On Calculating With Lower Order Chebyshev Splines

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Abstract: We develop a technique to calculate with Chebyshev Splines of order ≤ 4, based on the known derivative formula for Chebyshev splines and an Oslo type algorithm. We assume that one special Hermite interpolant can be constructed by stable explicit formulæ, and that splines in the reduced system are simple enough to calculate. Local basis of Chebyshev splines of order 3 and 4 can thus be evaluated as positive linear combinations of less smooth B–splines. The coefficients in such linear combinations are discrete Chebyshev splines, normalized so as to make a partition of unity.

There is a number of interesting special cases, such as Foley's ν-splines, Chebyshev polynomial splines (q-splines), and splines in tension (at least with uniform tension parameters), which can be calculated stably by such formulæ.

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