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On some questions concerning decompositions of shapes into Cartesian factors

Following K. Borsuk, a shape $\mathrm{Sh}(X)$ is said to be *prime*, if it is not trivial and can not be decomposed into the product of two nontrivial shapes.

In 1968, at the Topological Conference in Herceg-Novi, K. Borsuk asked: *Does there exist for every* $\mathrm{Sh}(X) \neq 1$ *a prime factor?*

The above problem was also published in a few of Borsuk's papers from the seventies and eighties, for example in [*Fund. Math.* 67 (1970), 221–240].

We answer this question, showing that there exists a continuum X such that $\mathrm{Sh}(X) \neq 1$ has no prime factor.

We also consider some other problems on decompositions of shapes into factors from Borsuk's papers and his monograph *Theory of Shape*, and from the collection [J. Dydak, A. Kadlof, S. Nowak, *Open Problems in Shape Theory*, Warsaw, 1981].

For example, we prove that for each integer $n \geq 3$, there exists a continuum X such that $\mathrm{Sh}(X) = \mathrm{Sh}^n(X)$, but $\mathrm{Sh}(X) \neq \mathrm{Sh}^{n-1}(X)$. This answers in the negative the following question: *Suppose that* $\mathrm{Sh}(X) = \mathrm{Sh}^n(X)$ *for some* $n \geq 3$*. Is it true that* $\mathrm{Sh}(X) = \mathrm{Sh}^2(X)$*?*

Similar problems in the homotopy category of CW-complexes, related to my previous work on homotopy dominations of polyhedra, are also discussed.