Stanisław Spież*, *Polish Academy of Sciences, Warsaw, Poland*

Embedding of compacta into product of curves

We present some results on n-dimensional compacta embeddable into n-dimensional Cartesian products of compacta. We pay special attention to compacta embeddable into products of 1-dimensional compacta. Our investigations have been inspired by some results in this direction established by Borsuk, Cauty, Dydak, Koyama and Kuperberg.

We prove that if X is an n-dimensional compactum with non-trivial Čech cohomology group $H^n(X)$ that embeds in a product of n *curves* (i.e. 1-dimensional continua) then there exists an algebraically essential map $X \to \mathbb{T}^n$ into the n-torus. The same is true if X embeds in the nth symmetric product of a curve. The existence of such a mapping implies that there exist elements $a_1, \cdots, a_n$ in $H^1(X)$ whose cup product $a_1 \cdots a_n$ is non-zero. Consequently, $\operatorname{rank} H^1(X) \geq n$ and $\operatorname{cat} X > n$. In particular, $\mathbb{S}^n$, $n \geq 2$, is not embeddable in the nth symmetric product of any curve. The case of $\mathbb{S}^2$ answers in the negative a question of Illanes and Nadler. Also, it follows that neither the projective plane nor the Klein bottle can be embedded in the second symmetric product of any curve.

We introduce some new classes of n-dimensional continua and show that embeddability of locally connected *quasi n-manifolds* into products of n curves also implies $\operatorname{rank} H^1(X) \geq n$. Applying this (with $n = 2$) to either the "Bing house" or the "dunce hat" we infer that neither is embeddable in a product of two curves. So, each is a 2-dimensional contractible polyhedron not embeddable in any product of two curves. On the other hand, we show that any collapsible 2-dimensional polyhedron (e.g. the cone over a graph) can be embedded in a product of two *trees* (i.e. acyclic graphs). We answer a question posed by Cauty proving that closed surfaces embeddable in products of two curves can be also embedded in products of two graphs. We prove that no closed surface $\neq \mathbb{T}^2$ lying in a product of two curves is a retract of that product.

*This is a joint work with Akira Koyama and Jozef Krasinkiewicz