A de Boor Type Algorithm for Tension Splines

Mladen Rogina* and Tina Bosner

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Abstract:

We consider a special knot insertion algorithm for splines which are piecewisely in the linear space spanned by $\{1, x, \mathrm{Sinh}(px), \mathrm{Cosh}(px)\}$, *i.e.* tension splines with uniform tension $p > 0$. Despite its unusual shape preserving and approximation properties widely used in removing extraneous inflexions and solution of singularly perturbed boundary value problems, little is said about the numerical representation. We show how existing direct and recursive algorithms use dangerous subtractions and can therefore be unstable, especially for calculating the B-spline basis. One possible remedy is to evaluate tension spline by inserting the evaluation point in the knot sequence with maximal multiplicity. We use facts from the theory of Chebyshev splines and treat tension splines as Chebyshev ones. However, various product representations of the associated differential operator $D^2(D^2 - p^2)$ exist, and we choose one that leads to hyperbolic splines in the first reduced Chebyshev system. An analysis follows, showing that certain combinations of hyperbolic functions can be evaluated to the machine precision. For $p = 0$ we obtain cubic splines and the well known de Boor's algorithm, but the linear case $p \to \infty$ is also covered by the instance of the same algorithm. The knot insertion matrices involved behave nicely with respect to the change in tension parameter within range $0 < p < \infty$, but also with respect to the knot sequence. Thus, we can also raise a multiplicity by the knot insertion, and the implementation of the algorithm can use de Boor's maxime 'anything times zero is zero'. Numerical experiments illustrate the theory.

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Mladen Rogina
Dept. of Mathematics, University of Zagreb
Bijenička 30, Zagreb, Croatia
rogina@math.hr

Tina Bosner
Dept. of Mathematics, University of Zagreb
Bijenička 30, Zagreb, Croatia
tinab@math.hr