction to the exotic derivative business
  - Weapons of mass destruction?
  - What are derivatives for?
  - What are exotic derivatives for?
  - Challenges of the exotic derivative business across asset classes
- The framework
  - Starting point: Black Scholes
  - Relaxing Black Scholes: the diffusive framework
  - Various kind of non-normality
  - Exotic risks
- Gap risk *case studies*
  - Barrier options
  - Margin calls
  - NRL (non recourse loans)
- Jump risk *case studies*
  - The crash put / stability note
- "Variance swap vs.. Straddle" *case study*
  - Volatility convexity
  - Dividend convexity
- Model risk *case study*
  - The Napoleon
- Cross-asset class risks *case study*
  - Credit and equity: the "Warren Buffett" puts
- "Autocall: too good to be true!" *case study*
  - Review of the autocallable family
  - Building an exotic payoff
  - The conundrum: How can the coupons be so high?
  - Five steps to the autocall
  - Step 1: the call
  - Step 2: the reverse convertible
  - Step 3: digital coupons
  - Step 4: the KI barrier
  - Step 5: the Worst-Of feature
  - Putting it together: additional risks generated
- The global business of exotic derivatives
  - History
  - Current situation
  - Retail structured flows
  - Institutional and HF flows
  - Broker OTC markets
  - Structural positions: typical exposures
- Summary and Conclusion

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A typical autocall

- **20%** p.a. **snowballing** coupon paid if the WO of 3 indices above 100% *("autocall barrier" or U&O barrier)* on set dates *(e.g. annually)*
- As soon as a coupon is paid, terminate *("Early termination")*
- If it reaches the end AND still not above U&O barrier, then short WOKI put *(e.g. ATM strike, 50% barrier)*

- KI is a long way away… coupon is very high (a lot higher than libor)… so very popular. Many, many billions of notional sold all over the world, especially in Europe and Asia

- Variations on this theme are legions (also called TARNs, Phoenix, etc)

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Yet another conundrum! *How to generate the cash to pay those coupons?*

- How is it possible to get such high coupons, for fairly little risk?
- The exotic trading desk takes the other side: what is the self-financing replication strategy ???

- skew
- Worst of
- But … does it add up?

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5 steps to the conundrum

1) Simple Black Scholes: *how a simple call is not a zero sum game*
2) The reverse convertible: *the emergence of skew*
3) Digital coupons: *mind the gap*
4) KI put: *the put that wasn't there…*
5) Equity correlation: *the worst-of*

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Step 1: a call option

both the dealer and the customer can win

The customer:

- Buy a call, hold it to expiry
- **If** spot above strike *(+premium)* at expiry, **WIN**
- *Enhanced by: low vol, low forward (high dividends, low rates)*

The desk:

- short the call (*received premium => "implied vol"*)
- "dynamic delta hedging" i.e. has to buy high, sell low => loose money! *("short gamma")*
- If the hedging loses less than the premium, WIN (~ *realized vol < "implied vol" received*)

Remember: often the ATM "implied vol" is higher than average "realized vol"

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Foundation of derivatives business model

Two critical ideas:

Existence of a replication strategy *(a.k.a. self financing portfolios, dynamic hedging)*

The cost of the replicating strategy is the price of the option (financial derivative product).

The market is "diffusive": continuous dynamic hedging is possible.

This allows to calculate the cost of the strategy once a specific model is chosen (e.g. BSM, stochastic vol, local vol, SLV,…)

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Step 2: the reverse convertible

1987: Black-Scholes-Merton dies: the skew is born

The customer:

- Sell a put low strike *(premium is invested to generate the coupon)*
- If spot above strike at expiry *(out-of-the-money)*, receive coupon: **WIN**
- *Enhanced by: high vol, low forward, high skew*

The desk:

- Now long a put: *paying high premium => "implied vol" much higher than the vol in step (1) – this is the skew)*
- "dynamic delta hedging" would be "long gamma" *(buy low sell high)*
- But that is likely to generate less than the premium: **LOSE!** (*realized vol < "implied vol" paid*)

So dynamic delta hedging not enough:
*the dealer now **MUST sell options**, at vol at least as high as the one he has paid.*
(selling vega, buying theta… welcome to volsurface dynamic hedging)

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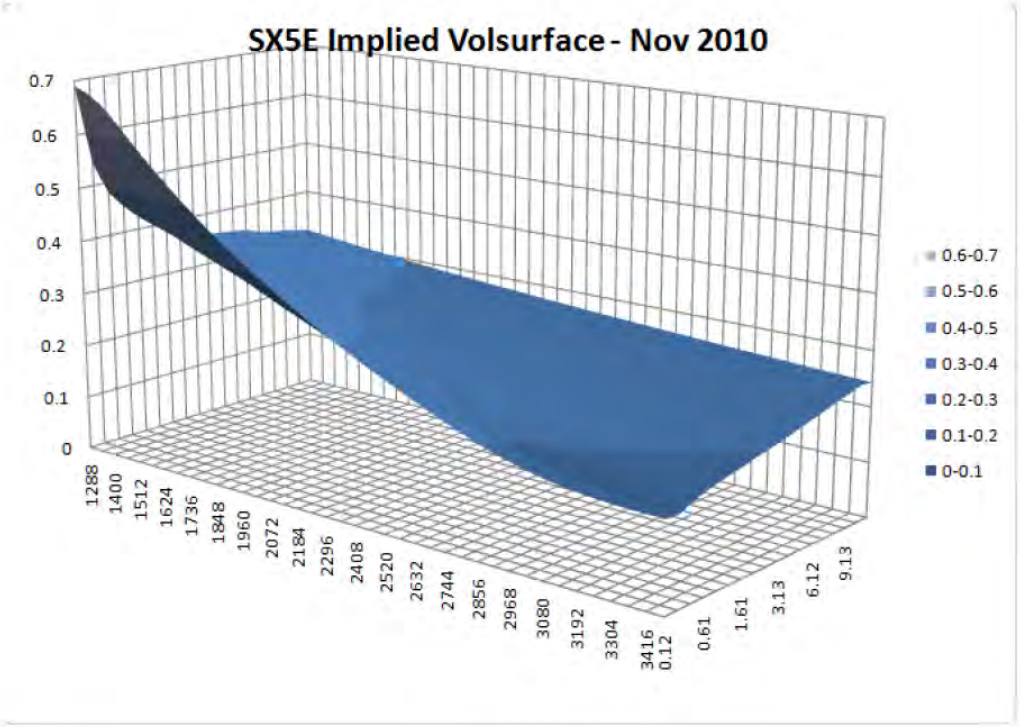
The volatility surface *(evidence of "non normality")*

- Calculating implied volatility from market prices of vanilla options at different strikes and expiries does not yield a constant, but a surface: the implied volatility surface



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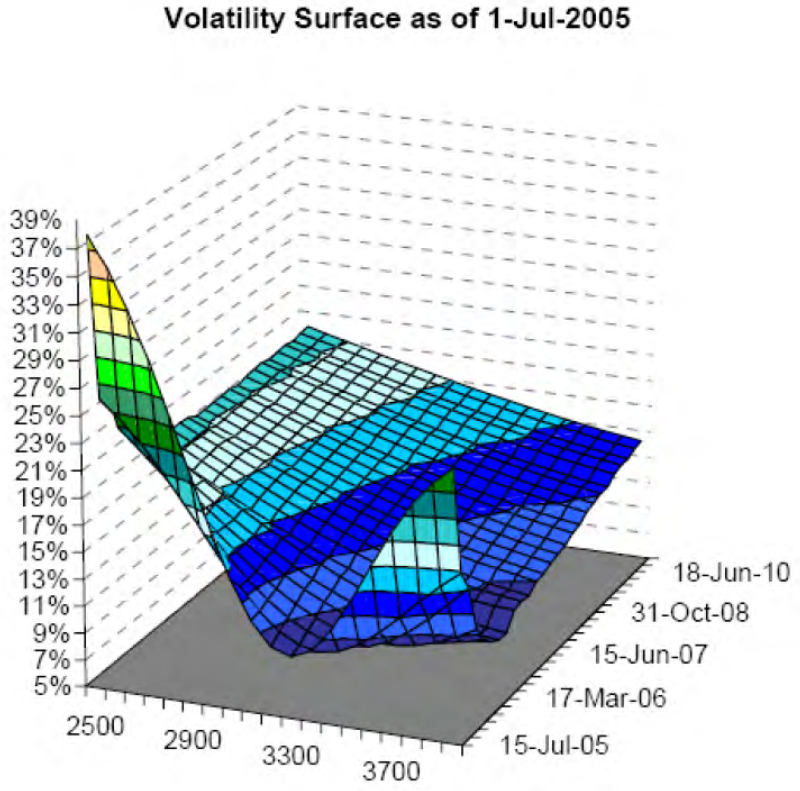
Volatility Surface as of 1-Jul-2005
39%
37%
35%
33%
31%
29%
27%
25%
23%
21%
19%
17%
15%
13%
11%
9%
7%
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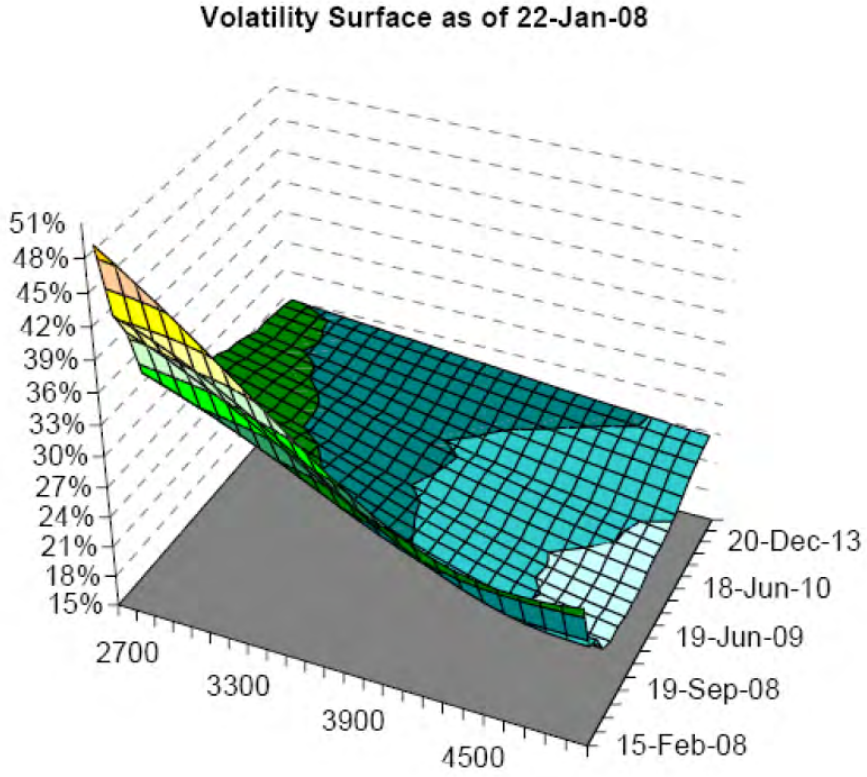
2900
3300
3700
18-Jun-10
31-Oct-08
15-Jun-07
17-Mar-06
15-Jul-05


Volatility Surface as of 22-Jan-08
51%
48%
45%
42%
39%
36%
33%
30%
27%
24%
21%
18%
15%
2700
3300
3900
4500
20-Dec-13
18-Jun-10
19-Jun-09
19-Sep-08
15-Feb-08

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Step 3: digital coupons

European barrier: introducing simple gap risk

The customer:

- Receive a coupon only if the spot is above a barrier on a date. Pay a "premium", of course much lower than PV of coupon
- If spot above barrier: **WIN**
- *Enhanced by: low forward, low skew*

The desk:

- "volsurface hedging": buying callspreads at same expiry would be a good hedge. BUT size and liquidity often makes it impossible *(even on liquid underlying, because of strike and expiry)*
- "dynamic delta hedging" reappears, both ways *(long and short gamma: tricky)*
- Jumps can lead to large losses *(e.g. a large move from way below the barrier to above, loss of nearly the whole coupon)*

Volsurface hedging and dynamic delta hedging are now of limited efficacy:
The dealer PL now is much more exposed to path dependence… aka randomness

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Step 4: the KI put

American barrier: continuous gap risk

The customer:

- "Reverse convert" with a twist: sell a put at-the-money, KI at low barrier *(premium is invested to generate the coupon)*
- always **WIN**, unless spot below strike at expiry **AND** below barrier during life
- The premium is high because of skew: if priced with BSM, it would be worth much less
- *Enhanced by: low forward, high vol, high skew*

The desk:

- long a put ? Maybe!
- Priced with "local vol" methods to include the skew and so pay higher premium
- At inception: long vega, long skew, but this may change
- MUST: "volsurface hedging": first, sell some lowish strike puts. This will have to be readjusted *(there is more to this…)*
- Just "dynamic delta hedging" would likely lose even more than in the vanilla reverse *(large theta leak)*
- A very large jump down would be profitable, a smaller one can still lead to losses

Dynamic Volsurface hedging comes into play: costly!
The dealer PL is even more exposed to "path dependence"

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Step 5: the WORST-OF

equity correlation at its "best"

The customer:

- Uses the previous building blocks, but the underlying is now the worst performing in a basket of assets
- The value is maximized by low correlation assets, and by increasing the size of the basket *(e.g. 2 assets a -100% corr: the worst-of is always going down)*
- *Enhanced by: low forward, high vol, high skew, low correlation*

The desk:

- **All previous issues**, plus…
- What is the underlying? *(e.g. the NKY SMI flip)*
- At inception, even longer skew, from the WO effect (but opposite vanna: UNHEDGEABLE)
- "volsurface hedging": on which underlying to sell vega ?
- Theta leak is even higher, of course
- Major new risk: SHORT CORRELATION
- *Also short CORRELATION SKEW*
- *See later for others…*

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A typical autocall

the conundrum: how to pay those coupons?

The customer:

- Using the previous building blocks + **early termination** (overall U&O barrier action)
- E.g. 20% p.a. snowballing coupon paid if the WO of 3 indices above 100% on given dates. If paid, terminate. If get to the end AND not paying, then short WOKI put (e.g. atm, 50% D&I barrier)
- *KI is a long way away… coupon is a lot higher than libor… so very popular, very likely to* ***WIN***

The desk:

- Will try to generate the money needed to pay the coupon by a combination of previous hedging strategies, all dynamical
- All the previous issues, but additional major problem: uncertain duration (i.e. Expected life)
- It enhances cross-risks: **spot moves change expected duration**, and hence vega bucketing, size of gamma, dividend exposure, etc
- If large size: large feedback effects: *breaking of diffusive framework*
- If hedging just one autocallable, highly path dependent, expected standard deviation of PL very high: random outcome *(especially through market regime changes)*

AND MORE…

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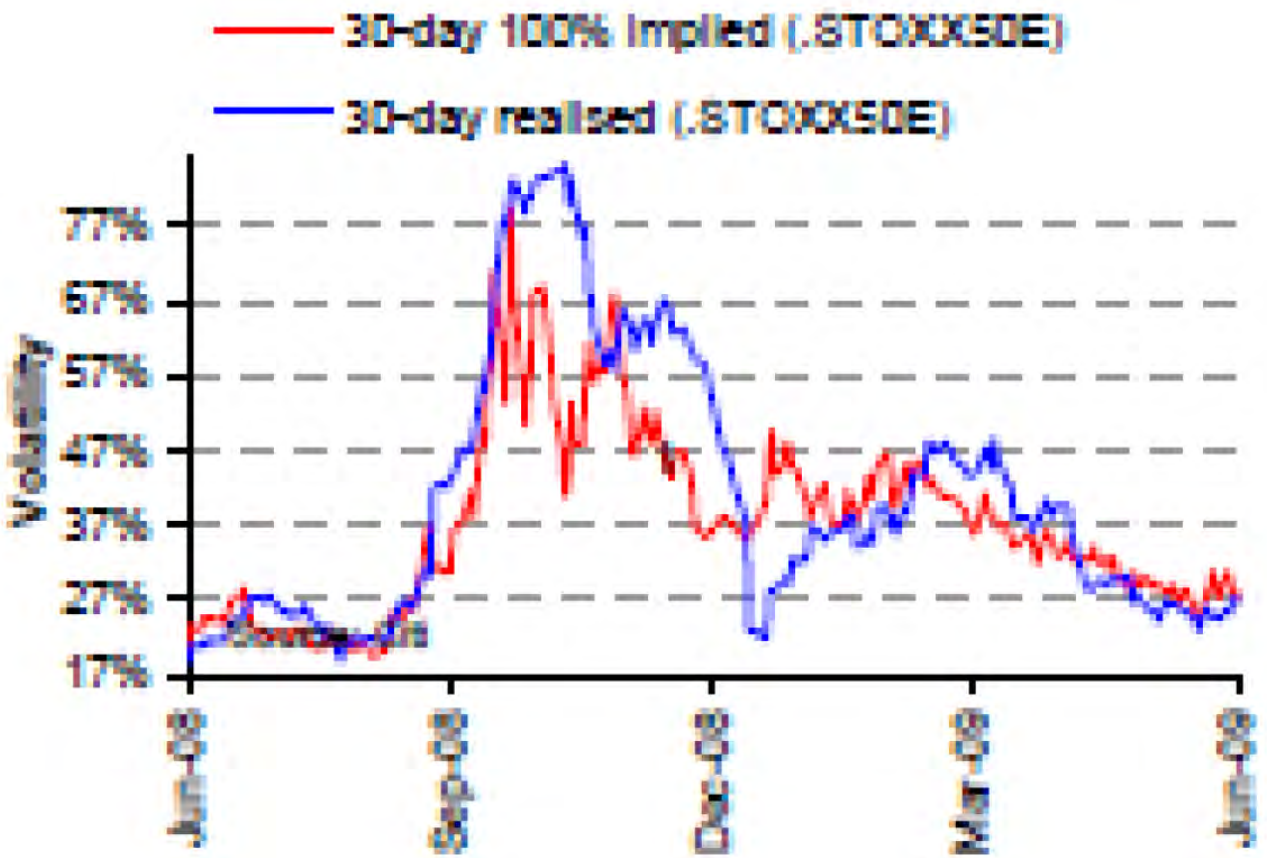
Non-normality

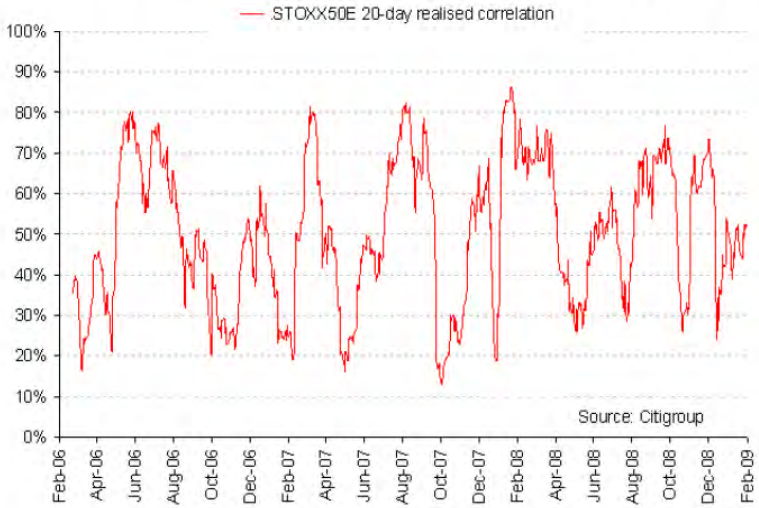
- Still diffusive:
  - Non constant volatility
  - Non constant correlation: certainly non-normal multivariate distributions of returns
  - Covariance is also non constant, and in fact amplifies the non normality, as correlation rises when vol does.
- Non diffusive
  - Feedbacks effects
  - Jumps: made worse by bunching

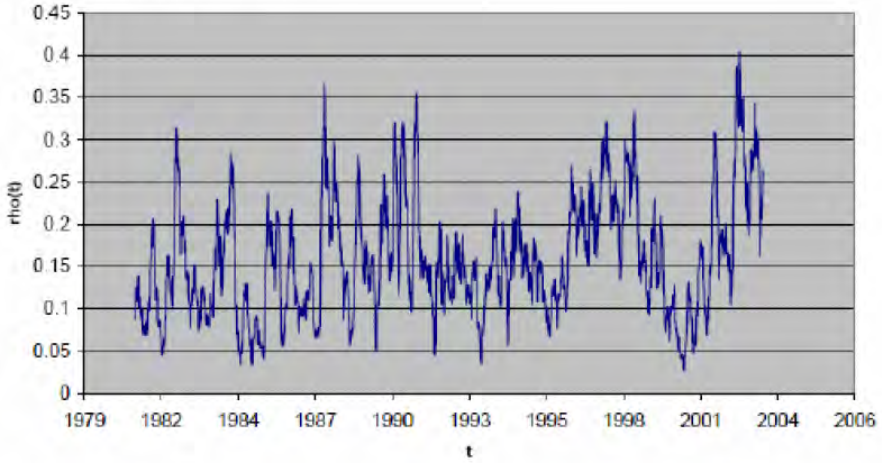
Constant volatility?



Is Correlation a good statistic?

- Statisticians tell us it is not robust
- It is most certainly not constant





- Its “volatility” can be higher than the volatility of the underlyings

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But also:

- Quanto crossgammas feedback:
  - E.g. NKY quanto EUR
  - When the yen goes up, desks get longer NKY so need to sell futures
  - The positions are so large (including prop ones) that the feedback in 2008 was considerable.
  - Additionally, the short cross gamma on SX5E-NKY then drove down SX5E
- Spot – Dividends feedback

Some other "exotic risks"

- Gaps *(e.g. through digitals)*
- Skew and surface shape
- Spot vs. vol
- Vol of vol
- Equity correlation
- Equity/fx corr. *(quanto)*
- Equity/Rates correl
- Rate/ forex correl
- Dividend/Spot correl

A portfolio approach is needed, and most likely an "insurance" approach as well

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Conclusions (1)

Derivatives business model is based on existence of replication strategy and a diffusive framework.

Even in diffusive framework, complete set of vanilla prices from market is not enough to price exotic options uniquely

Self-financing replication is not assured, threatening the derivatives business model

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Conclusions (2)

In-framework risks (..., cross-risks, gap risk, un-hedgeability)

Out-of-framework risks (jumps, feedback loops, model risk, ...)

Some of the above issues are often disregarded

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