Non-uniform Exponential Tension Splines

Tina Bosner (tinab@math.hr) and Mladen Rogina (rogina@math.hr)
Department of Mathematics, University of Zagreb

Abstract. We describe explicitly each stage of a numerically stable algorithm for calculating with exponential tension B-splines with non-uniform choice of tension parameters. These splines are piecewisely in the kernel of $D^2(D^2 - p^2)$, where D stands for ordinary derivative, defined on arbitrary meshes, with a different choice of the tension parameter p on each interval. The algorithm provides values of the associated B-splines and their generalized and ordinary derivatives by performing positive linear combinations of positive quantities, described as lower-order exponential tension splines. We show that nothing else but the knot insertion algorithm and good approximation of a few elementary functions is needed to achieve machine accuracy. The underlying theory is that of splines based on Chebyshev canonical systems which are not smooth enough to be ECC-systems. First, by de Boor algorithm we construct Chebyshev exponential tension spline with known jumps in the second derivative, and then use quasi-Oslo type algorithms to evaluate classical non-uniform tension exponential splines.

Keywords: Chebyshev theory, exponential tension splines, knot insertion, generalized de Boor algorithm, generalized Oslo algorithm

AMS subject classification: 41A50, 65D07, 65D17

1. Introduction

Exponential tension splines have proved as a remarkable tool for solving problems in which the solution is expected to grow exponentially, a situation where inferiority of polynomial splines is easily recognized. Three major areas of application exist: projection methods for singularly perturbed BVP's and integral equations, monotone and convex data fitting, and CAGD applications. There is a vast number of references regarding projection methods, mostly collocation for singular perturbation problems [10, 16, 17] and some for Volterra integral equations (see [9] and references therein). It is common for these methods that they try to determine the boundary or internal layer in which the solution grows exponentially. To preserve consistency one must include polynomials, and to catch the exponential growth one can add exponentials of the form $\exp(\pm p_i x)$. The tension parameters $p_i > 0$ are determined in a certain way by the application, and numerical parameters such as knot position and multiplicity also play an important role. In fact, one can use variable meshes and polynomial splines for the same kind of problems, or combine both approaches. The special choice of Shishkin