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Loading of Granular Material on Silo Walls

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Abstract. Macroscopic behaviour of granular material is complex and diverse. Classical mathematical models are appropriate only in some situations. Recently, some new methods were introduced, based on discontinuous concept. In this paper two discrete mathematical models are briefly described: Discrete Element Method (DEM) and Cellular Automata (CA).

AMS subject classification: 70–04, 70–08, 70C20, 70F25

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1. Introduction

Although all substances consist of many discrete particles, models of classical mathematical physics are based on the ground of idealized concept of continuum. This concept is very successful, and models of almost all applications in engineering are based on it.

However, in the 20$^{\text{th}}$ century began the research of some strongly discontinuous phenomena, where discrete models are necessary. The most famous example is quantum mechanics. Other discrete concepts with practical applications are chaos theory, percolation theory, fracture mechanics, discrete element method, cellular automata, etc.

2. Behaviour of granular materials

Granular materials such as: gravel, sand, dust, cement, cereals, sugar, salt, raw materials, different industrial products, etc., are important for civil engineers as soils for foundations, building materials and loadings on structures.

Each grain is modelled as a simple deformable body, and interactions between two bodies are well-known. Nevertheless, behaviour of a heap with many grains and many interactions is unpredictable and diverse.

In the USA and Canada about 1000 containers for granular material are destroyed or damaged every year, mostly because of insufficient knowledge of mechanics of granular material.

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Behaviour of granular material is distinctly discontinuous, nonlinear and stochastic. Depending on conditions, it is sometimes to some degree comparable with behaviour of solid, liquid or gas, and sometimes it is essentially different from all the above mentioned states.

Some characteristic phenomena will be mentioned.

1. Solid–like behaviour. Under some conditions granular material can withstand nonuniform static and dynamic load. For example, foundations of many buildings lay on sand or gravel. For moderate loads, deformations are relatively small, comparable to deformations of solid bodies. Arrangement of grains is approximately preserved, which means that two grains in mutual contact before deformation mostly remain in contact after deformation. Nevertheless, sufficiently big load breaks the equilibrium and arrangement of grains.

In granular material, arches and domes can form with cavities inside. Moisture increases stability of these structures. (Children build complicated structures of wet sand.) In dry material they usually exist only for a short period of time in unstable state. The cavities can appear during discharge of a silo from below. The fall of arches and domes causes dynamic loading — strong irregular blows.

2. Liquid–like behaviour. During filling, material adapts to the shape of a container like a liquid. In contrast to normal liquids, statical stress in some point of granular material in equilibrium is not necessarily equal in all directions. The ratio between maximal and minimal stress in some point is limited by internal friction angle. In the equilibrium state upper free surface is not necessarily horizontal as in liquids, but usually forms a heap whose shape depends on history of filling. The maximal angle of stable slope of dry material is "angle of repose", and is approximately equal to the angle of internal friction. An attempt to put additional material on top of the heap with maximal angle often results with avalanches in the surface layer. The forces on different grains of uniformly loaded material are uneven. In the material force chains are formed. The grains outside of these chains have only small forces. Outer frictional forces between the material and the container wall reduce average stress in material and on walls. In liquids without friction on walls, hydrostatic pressure is proportional to depth, but in granular material, as the depth increases, all stress components asymptotically tend to a constant value. Explanation: in the limit depth, the additional weight of some layer of material is balanced with resultant of friction forces on boundary part of this layer in contact with silo walls.

Forced vibrations (caused, for example, by earthquakes or some machines) reduce the internal and outer friction, and increase stresses.

During discharge through the bottom opening, the material flows as fluid. Sometimes the material in the whole container flows (as in sand-glass), but sometimes only the material in one part of the container flows ("funnel flow"), and the rest of it has only small displacements similar to the deformation of solids. Between these two parts of the container a boundary layer is formed. The funnel flow can sometimes be relatively smooth, but sometimes has the form of "internal avalanches". In repeated experiments the mode of discharge is not necessarily repeated. Macroscopically, symmetric boundary and initial conditions don't guarantee symmetry of the flow.

3. Gas–like behaviour. For sufficiently big amplitude and for some frequencies, forced vibrations of granular material can cause chaotic movements of particles. Two or more different granular materials in this situation mix similarly to gas diffusion.

4. Some other phenomena. Under some conditions a segregation ("unmixing") of different materials, or different grain shapes and sizes from the mixture is possible. For example, in filling the heap from above, bigger, smoother and "more round" grains tend to concentrate on outer parts of the heap. This is explained by easier rolling of these grains over rough surface of the heap, than that of the smaller and irregular ones. Depending on the amplitude and frequency, forced vibrations can cause not only mixing, but also segregation.

Large nonlinear shear deformations in part of granular material (without big deformations of individual grains) produce enlargement of volume. Namely, the grains in the equilibrium state before deformation are relatively densely packed, but deformation causes unpacking.

5. Examples of phenomena characteristic for particular materials. Under loading some grains break. Electrostatic and magnetic properties and air captured inside the heap influence the behaviour of particles. The moisture in cereals causes swelling of grains, significantly increasing the load on container walls.

The above mentioned phenomena, as well as some other, are known mostly qualitatively, with small number of numerical data. In granular materials some other unknown phenomena probably exist, but present knowledge is insufficient for applications.

Experimental research of dynamic behaviour of granular material is difficult. Measuring instruments inside material disturb the observed phenomena. For that reason some experiments with glass grains in glass containers or other transparent materials, without any instruments in the material, were proposed. In some recent experiments magnetic resonance and computer tomography were also applied.

3. Continuum models for granular material

Classical linear or nonlinear models approximate granular material as homogeneous continuum, but are restricted only to some static problems and small linearized oscillations around the equilibrium state, like, for example, the model of foundation of buildings on granular soil. Some new continuum models use improved constitutive equations with "smeared granularity", but they also have limited applications.

Horizontal pressure of granular material on silo wall is computed by classical Janssen–Koenen theory dating from the end of the 19^{th} century, based also on continuum concept. Numerical results are in agreement with experiments and with the experience, for the rest state and filling of material, but only for the cylindrical shape of silo. Filling is much simpler than discharge. In filling new material is added only on top and old material has only small deformations. On the contrary, discharge from below causes flow in the whole silo, or in a large part of it. In engineering practice, for discharge and even for the equilibrium state with noncircular cross-sections, results of Janssen–Koenen concept are modified by using empirical correction factors from building codes, without theoretical interpretation. Unreliability of results is compensated with large safety factors. Silo structures built according to the present building codes are generally uneconomic and sometimes even unsafe.

4. Discrete Element Method (DEM)

The basic concept of DEM is theoretically simple and straightforward, but the method is extremely demanding for computer implementation. The granular material is modelled as a system of discrete elements. Each element represents one grain and has defined shape and properties — mass, moments of inertia, stiffness, damping, etc. Friction coefficients between two grains and between a grain and a wall, cohesion, distribution of element shapes and sizes, and boundary and initial conditions are also defined. In different models, data are idealized and simplified in different ways, according to suppositions. The system of differential equations of motion is solved by some numerical methods. The big problem in implementation is to take into account collisions and contacts of elements. Of course, discrete elements must be much bigger than real grains to speed up computing. Nevertheless, the necessary computer time is much longer than for any standard numerical method in engineering, and sometimes can be measured in months. One of the most demanding problems is contact detection. Several algorithms are known, but are not very efficient yet. Due to high computing times, majority of models in the past were two-dimensional, but their results were not particularly useful for engineers. Most of the modern algorithms are three-dimensional, but feasible number of elements is not yet sufficient. With further improvements in computer technology and algorithms, an expansion of DEM can be expected.

5. Cellular automata (CA)

The CA method is very simple (particularly for programming) and fast, but normally gives only qualitative results. The space is simplified by using regular square, triangular or hexagonal net. The method is efficient for analysis of many diverse discrete problems and for discrete approximation of continuous ones. Even a classical finite difference method can be interpreted as CA. Some models from literature connected with mechanics of granular material are:

- model for two-dimensional approximation of silo loading,
- tetris-like two-dimensional model for silo discharge,
- model of avalanches on a heap of granular material.

6. Our numerical experiments

6.1. Statical DEM model

Spherical rigid body discrete elements are used. The arrangement of elements is modelled by simulation of filling. Elements are filled one by one from the top opening. They have different sizes, calculated using random number generator. (Uniform elements tend to form unrealistic "crystalized" regular patterns.) Every element rolls over heap surface until it finds its equilibrium position. Simplification consists in freezing old elements — displacements and disturbance in equilibrium of old elements (and possibility of avalanches), under action of the current one, are neglected. The model of silo wall consists also of rigid spherical discrete elements ("grains"). In the equilibrium position, every regular grain lies on three "older" grains (filled earlier) for which we have previously determined the state of equilibrium in the same way. During rolling there are possibilities of unstable equilibrium on one or two grains. In computer program, such a grain is automatically disturbed, and finds another stable position. If the new grain lies on more then three old ones, the extra contacts are neglected.

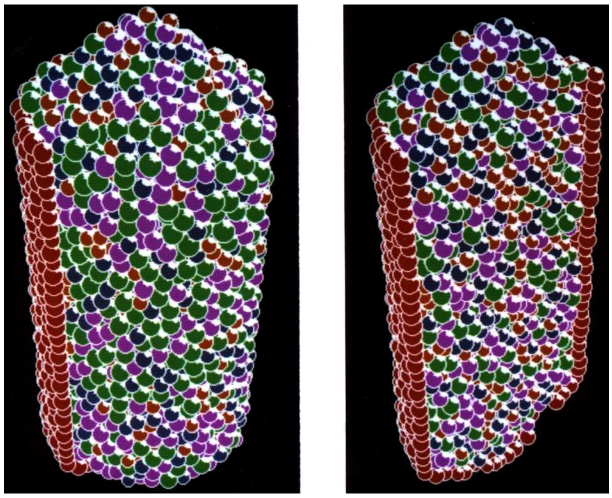

Figure 1. Statical DEM model:

half of silo wall is removed (left), half of model is removed (right).

The system of grains in the model is statically determinate. It means that contact forces are known only from the equilibrium equations. These are determined in the reverse order of filling, starting from the last grain. The weight of this grain is decomposed into three components — contact forces with its base grains. In the case without friction, the direction of contact force lies on the line connecting the centers of grains. If the friction is taken into account, the direction of force lies on a line through contact point of grains, but not in the normal direction. At least one component has maximal friction angle with respect to normal. Only friction in vertical plane through centers of grains is included. The components on other base grains are determined from equilibrium conditions. Every grain transmits forces of "newer" grains and its own weight to "older" grains. Silo walls are modelled as a system of fixed spherical grains. Except between themselves, material grains also transmit forces on silo walls. The load on silo walls consists of these contact forces.

6.2. Dynamical DEM model

The elements are spherical and deformable. The elastic contact force is linearized. Instead of geometric nonlinearity from Hertz contact theory, it linearly depends on overlapping. (This is a good approximation of spherical element with rigid core and linearly elastic thin surface layer.) The local friction (grain to grain) is neglected. It is a significant simplification, because rotational degrees of freedom in dynamical model are excluded. (The global internal friction in the heap, nevertheless, exists.) Viscose (velocity proportional) damping and small cohesion are included. The equations of motion are solved with one variant of predictor–corrector method using small time steps (10^{-4} s). Despite of simplifications, every computer run (with about $3 \cdot 10^4$ elements) lasts several months. The unbalanced forces on material grain cause acceleration, and the ones on silo wall cause loading of silo. Qualitatively, results of examples are in agreement with experience and literature. Some of the above mentioned phenomena of real material are observed in the numerical model.

6.3. Statical CA model

In this model the elements are regularly arranged and their shape is undefined. The model is interpreted as a modified cellular automaton with three real numbers — components of lattice site values — forces on every grain. The lattice is regular hexagonal, and represents one layer of granular material. A role of the "next time step", usual in cellular automata, is replaced by the "next layer". This model is a simplification and idealization of the statical DEM model. The difference is that grains have the same radius and are arranged in a regular pattern. After transmission of forces to the next layer, the upper layer is "forgotten". Decomposition on six base grains is not unique.

The forces from one layer are transmitted to the next one, in deterministic or probabilistic way (but satisfying the equilibrium conditions), increasing the vertical force component for the weight of one element. The silo wall is modelled by fixed grains on the boundary in arrangement of approximate geometric form of silo cross-section. The forces on the fixed wall grains are not transmitted to the next layer. The main difference is that the elements are arranged in a regular pattern. In this case "crystallization" is not the problem, because the angle of internal friction must be explicitly prescribed. This method is very efficient, but must be used in combination with other numerical or experimental methods, because it cannot produce the complete solution.

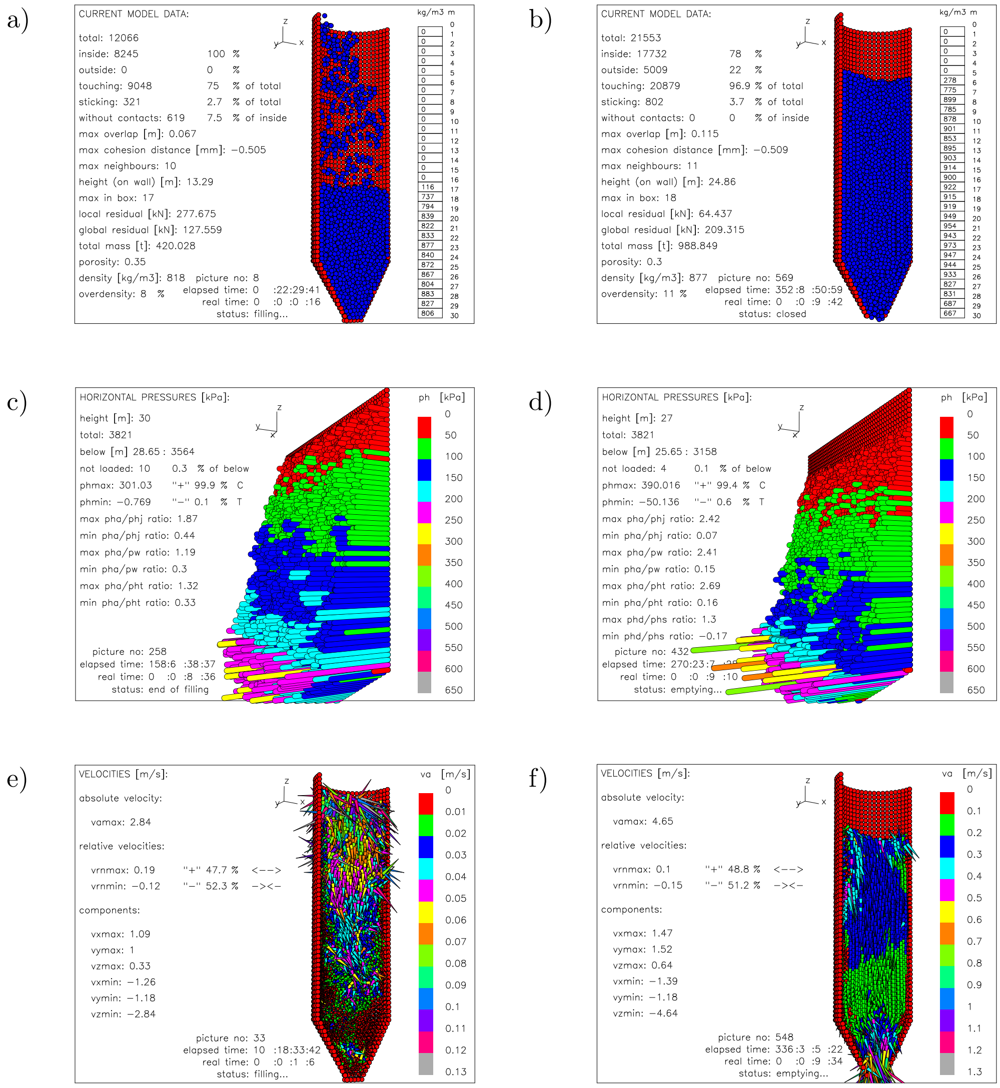

Figure 2. Dynamical DEM model:

a) filling,	b) discharge,
c) filling pressures,	d) discharge pressures,
e) filling velocities,	e) discharge velocities.

7. Concluding remarks

DEM is too inefficient for recent generation of computer hardware, but in the future an expansion can be expected. The mathematical problems not yet properly solved include: convergence with element size, optimal time step for numerical integration for dynamical problems (probably different steps for fast and slow elements), numerical stability, contact detection algorithms, etc.

CA is efficient, but much more restricted. With appropriate correction factors it can become a practical method for engineering applications.

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