Sibe Mardešić, *University of Zagreb, Croatia*

Functoriality of the standard resolution of the Cartesian product of a compactum and a polyhedron

To study the shape of the Cartesian product $X \times P$ of a compact Hausdorff space X and a polyhedron P, the author has introduced in [2] a resolution $\boldsymbol{q} \colon X \times P \to \boldsymbol{Y}$, here called the standard resolution of $X \times P$. It consists of paracompact spaces having the homotopy type of polyhedra and is completely determined by the limit $\boldsymbol{p} \colon X \to \boldsymbol{X}$ of a cofinite inverse system $\boldsymbol{X}$ of compact polyhedra and by a triangulation K of P. Now the construction is considerably enriched by showing that the standard resolution is a functor. More precisely, with every homotopy class $[\boldsymbol{f}]$ of coherent mappings $\boldsymbol{f} \colon \boldsymbol{X} \to \boldsymbol{X}'$, one can associate a homotopy class $[\boldsymbol{g}]$ of homotopy mappings $\boldsymbol{g} \colon \boldsymbol{Y} \to \boldsymbol{Y}'$ between the corresponding standard resolutions such that $[\boldsymbol{f}] = 1$ implies $[\boldsymbol{g}] = 1$ and $[\boldsymbol{f}''] = [\boldsymbol{f}'][\boldsymbol{f}]$ implies $[\boldsymbol{g}''] = [\boldsymbol{g}'][\boldsymbol{g}]$. The proof uses in an essential way particular cellular decompositions of simplicial complexes and their properties.

Among the consequences of the functoriality of the standard resolution is the existence of a functor R from the strong shape category of compact Hausdorff spaces to the shape category Sh of topological spaces such that $R(X) = X \times P$. This resut is nontrivial, because $X \times P$ need not be the product of X and P in the shape category Sh, as demonstrated in [1]. The functor R plays an essential role in proving the theorem that, for compact Hausdorff spaces X, X' such that X is strong shape dominated by X', $X \times P$ is a product in Sh whenever $X' \times P$ is a product in Sh. An easy consequence of the latter result and a result from [3] is Kodama's theorem from 1973 that, for X an FANR, $X \times Y$ is a product in Sh, for every topological space Y.

References

[1] J. Dydak and S. Mardešić. A counterexample concerning products in the shape category, *Fund. Math.*, 186 (2005), 39–54.

[2] S. Mardešić. A resolution for the product of a compactum with a polyhedron, *Topology and its Appl.*, 133 (2003), 37–63.

[3] S. Mardešić. Products of compacta with polyhedra and topological spaces in the shape category, *Mediterr. J. Math.*, 1 (2004), 43–49.