Absolute continuity of non-commutative measures

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There are many principles in measure theory saying that pointwise absolute continuity of a suitable system of measures implies, surprisingly, uniform absolute continuity. Prominent ones are the Vitali-Hahn-Saks Theorem and the Brooks-Jewett Theorem. In this note we summarize basic results on extensions of these theorems to von Neumann algebras and C^*-algebras. In particular, we show that the Vitali-Hahn-Saks Theorem holds precisely for finite von Neumann algebras. Moreover, we state that the Brooks-Jewett Theorem is valid for a C^*-algebra A if, and only if, A is a Grothendieck space and all irreducible representations of A are finite-dimensional. In connection with these results we discuss absolute continuity of non-commutative scalar as well as vector-valued measures and weakly compact operators acting on operator algebras.

References

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