Tension spline collocation methods for singularly perturbed Volterra integro-differential and Volterra integral equations

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Abstract. We consider a discretization of the singularly perturbed Volterra integro-differential equations (VIDE)

$$\begin{aligned} \varepsilon y'(t) &= q(t) - p(t)y(t) + \int_0^t K(t,s)y(s)ds \quad t \in I := [0,T] \qquad (1) \\ y(0) &= y_0, \end{aligned}$$

and Volterra integral equations (VIE)

$$\varepsilon y(t) = g(t) - \int_0^t K(t,s)y(s)ds \quad t \in I := [0,T], \qquad (2)$$

by spline collocation methods in tension spline spaces, where ε is a small parameter satisfying $0 < \varepsilon \ll 1$, and q, p, g, and K are sufficiently smooth for the equations (1) and (2) to possess a unique solution. We construct the appropriate finite dimensional spaces of powers in tension, consider the numerical difficulties in this construction, and then proceed to an analysis of the global convergence properties of the collocation solution. The existing theory for $\varepsilon = 1$ is extended to the singularly perturbed case, and the numerical examples support the theoretical results.

References

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