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On Addition Theorems for Inductive Dimensions

The problem discussed here is: *Given a space X which is represented as the union of two subsets X_1 and X_2 of known dimension, what can be said about the dimension of X?* Results giving an estimate of the dimension of the union of two subspaces are known as *addition theorems.*

There are classical addition theorems for dimensions *ind* and Ind if X is hereditarily normal. Namely, $\operatorname{ind} X \leq \operatorname{ind} X_1 + \operatorname{ind} X_2$ and $\operatorname{Ind} X \leq \operatorname{Ind} X_1 + \operatorname{Ind} X_2$. The inequalities are known as Menger-Urysohn formulas. Here we present different addition theorems for these dimensions in more general cases if $\operatorname{Ind} X_1 = m$ and $\operatorname{Ind} X_2 = n$. For example, if X is normal then $\operatorname{ind} X \leq 2(m + n + 1)$.

The above result raises the problem of estimating $\operatorname{ind} X$ in terms of $\operatorname{ind} X_1$ and $\operatorname{ind} X_2$. In particular one question is whether $\operatorname{ind} X$ is finite when both $\operatorname{ind} X_1$ and $\operatorname{ind} X_2$ are finite. The answer is negative if instead of ind one considers inductive dimensions ind_0 or Ind_0 introduced by Charalambous and Filippov. In particular, a hereditarily normal compact space which is the union of two dense zero-dimensional subspaces can be infinite-dimensional in the sense of these dimensions.

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