DOUBLY REGULAR DIOPHANTINE QUADRUPLES

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Abstract. For a nonzero integer n, a set of m distinct nonzero integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$, is called a $D(n)$-m-tuple. In this paper, by using properties of so-called regular Diophantine m-tuples and certain family of elliptic curves, we show that there are infinitely many essentially different sets consisting of perfect squares which are simultaneously $D(n_1)$-quadruples and $D(n_2)$-quadruples with distinct nonzero squares n_1 and n_2.

1. Introduction

For a nonzero integer n, a set of distinct nonzero integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$, is called a Diophantine m-tuple with the property $D(n)$ or $D(n)$-m-tuple. Sometimes it is convenient to allow that $n = 0$ in this definition. The $D(1)$-m-tuples are called simply Diophantine m-tuples, and sets of nonzero rationals with the same property are called rational Diophantine m-tuples. The first rational Diophantine quadruple, the set $\left\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right\}$, was found by Diophantus of Alexandria. By multiplying elements of this set by 16 we obtain the $D(256)$-quadruple $\{1, 33, 68, 105\}$. The first Diophantine quadruple, the set $\{1, 3, 8, 120\}$, was found by Fermat. In 1969, Baker and Davenport [2], proved that Fermat's set cannot be extended to a Diophantine quintuple. Recently, He, Togbé and Ziegler proved that there are no Diophantine quintuples [14]. Euler proved that there are infinitely many rational Diophantine quintuples. The first example of a rational Diophantine sextuple, the set $\{11/192, 35/192, 155/27, 512/27, 1235/48, 180873/16\}$, was found by Gibbs [13], while Dujella, Kazalicki, Mikić and Szikszai [7] recently proved that there are infinitely many rational Diophantine sextuples (see also [6, 8, 9]). It is not known whether there exists a rational Diophantine septuple. Gibbs' example shows that there exists a $D(2985984)$-sextuple. It is not known whether there exist a $D(n)$-septuple for some $n \neq 0$. Moreover, it is not known whether there exist a $D(n)$-sextuple for any n which is not a perfect square. For an overview of results on Diophantine m-tuples and its generalizations see [5].

In [15], A. Kihel and O. Kihel asked if there are Diophantine triples $\{a, b, c\}$ which are $D(n)$-triples for several distinct n's. In [1], several infinite families of Diophantine triples were presented which are also $D(n)$-sets for two additional n's. Furthermore, there are examples of Diophantine triples which are $D(n)$-sets for three additional n's. If we omit the condition that one of the n's is equal to 1, then the size of a set N for which there exists a triple $\{a, b, c\}$ of nonzero integers which is a $D(n)$-set for all $n \in N$ can be arbitrarily large.

In [11], we proved that there are infinitely many nonequivalent sets of four distinct nonzero integers $\{a, b, c, d\}$ with the property that there exist two distinct nonzero integers n_1 and n_2 such that $\{a, b, c, d\}$ is a $D(n_1)$-quadruple and a $D(n_2)$-quadruple (we called equivalent a quadruple $\{a, b, c, d\}$ with properties $D(n_1)$ and

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$D(n_2)$ and a quadruple $\{au, bu, cu, du\}$ with properties $D(n_1u^2)$ and $D(n_2u^2)$ for a nonzero rational u). We presented two constructions of infinite families of such quadruples. The first of them contains pairs $\{a, b\}$ such that $a/b = -1/7$, while in the second family we allowed that $n_1 = 0$.

In this paper, we will improve results of [11] by considering so-called regular Diophantine m-tuples. A (rational) $D(n)$-quadruple $\{a, b, c, d\}$ is called *regular* if

$$n(d + c - a - b)^2 = 4(ab + n)(cd + n). \tag{1}$$

Equation (1) is symmetric under permutations of a, b, c, d. Since the right hand side of (1) is a square, it is clear that a regular $D(n)$-quadruple may exist only if n is a perfect square. On the other hand, if $n = \ell^2$ is a perfect square, then e.g. $\{\ell, 3\ell, 8\ell, 120\ell\}$ is a regular $D(\ell^2)$-quadruple. A $D(\ell^2)$-quadruple $\{a, b, c, d\}$ is regular if and only if the rational $D(1)$-quadruple $\{a/\ell, b/\ell, c/\ell, d/\ell\}$ is regular.

In this paper, we consider the question is it possible that a quadruple $\{a, b, c, d\}$ is simultaneously a regular $D(u^2)$-quadruple and a regular $D(v^2)$-quadruple for $u^2 \neq v^2$ (we called such sets *doubly regular Diophantine quadruples*). We will give an affirmative answer to this question. Moreover, in our solution all elements a, b, c, d will be perfect squares. So, if we allow $n = 0$ in the definition of $D(n)$-m-tuples, we get quadruples which are simultaneously $D(n_1)$-quadruples, $D(n_2)$-quadruples and $D(n_3)$-quadruples, with $n_1 \neq n_2 \neq n_3 \neq n_1$, thus improving the results from [11].

Our main result is

Theorem 1. *There are infinitely many nonequivalent sets of four distinct nonzero integers $\{a, b, c, d\}$ which are regular $D(n_1)$ and $D(n_2)$-quadruples for distinct nonzero squares n_1 and n_2. Moreover, we may take that all elements of these sets are perfect squares, so they are also $D(0)$-quadruples.*

The construction of sets with the properties from Theorem 1 use a parametrization of rational Diophantine triples and properties of certain family of elliptic curves (for other connections between Diophantine m-tuples and elliptic curves see e.g. [4, 10]).

2. Construction of doubly regular Diophantine quadruples

As we mentioned in the introduction, in [11] we constructed two infinite families of such quadruples which are $D(n_1)$ and $D(n_2)$-quadruples with $n_1 \neq n_2$. We also listed some sporadic examples which do not fit in these two infinite families. None of these examples is such that n_1 and n_2 are both nonzero squares. However, in some of them one of the numbers n_1, n_2 is a square. For example, $\{28, 6348, 18750, 88872\}$ is a $D(330625)$ and $D(38101225)$-quadruple and $330625 = 575^2$. Moreover, $\{28, 6348, 18750, 88872\}$ is a regular $D(330625)$-quadruple.

Assume now that $\{a_1, b_1, c_1, d_1\}$ is a regular $D(u^2)$-quadruple and regular $D(v^2)$-quadruple. Then $\{a, b, c, d\}$, where $a = a_1/u$, $b = b_1/u$, $c = c_1/u$, $d = d_1/u$, is a regular rational $D(1)$-quadruple, and $\{a/x, b/x, c/x, d/x\}$, where $x = v/u$, is also a regular rational $D(1)$-quadruple.

We will use a parametrization of rational $D(1)$-triples which is a slight modification of the parametrization due to L. Lasić [17] (see also [9]). Lasić's parametrization is

$$a = \frac{2t_1(1 + t_1t_2(1 + t_2t_3))}{(-1 + t_1t_2t_3)(1 + t_1t_2t_3)},$$
$$b = \frac{2t_2(1 + t_2t_3(1 + t_3t_1))}{(-1 + t_1t_2t_3)(1 + t_1t_2t_3)},$$
$$c = \frac{2t_3(1 + t_3t_1(1 + t_1t_2))}{(-1 + t_1t_2t_3)(1 + t_1t_2t_3)}.$$

From the condition that $\{a, b, c, d\}$ is a regular $D(1)$-quadruple, we compute d and we obtain

$$d = \frac{2(1 + t_1t_2t_3)(t_1t_2 + 1 + t_2)(t_1 + 1 + t_3t_1)(1 + t_3 + t_2t_3)}{(-1 + t_1t_2t_3)^3}.$$

By inserting these values of a, b, c, d in the condition of regularity of quadruple $\{a/x, b/x, c/x, d/x\}$, we obtain the following quartic equation in x:

$$4x^4 + (-a^2 + 2ab + 2ad - b^2 + 2bc + 2ac - c^2 + 2cd - d^2 + 2bd)x^2 + 4abcd = 0. \tag{2}$$

By inserting the condition (1) with $n = 1$ in the x^2-term in (2), we obtain

$$4(x^2 - 1)(x^2 - abcd) = 0.$$

Since we are interested in solutions with $u^2 \neq v^2$, i.e. $x^2 \neq 1$, we get that $x^2 = abcd$. Thus, $abcd$ should be a perfect square, which leads to the condition that

$$t_1t_2t_3(1+t_3+t_2t_3)(t_1+1+t_3t_1)(t_1t_2+1+t_2)(t_1t_2+t_1t_2^2t_3+1)(t_2t_3^2t_1+1+t_2t_3)(t_3t_1^2t_2+1+t_3t_1)$$

is a perfect square.

To solve the last condition, we introduce the following substitutions:

$$t_1 = \frac{k}{t_2t_3},$$
$$t_2 = m - \frac{1}{t_3}.$$

Now the condition becomes

$$kt_3(1+m)(k+mt_3-1+kt_3)(k+t_3+mt_3-1)(km+1)(k+m)(k^2+mt_3-1+kt_3) = w^2,$$

which can be considered as a quartic in t_3:

$$\begin{aligned} &k(m + 1)^2(km + 1)(k + m)^3t_3^4 \\ &+ k(m + 1)(k - 1)(km + 3m + 2k + 2)(km + 1)(k + m)^2t_3^3 \\ &+ k(m + 1)(k - 1)^2(km + 1)(k + m)(k^2 + 2km + 3k + 3m + 1)t_3^2 \\ &+ k(m + 1)(k + 1)(k - 1)^3(k + m)(km + 1)t_3 = w^2. \end{aligned} \tag{3}$$

The quartic (3) has an obvious rational point $[t_3, w] = [0, 0]$, so it can be, in the standard way (see e.g. [3, Section 1.2]), transformed in an elliptic curve. To ensure that this curve has positive rank, we will force (3) to have an additional rational point. A good candidate for an additional point is $t_3 = 1/m$, since it is a root of the discriminant of the left hand side of (3) with the respect to k. By inserting $t_3 = 1/m$ in (3), we get the condition that $k(km+1)(k+m)$ is a perfect square (note that this condition is equivalent to ab being square). From $k(km + 1)(k + m) = (km + z)^2$, we get $m = \frac{k^2-z^2}{k(-1-k^2+2z)}$. Here we take for the simplicity that $z = 2$.

By transforming the quartic, with the substitution

$$t_3 = k(k - 1)(k + 1)(k^2 - 3)(k^3 - k^2 - 3k + 4)/X, \tag{4}$$

we obtain the following elliptic curve over $\mathbb{Q}(k)$:

$$\begin{aligned} Y^2 = &(X + (k^3 - k^2 - 3k + 4)(k^2 - 2)^2)(X + (k + 1)(k^3 - k^2 - 3k + 4)(k^2 - 2)^2) \\ &\times (X + (k + 1)(k^3 - k^2 - 3k + 4)^2) \end{aligned} \tag{5}$$

with 2-torsion points

$$T_1 = [-(k + 1)(k^3 - k^2 - 3k + 4)^2, 0],$$
$$T_2 = [-(k + 1)(k^3 - k^2 - 3k + 4)(k^2 - 2)^2, 0],$$
$$T_3 = [-(k^3 - k^2 - 3k + 4)(-2 + k^2)^2, 0],$$

and an additional rational point

$$P = [-(k-2)(k+2)(k+1)(k^3-k^2-3k+4)(k-1), k^2(k+1)(k^3-k^2-3k+4)^2].$$

The point P does not give the desired solution because it corresponds to $t_3 = 1/m$ which leads to $t_2 = 0$. A point $[X, Y]$ would give us a solution if the corresponding quadruple $\{a, b, c, d\}$ satisfies that $ab + x^2, \dots, cd + x^2$ are all perfect squares. However, since $x^2 = abcd$ and $ab + x^2 = ab(cd + 1)$, we see that the conditions are equivalent to ab, ac, ad, bc, bd, cd being perfect squares (i.e. to the condition that $\{a, b, c, d\}$ is a $D(0)$-quadruple). Since $ab = \frac{4(k^2-1)^2}{(k+1)^2(k-1)^2(k^2-3)^2}$ is a perfect square, and $ad = ac \cdot cd/c^2 = ac \cdot abcd/(c^2 \cdot ab)$, it suffices to satisfy the condition that ac is a perfect square. The condition is

$$t_3(4t_3 - 4t_3k^2 - 4k^3 + 3k + t_3k^4 + k^5) = \square,$$

which under substitution (4) becomes

$$(k^3-k^2-3k+4)(X + (k^3-k^2-3k+4)(k^2-2)^2) = \square.$$

Since this condition is satisfied for the X-coordinate of the point P, and $(X + (k^3 - k^2 - 3k + 4)(k^2 - 2)^2)$ is one of the factors of the right hand side of (5), by the 2-descent argument (see [16, Theorem 4.2]), it is satisfied for the values of t_3 which correspond to X-coordinates of points of the form $P + 2T$, hence it is satisfied for all odd multiples of the point P.

In particular, we may take the point

$$\begin{aligned} 3P = \Big[& \frac{1}{(k^6-6k^5-3k^4+28k^3-8k^2-32k+16)^2} \times (k-2)(k+2)(k-1)(k+1) \\ & \times (3k^6-2k^5-13k^4+8k^3+16k^2-16)(5k^6-6k^5-27k^4+40k^3+32k^2-64k+16) \\ & \times (k^3-k^2-3k+4), \\ & \frac{-1}{(k^6-6k^5-3k^4+28k^3-8k^2-32k+16)^3} \times k^2(k+1) \\ & \times (4k^7-7k^6-22k^5+49k^4+20k^3-88k^2+32k+16) \\ & \times (4k^7-5k^6-26k^5+39k^4+48k^3-88k^2-16k+48) \\ & \times (k^6+2k^5-7k^4+8k^2-16k+16)(k^3-k^2-3k+4)^2 \Big] \end{aligned}$$

which corresponds to

$$t_3 = \frac{k(k^2-3)(k^6-6k^5-3k^4+28k^3-8k^2-32k+16)^2}{(k-2)(k+2)(3k^6-2k^5-13k^4+8k^3+16k^2-16)(5k^6-6k^5-27k^4+40k^3+32k^2-64k+16)}.$$

By solving the quadratic equation in x, we obtain $x = x_1/x_2$, where

$$\begin{aligned} x_1 = {} & (k^2-2)(k^6+2k^5-7k^4+8k^2-16k+16)(k^6-6k^5-3k^4+28k^3-8k^2-32k+16) \\ & \times (4k^7-5k^6-26k^5+39k^4+48k^3-88k^2-16k+48) \\ & \times (4k^7-7k^6-22k^5+49k^4+20k^3-88k^2+32k+16), \\ x_2 = {} & 2(k+1)(k^2-3)(k^3-k^2-2k+4)(2k^4-k^3-7k^2+4k+4)(k-2)^2(k+2)^2(k-1)^3 \\ & \times (3k^6-2k^5-13k^4+8k^3+16k^2-16)(5k^6-6k^5-27k^4+40k^3+32k^2-64k+16). \end{aligned}$$

By getting rid of denominators in a, b, c, d, x we obtain the following proposition, which clearly implies the statements of Theorem 1.

Proposition 2. *Let k be an integer such that $k \neq 0, \pm 1, \pm 2$, and let*

$$
\begin{aligned}
a &= (k-1)^2(k-2)^2(k+2)^2(3k^6-2k^5-13k^4+8k^3+16k^2-16)^2 \\
&\quad \times (5k^6-6k^5-27k^4+40k^3+32k^2-64k+16)^2, \\
b &= 64k^2(k-1)^2(k-2)^2(k+2)^2(k^3-k^2-3k+4)^2(k^2-2)^2 \\
&\quad \times (k^3-k^2-2k+4)^2(2k^4-k^3-7k^2+4k+4)^2, \\
c &= k^2(k-1)^2(k^2-3)^2(k^6-6k^5-3k^4+28k^3-8k^2-32k+16)^2 \\
&\quad \times (4k^7-5k^6-26k^5+39k^4+48k^3-88k^2-16k+48)^2, \\
d &= (k+1)^2(k^3-k^2-3k+4)^2(k^6+2k^5-7k^4+8k^2-16k+16)^2 \\
&\quad \times (4k^7-7k^6-22k^5+49k^4+20k^3-88k^2+32k+16)^2.
\end{aligned}
$$

Then $\{a,b,c,d\}$ is a $D(n_1)$, $D(n_2)$ and $D(n_3)$-quadruple, where

$$
\begin{aligned}
n_1 &= 16k^2(k+1)^2(k-2)^4(k+2)^4(k-1)^6(k^2-3)^2 \\
&\quad \times (k^3-k^2-2k+4)^2(k^3-k^2-3k+4)^2(2k^4-k^3-7k^2+4k+4)^2 \\
&\quad \times (3k^6-2k^5-13k^4+8k^3+16k^2-16)^2 \\
&\quad \times (5k^6-6k^5-27k^4+40k^3+32k^2-64k+16)^2, \\
n_2 &= 4k^2(k^2-2)^2(k^3-k^2-3k+4)^2(k^6+2k^5-7k^4+8k^2-16k+16)^2 \\
&\quad \times (k^6-6k^5-3k^4+28k^3-8k^2-32k+16)^2 \\
&\quad \times (4k^7-5k^6-26k^5+39k^4+48k^3-88k^2-16k+48)^2 \\
&\quad \times (4k^7-7k^6-22k^5+49k^4+20k^3-88k^2+32k+16)^2, \\
n_3 &= 0.
\end{aligned}
$$

For example, by taking $k = 3$ in Proposition 2, we obtain that

$$\{1066758050, 7214407200, 8024417928, 44219811272\}$$

is a $D(90467582183447040000)$, $D(30185892484109116209)$ and $D(0)$-quadruple.

Other points $[X, Y]$ will not necessarily satisfy all required conditions. However, for the point

$$
\begin{aligned}
P + T_1 = \Big[&-\frac{1}{(k^3-k^2-2k+4)^2} \times (k+1)(k^6+2k^5-7k^4+8k^2-16k+16) \\
&\times (k^3-k^2-3k+4)^2, \\
&-\frac{2}{(k^3-k^2-2k+4)^3} \times k^2(k-2)(k+2)(k+1)(k^2-3) \\
&\times (2k^4-k^3-7k^2+4k+4)(k-1)^2(k^3-k^2-3k+4)^2/((k^3-k^2-2k+4)^3) \Big]
\end{aligned}
$$

the corresponding a, b, c, d, x satisfy that $ab + x^2$, $cd + x^2$ are squares, while $ac + x^2$, $ad + x^2$, $bc + x^2$ $bd + x^2$ are $(-k) \times$ squares. By taking $k = -u^2$, we see that all conditions are satisfied, and we obtain the following result.

Proposition 3. *Let u be an integer such that $u \neq 0, \pm 1$, and let*

$$\begin{aligned} a &= 2(u^6+2u^5+u^4-4u^2-4u-4)^2(u^6-2u^5+u^4-4u^2+4u-4)^2\\ &\quad\times(u^3-u^2+u-2)^2(u^3+u^2+u+2)^2,\\ b &= 2(2u^7-u^6+2u^5-u^4-6u^3+4u^2-8u+4)^2\\ &\quad\times(2u^7+u^6+2u^5+u^4-6u^3-4u^2-8u-4)^2(u^4-2)^2,\\ c &= 2(u^2+1)^2(2u^8+u^6-7u^4-4u^2+4)^2\\ &\quad\times(u^6+u^4-2u^2-4)^2u^2(u^4-3)^2,\\ d &= 8(u-1)^2(u+1)^2u^2(u^4-3)^2(u^3-u^2+u-2)^2\\ &\quad\times(u^3+u^2+u+2)^2(u^2+1)^4(u^2+2)^2(u^2-2)^2. \end{aligned}$$

Then $\{a,b,c,d\}$ is a $D(n_1)$, $D(n_2)$ and $D(n_3)$-quadruple, where

$$\begin{aligned} n_1 &= (u-1)^2(u+1)^2(u^4-3)^2(u^2+1)^2(2u^7-u^6+2u^5-u^4-6u^3+4u^2-8u+4)^2\\ &\quad\times(2u^7+u^6+2u^5+u^4-6u^3-4u^2-8u-4)^2(u^6+2u^5+u^4-4u^2-4u-4)^2\\ &\quad\times(u^6-2u^5+u^4-4u^2+4u-4)^2(u^3-u^2+u-2)^2(u^3+u^2+u+2)^2,\\ n_2 &= 64(u^2+1)^4(-2+u^4)^2(2u^8+u^6-7u^4-4u^2+4)^2(u^6+u^4-2u^2-4)^2\\ &\quad\times u^4(u^2+2)^2(u^2-2)^2(u^4-3)^2(u^3-u^2+u-2)^2(u^3+u^2+u+2)^2,\\ n_3 &= 0. \end{aligned}$$

For example, by taking $u=2$ in Proposition 3, we obtain that

$$\{861184, 734247409, 15591268225, 8760960000\}$$

is a $D(30668429385921600)$, $D(2816306908047360000)$ and $D(0)$-quadruple.

Somewhat simpler examples can be found by a brute force search for solutions k, m, t_3 of (3) with small numerators and denominators. Here are some examples obtained in that way:

$\{a,b,c,d\}$	n_1, n_2, n_3
$\{1458, 66248, 5000, 14112\}$	16769025, 406425600, 0
$\{451584, 25921, 12996, 950625\}$	30234254400, 4783105600, 0
$\{985608, 11858, 57800, 352800\}$	49177497600, 4846248225, 0
$\{105625, 50176, 72900, 1002001\}$	2981160000, 129859329600, 0
$\{693889, 116964, 47089, 1982464\}$	144284503104, 52510639104, 0
$\{74529, 2832489, 122500, 1115136\}$	134336910400, 214665422400, 0
$\{438048, 3246152, 187272, 451250\}$	618173337600, 194388401025, 0
$\{349448, 120050, 930248, 3645000\}$	493141017600, 288449555625, 0
$\{31752, 45125000, 3426962, 18727200\}$	1409028350625, 65260546560000, 0
$\{27766152, 1059968, 1820232, 61051250\}$	26694995558400, 122518001376225, 0

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