A level set method for fluid displacement in realistic porous media

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Scaling up and modeling for transport and flow in porous media
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Joint work with

Steven Bryant, The University of Texas at Austin

Support

- US Department of Energy, grant "Mechanisms leading to coexistence of gas and hydrates in ocean sediments"
- US Department of Agriculture, grant "Quantifying the mechanisms of pathogen retention in unsaturated soils“

Computational resources

- Texas Advanced Computing Center (TACC)

Outline

- **Introduction**
- Modeling
  - Level Set Method
  - PQS Algorithm (Prodanović/Bryant ‘06)
  - Contact angle modeling
- Results
  - 2D
  - 3D
- Conclusions

Pore scale immiscible fluid displacement

- Fluid-fluid interface (meniscus) at equilibrium with constant capillary pressure ***Pc*** and interfacial tension ***σ*** **satisfies Young-Laplace equation**

$$P_c = P_{nw} - P_w = \sigma \kappa$$

- Terminology: **wetting, non-wetting fluid, drainage, imbibition**
- We assume **quasi-static displacement:** at each stage interfaces are **constant mean curvature *(κ)* surfaces**

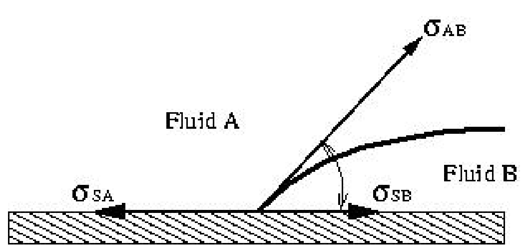

Fig.1. Contact angle at equilibrium satisfies $\sigma_{AB} cos\theta = \sigma_{SA} - \sigma_{SB}$

Statement of the problem

- **Goal**
  - **Accurately** model capillarity dominated fluid displacement in porous media
- **What is the big deal?**
  - Calculating constant curvature surfaces
  - Modeling in irregular pore spaces
  - Accounting for the splitting and merging of the interface within the pore space
- **What do we do?**
  - Adapt the **level set method** for quasi-static fluid displacement

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Level set method

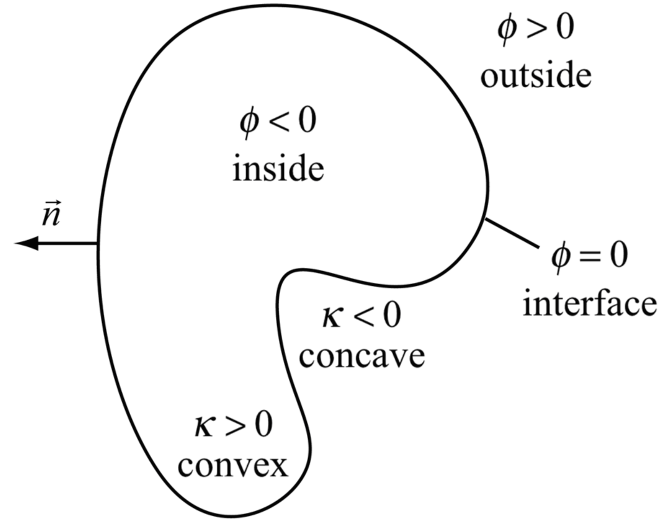
- Osher & Sethian, ’88: embed the moving interface as the zero level set of function Φ
- **The evolution PDE:**

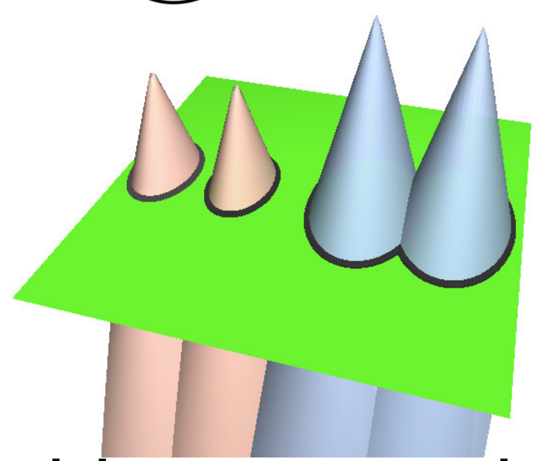
$$\phi_t + F|\nabla \phi| = 0, \quad given\ \phi(\vec{x}, 0)$$

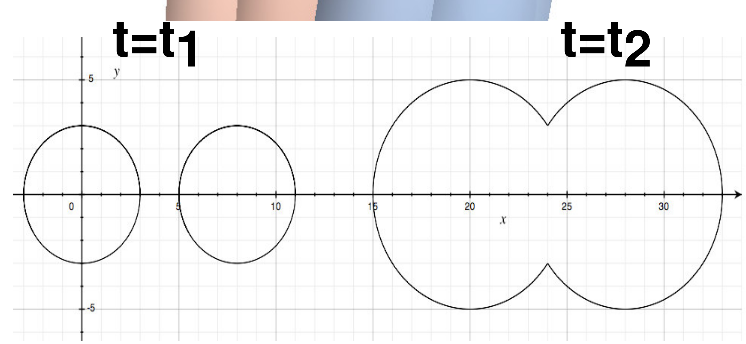
- ***F*** is particle speed in the normal direction, e.g.

$$F(x,t) = p_c - \sigma\kappa(x,t)$$

- Benefits:
  - works in any dimension
  - no special treatment needed for topological changes
  - (above F) finding const. curvature surface by solving a PDE







Progressive quasi-static algorithm (PQS)

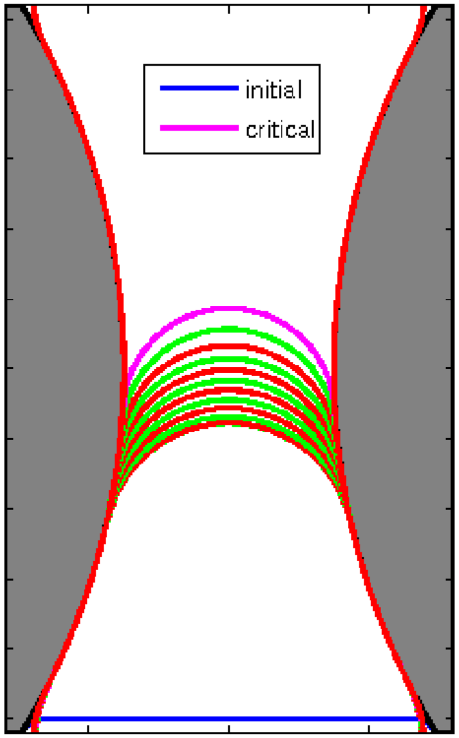
- **Drainage**
  - **Initialize with a planar front**
  - **Solve evolution PDE with slightly compressible curvature model for F until steady state:**

$$F(\vec{x},t) = p_c exp[f(1-\frac{V(t)}{V_m})] - \sigma\kappa(\vec{x},t)$$

  - **Iterate**
    - **increment curvature**
    - **Find steady state of prescribed curvature model**

$$F(x,t) = p_c - \sigma\kappa(x,t) = p_c - \sigma\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|}$$

- **Imbibition** starts from drainage endpoint and decrements curvature
- Zero contact angle: **wall BC** $\phi = max(\phi,\psi)$



M. Prodanović and S. L. Bryant. **A level set method for determining critical curvatures for drainage and imbibition.** *Journal of Colloid and Interface Science*, 304 (2006) 442--458.

Progressive quasi-static algorithm
non-zero contact angle

- Drainage
  - Initialize with a planar front
  - Solve evolution PDE with slightly compressible curvature model for F until steady state:

$$F(\vec{x},t) = p_c exp[f(1 - \frac{V(t)}{V_m})] - \sigma\kappa(\vec{x},t)$$

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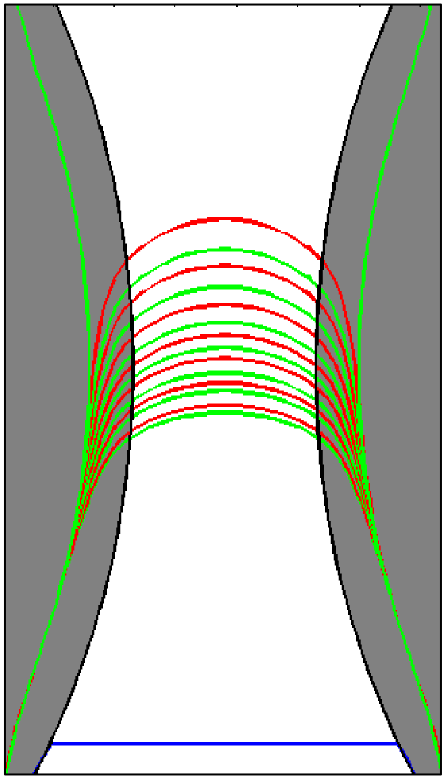

- **Contact angle model**

$$F(\vec{x},t) = p_c H(-\psi) - \nabla \cdot (\sigma(\psi)\frac{\nabla\phi}{|\nabla\phi|})$$

$$\sigma(\psi) = \begin{cases} |\sigma_{SA} - \sigma_{SB}|, & \text{if} \quad \psi \geq 0 \\ \sigma_{AB}, & \text{if} \quad \psi < 0. \end{cases}$$

$$\sigma_{AB} cos\theta = |\sigma_{SA} - \sigma_{SB}|$$

Software available

- LSMLIB Level Set Method Library
  - K. T. Chu / M. Prodanović
  - free for research, next release Jan 2009
  - C/C++/Fortran (serial & parallel), Unix-like env.
  - http://www.princeton.edu/~ktchu/software/lsmlib/index.html
- LSMPQS (Progressive Quasi-static alg.)
  - first release planned Feb 2009

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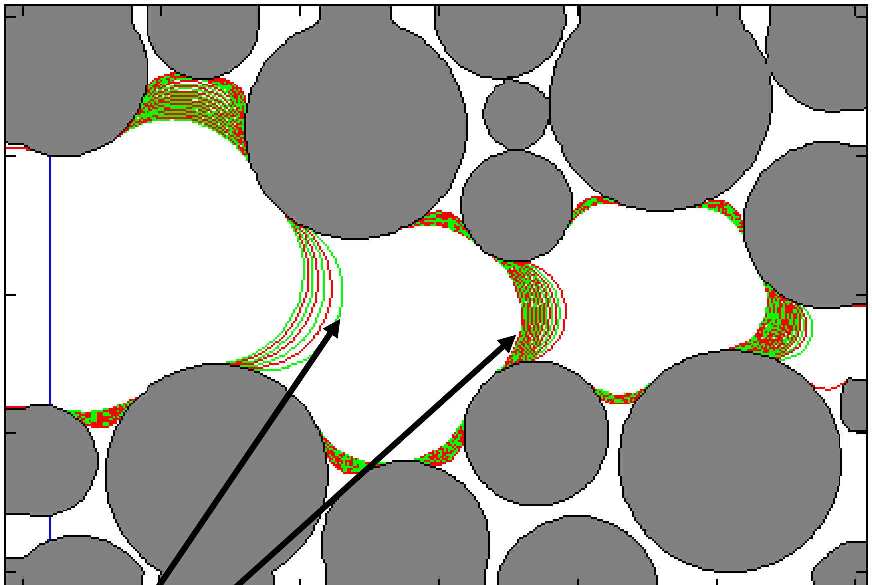
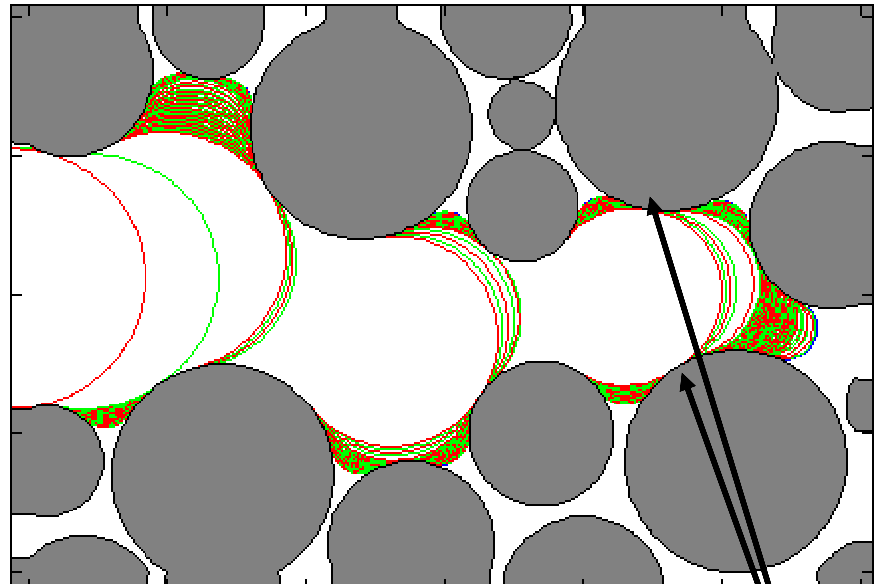
2D Fracture (θ=0)

drainage (controlled by throats)



imbibition (controlled by pores)



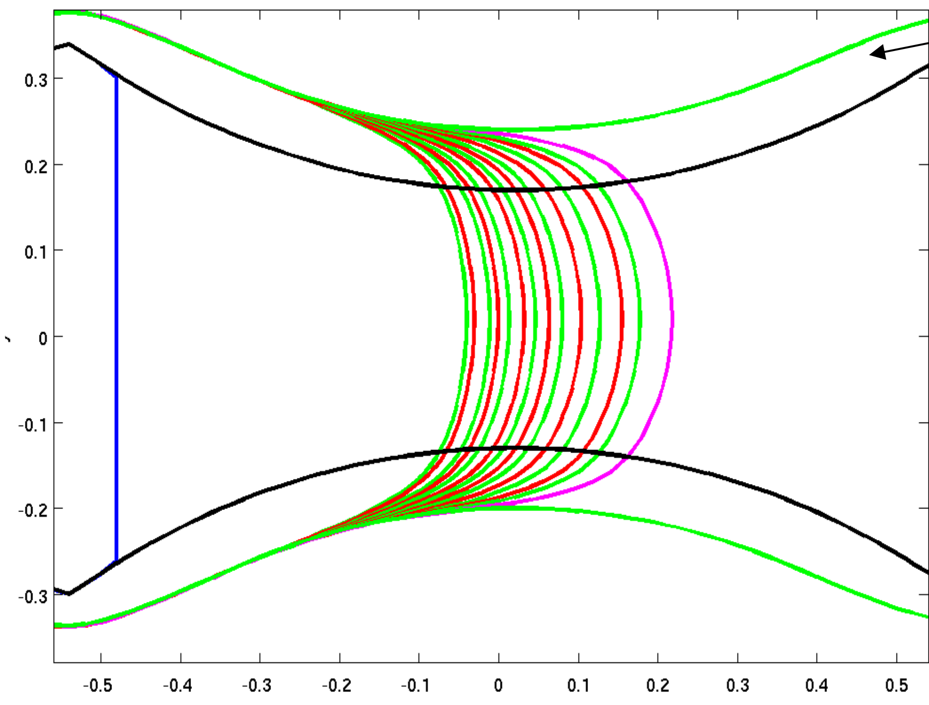
Simulation steps (alternating red and green colors). All <= 2% rel.abs.err.

Haines jump

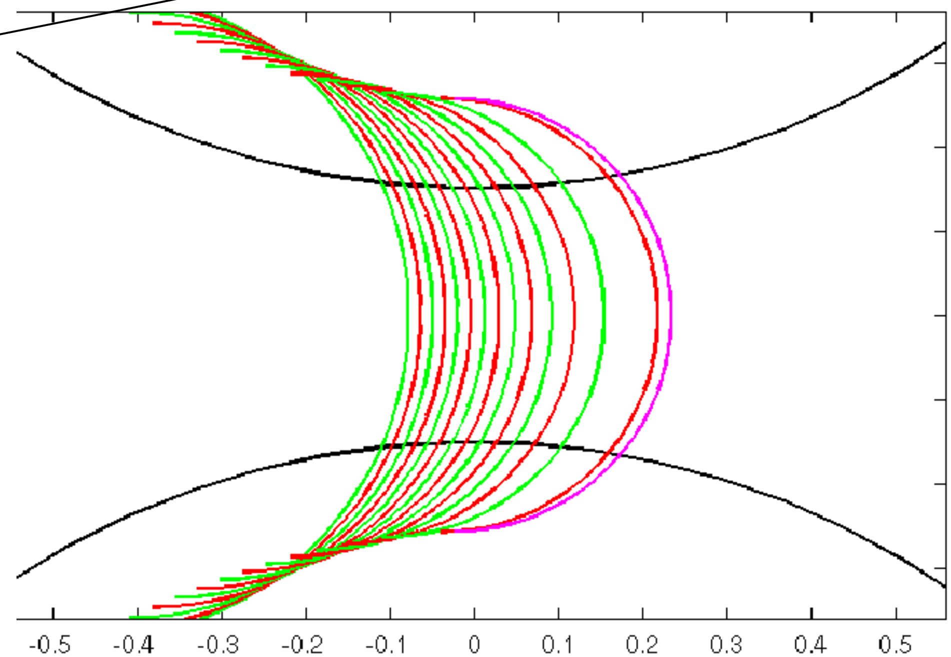
Melrose criterion

2D Throat: θ=60

Some overlap with solid allowed in order to form contact angle



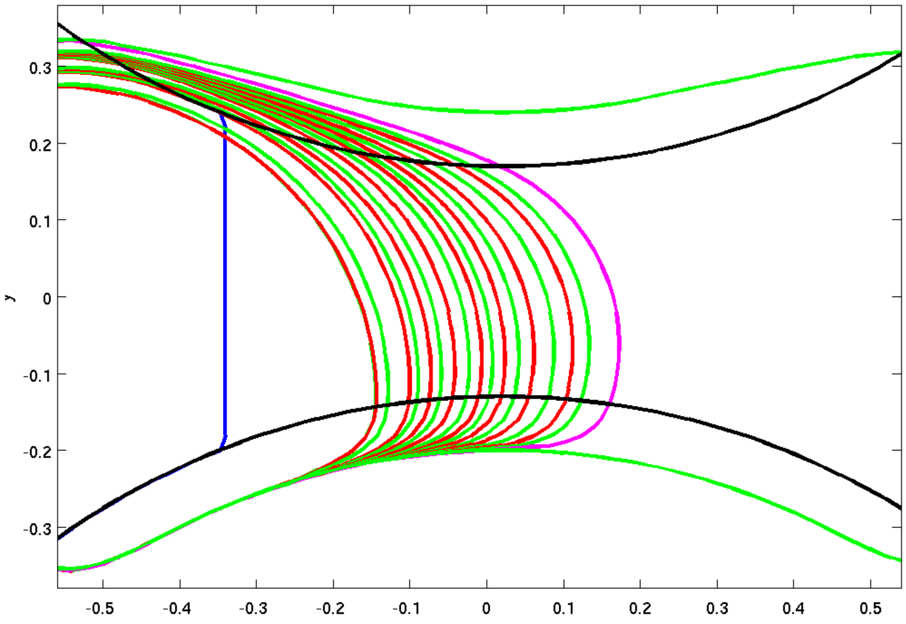
simulation, C=3.88



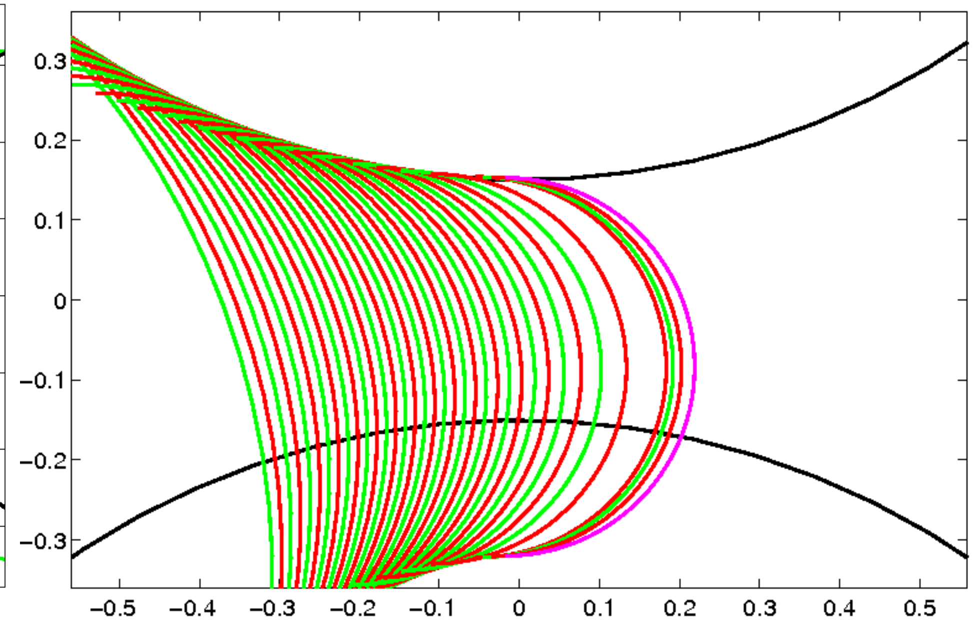
Analytic solution, C=3.89

The last stable meniscus shown in purple: not at geometrical throat!

Fractional wettability: θ=10 and 80



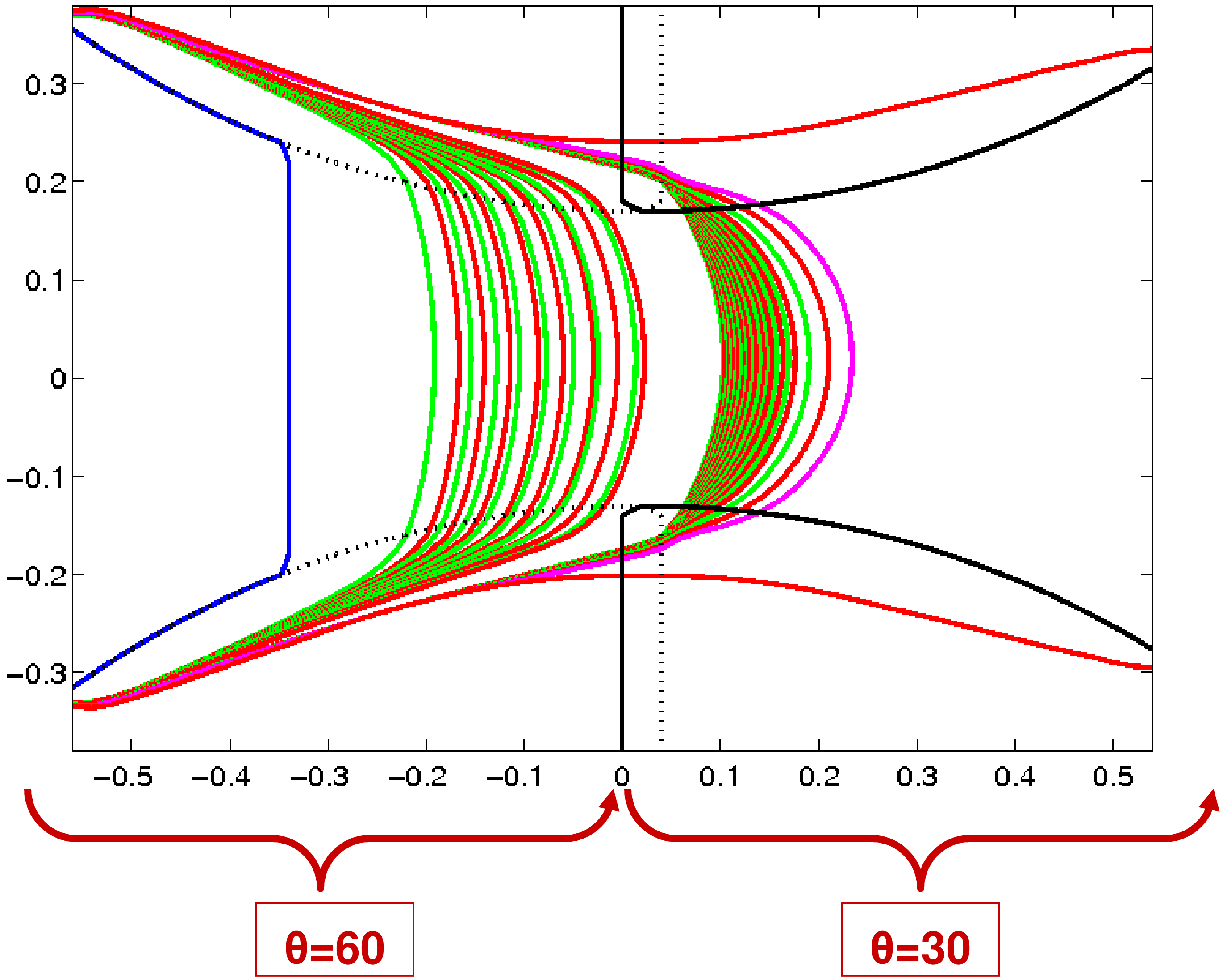
Simulation: C=4.16



Analytic solution: 4.23

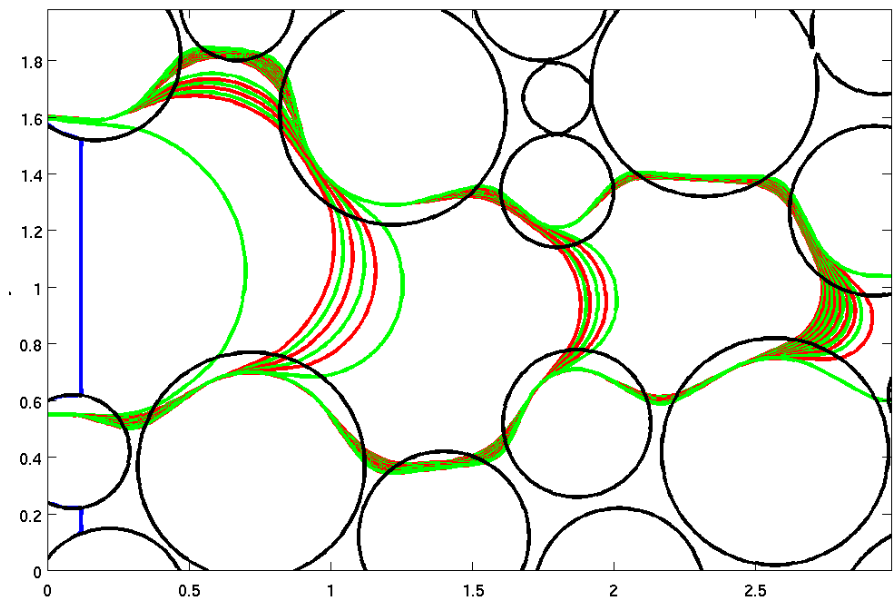
- Last stable meniscus shown in purple

Mixed wettability: θ=60 and 30

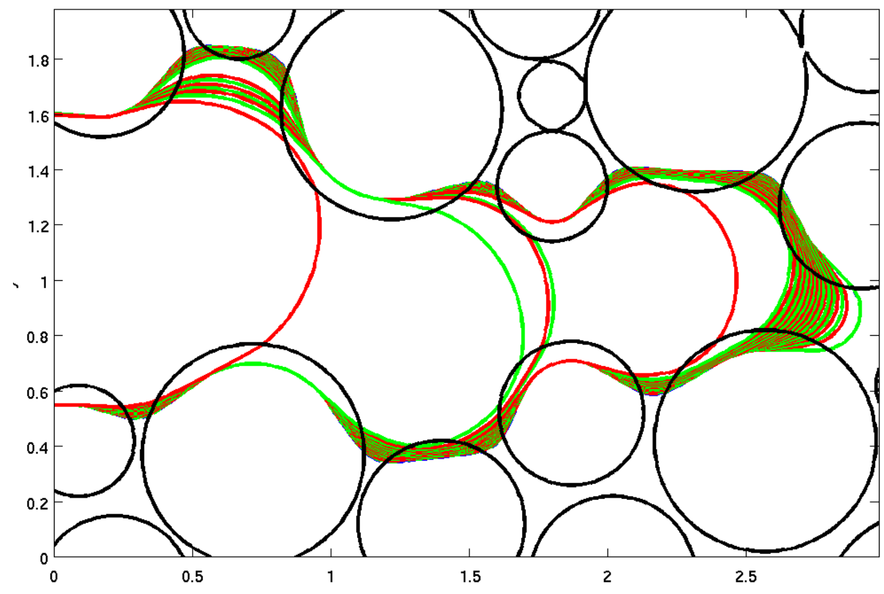


C=5.73

2D Fracture: θ=30



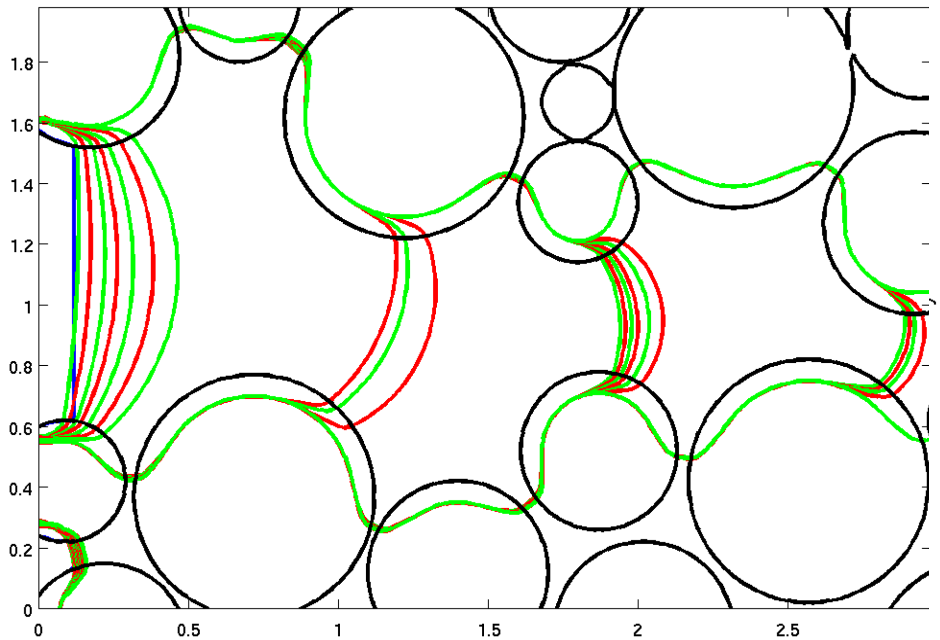
Drainage



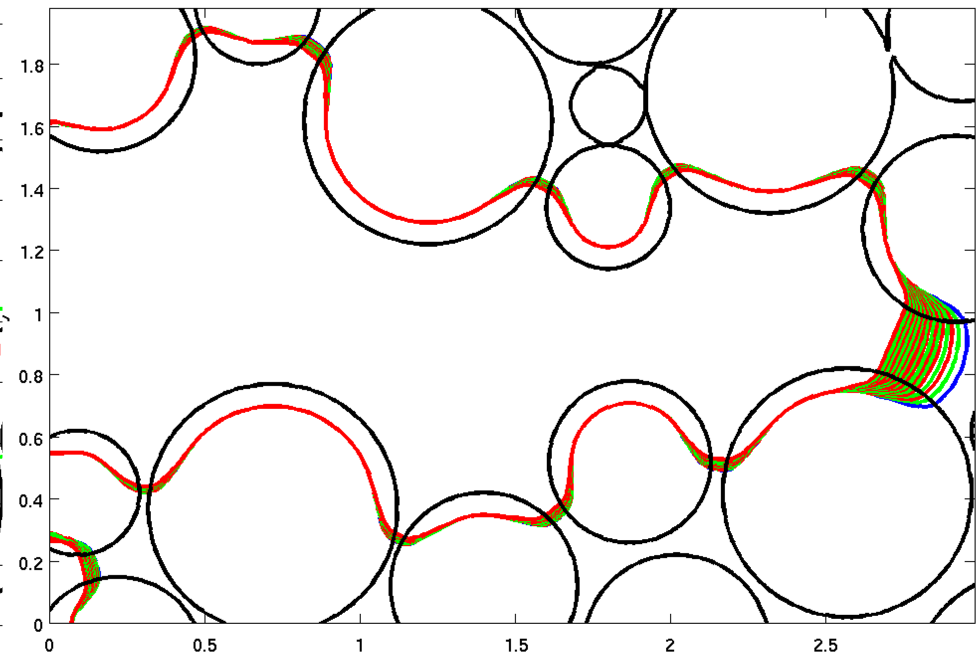
Imbibition

- LSMPQS steps shown in alternate red and green colors

2D Fracture: θ=80



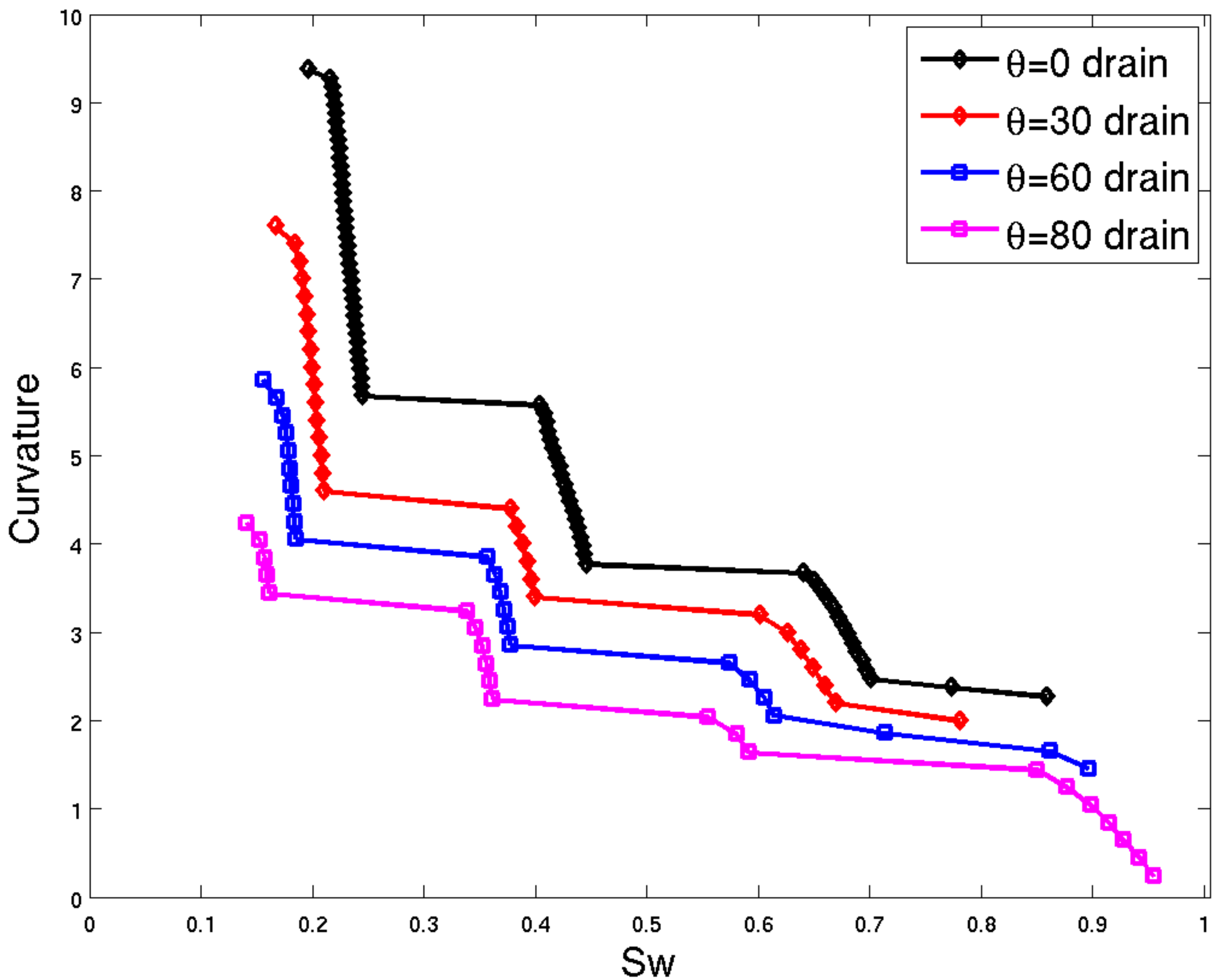
Drainage



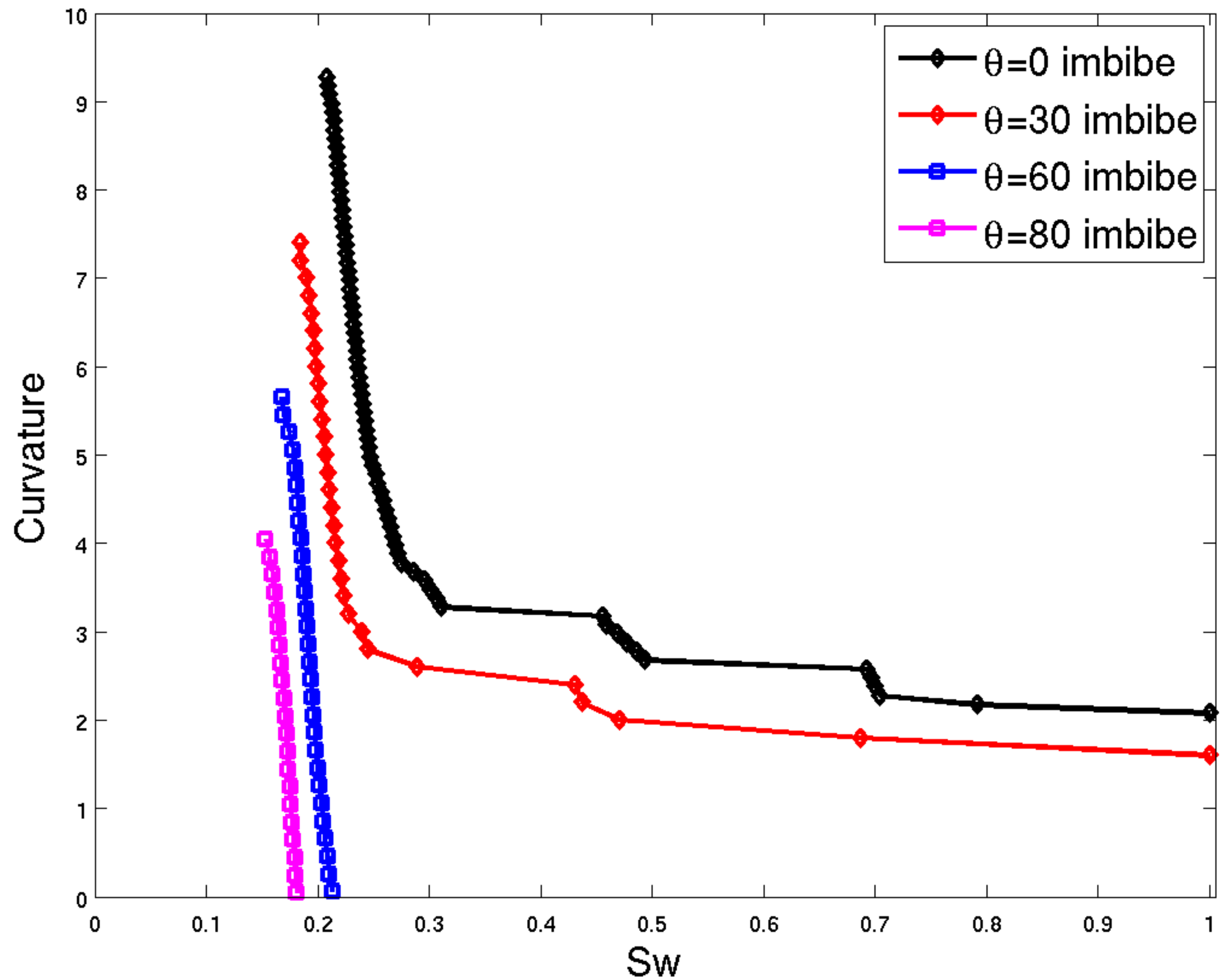
Imbibition: does not imbibe at a positive curvature!

- LSMPQS steps shown in alternate red and green colors

2D Fracture: drainage curves



2D Fracture: imbibition curves



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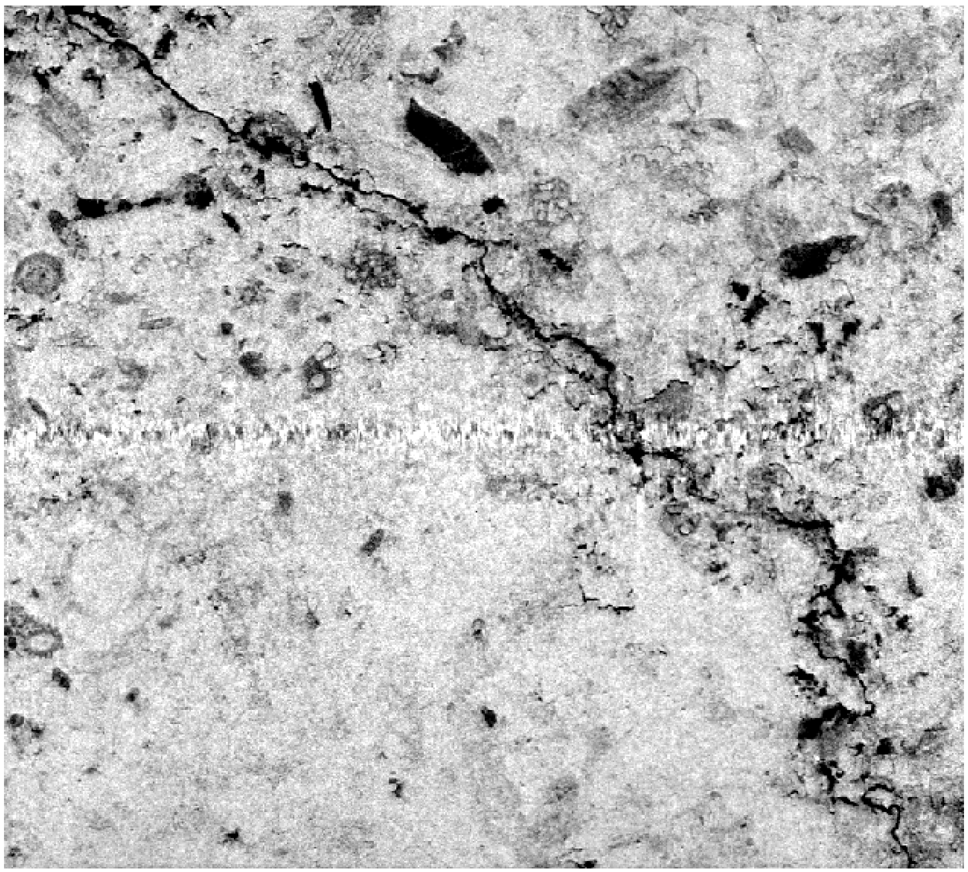
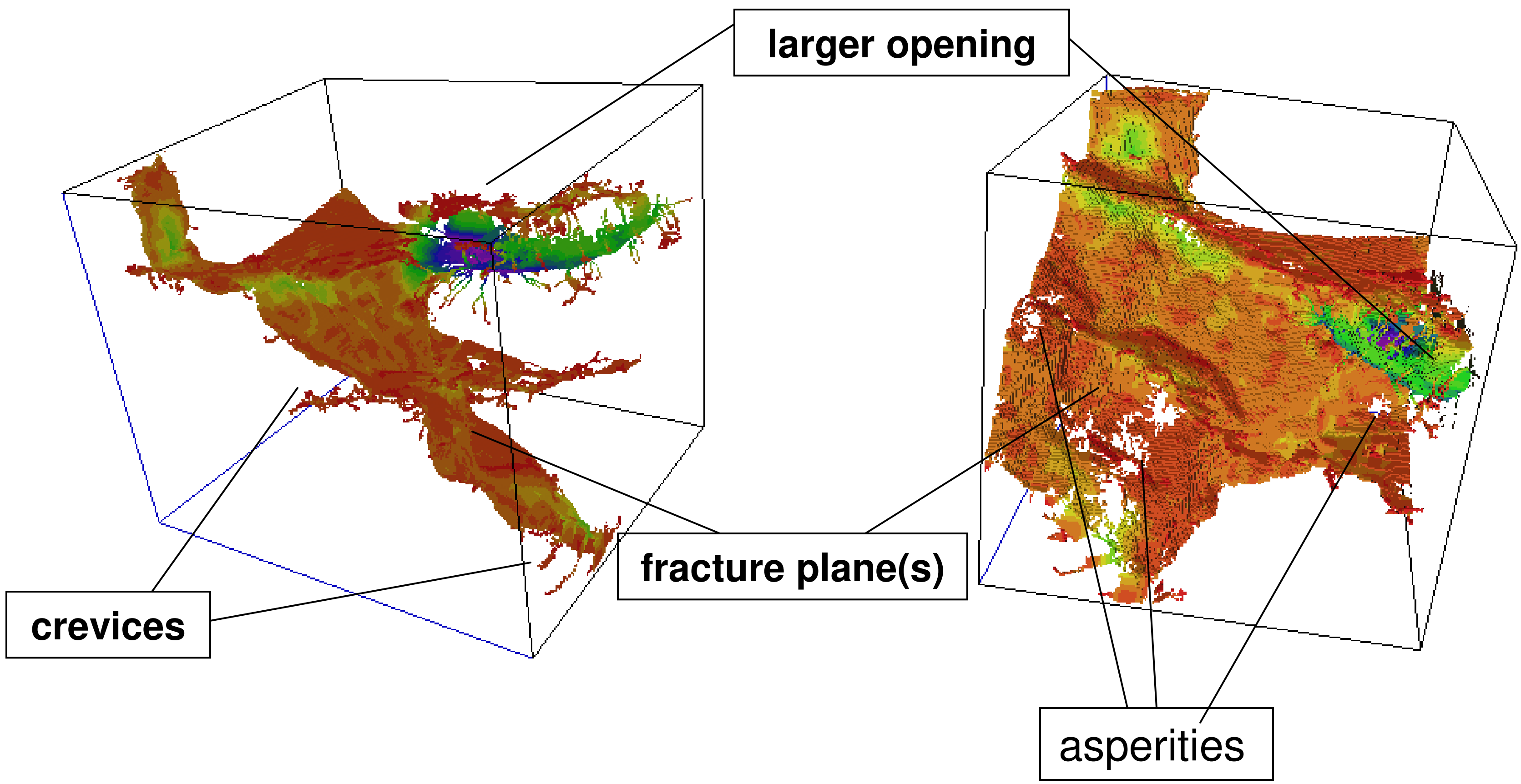
Naturally Fractured Carbonate



- original size 2048^3
- dx $=3.1\mu m$

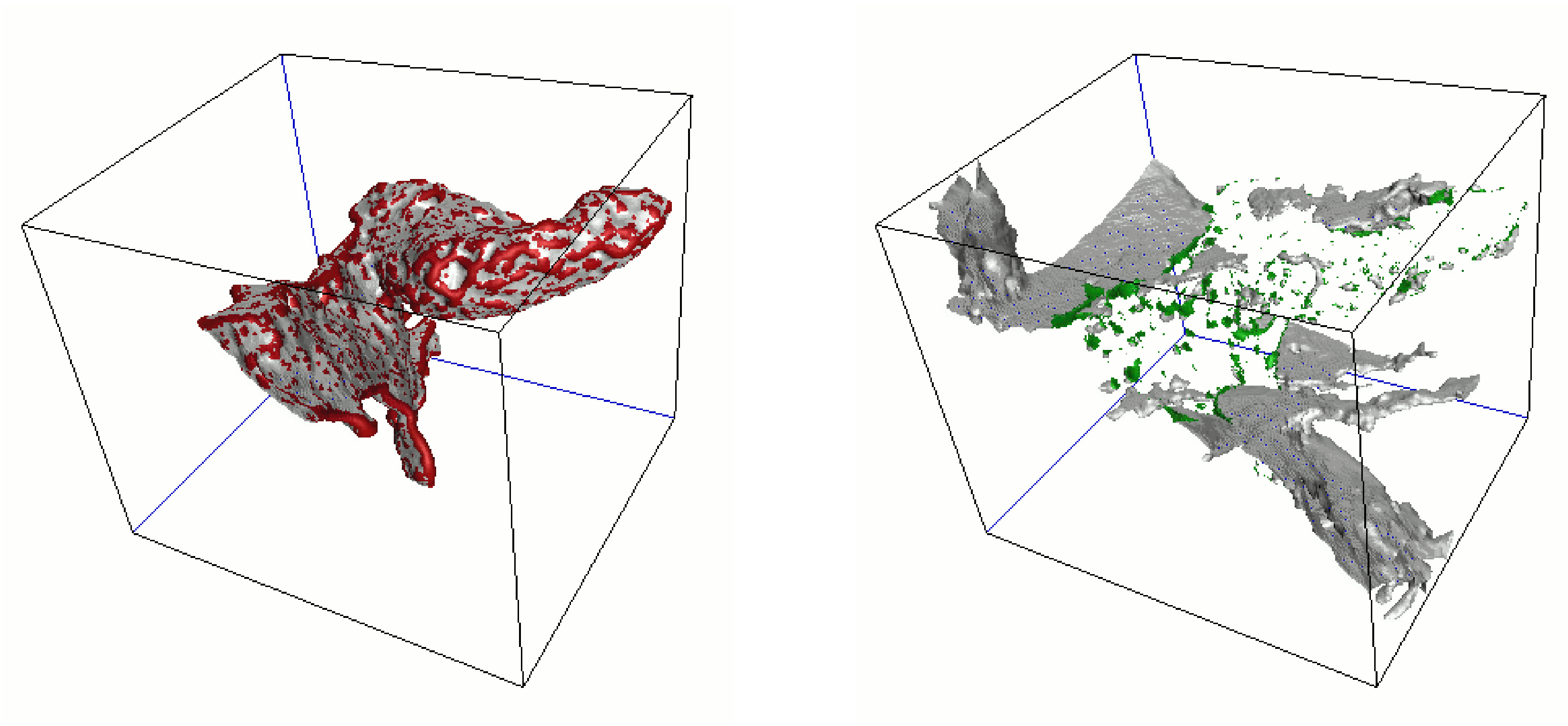
Image courtesy of Drs. M. Knackstedt & R. Sok, Australian National University

Fractured Carbonate Geometry



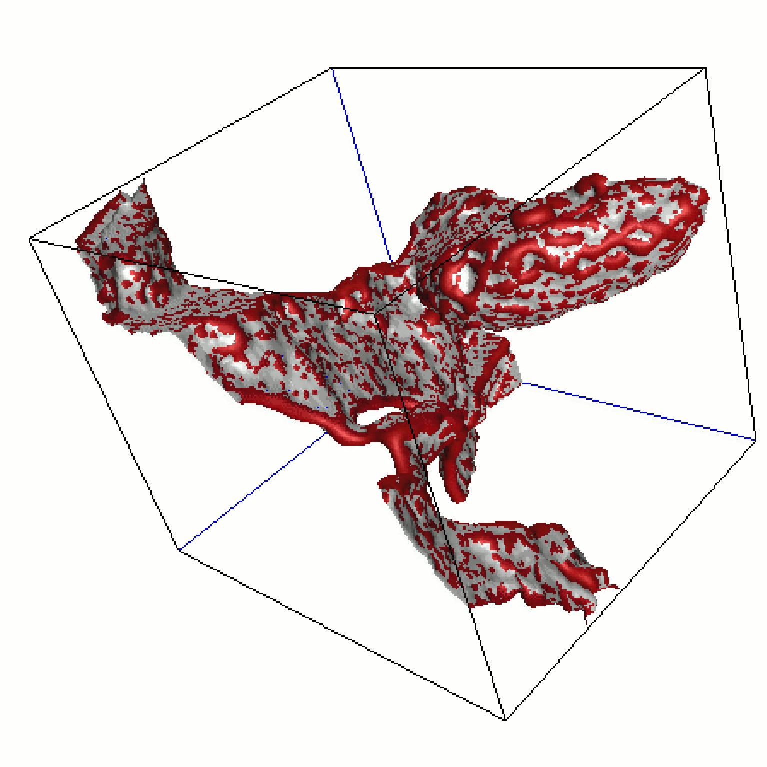
Medial surface of 200x230x190 subsample, rainbow coloring indicates distance to the grain (red close, velvet far)

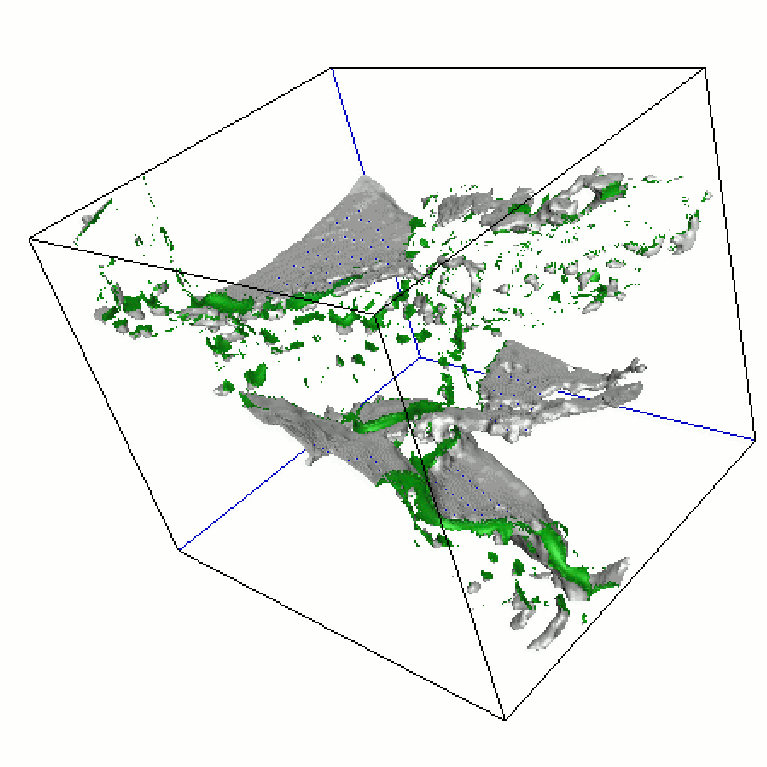
Fractured Carbonate Drainage



Non-wetting (left) and wetting phase surface (right) at $C_{16}=0.11\mu m^{-1}$

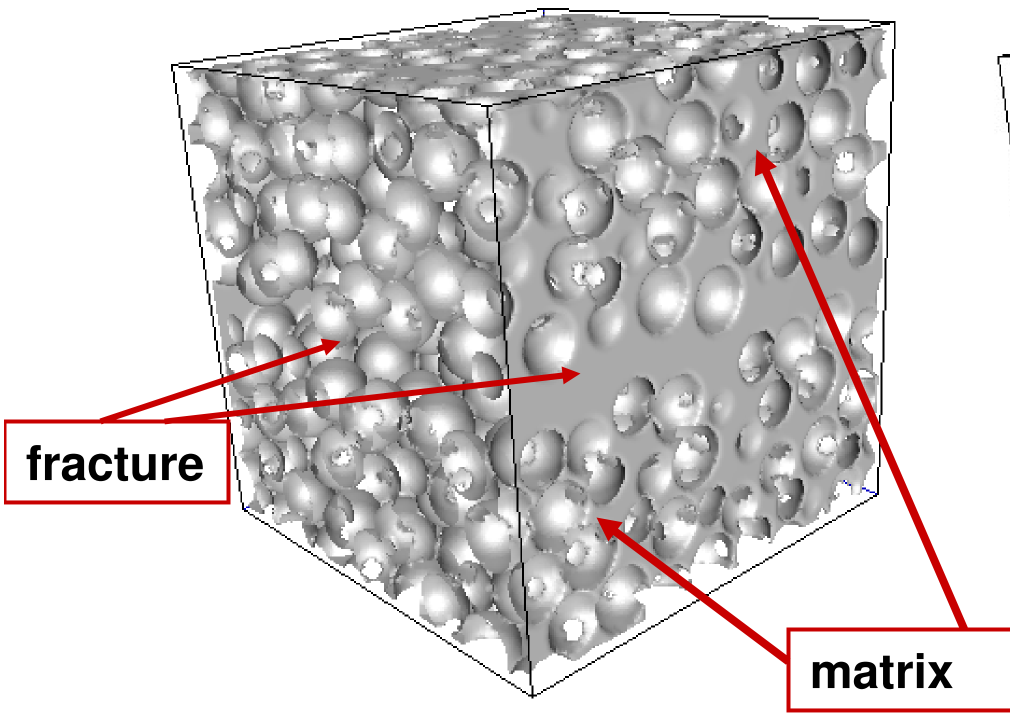
Fractured Carbonate Imbibition



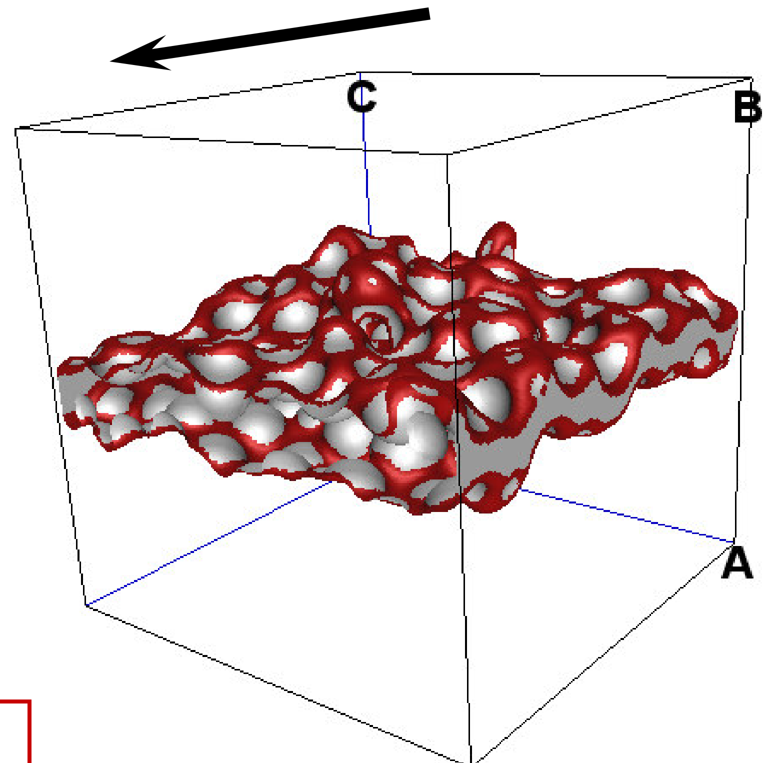


Non-wetting fluid (left) and wetting fluid (right) surface, $C_{15}=0.09\mu m^{-1}$

Fractured Sphere Pack

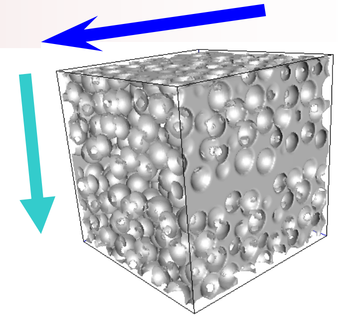


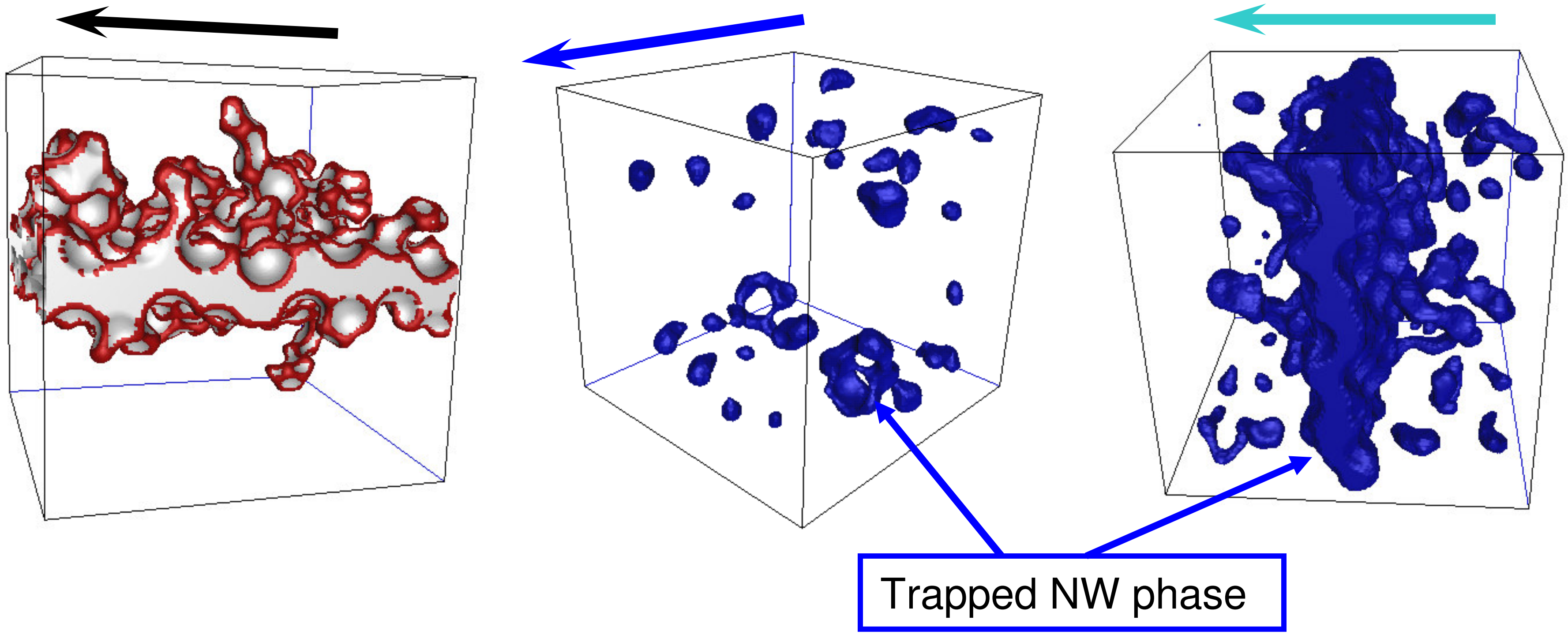
Pore-grain surface
sphere radii R=1.0
Image size 160^3 (dx=0.1)



NW phase surface in fracture
(drainage beginning)

Drainage and Imbibition



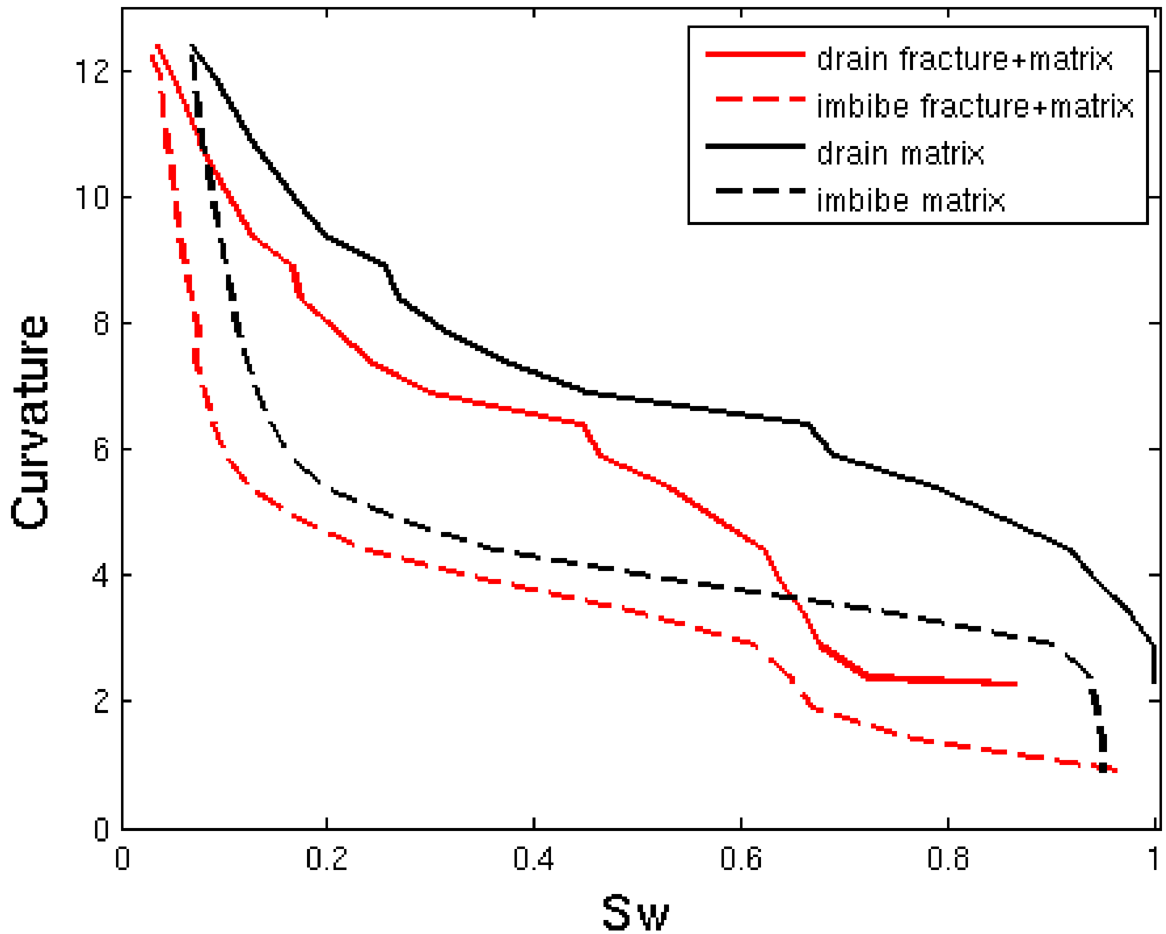


Drainage, C=4.9

imbibition, C=0.24

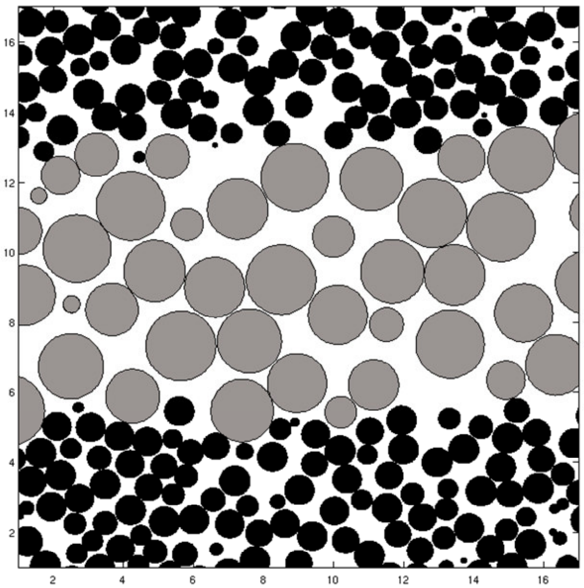
imbibition – rotated C=2.15

Simulated Pc-Sw: Fractured Sphere Pack

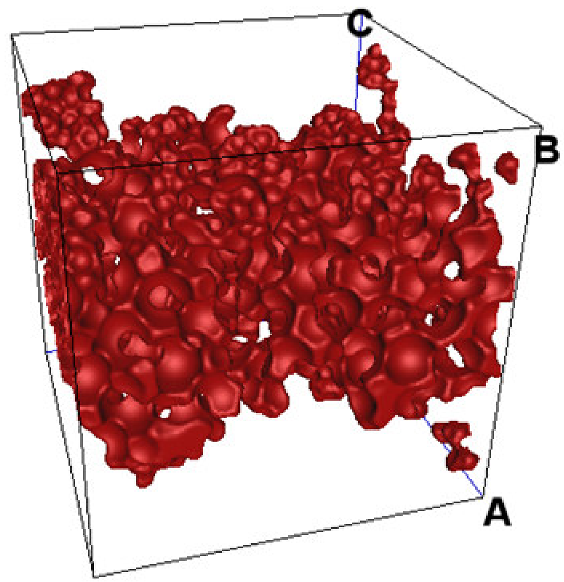


- In a reservoir simulation fracture+matrix curve might serve as an upscaled input (for a fractured system)

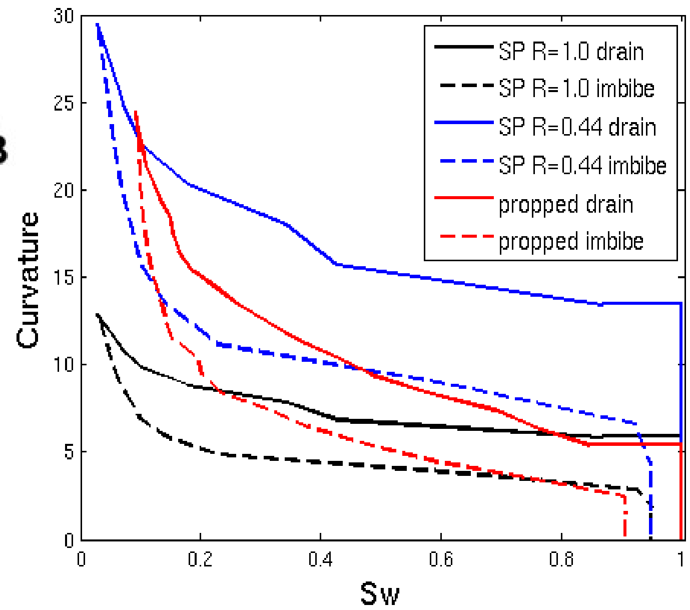
Fracture With Proppant: Drainage and Imbibition



(a)



(b)



$R_1 = 1.0$

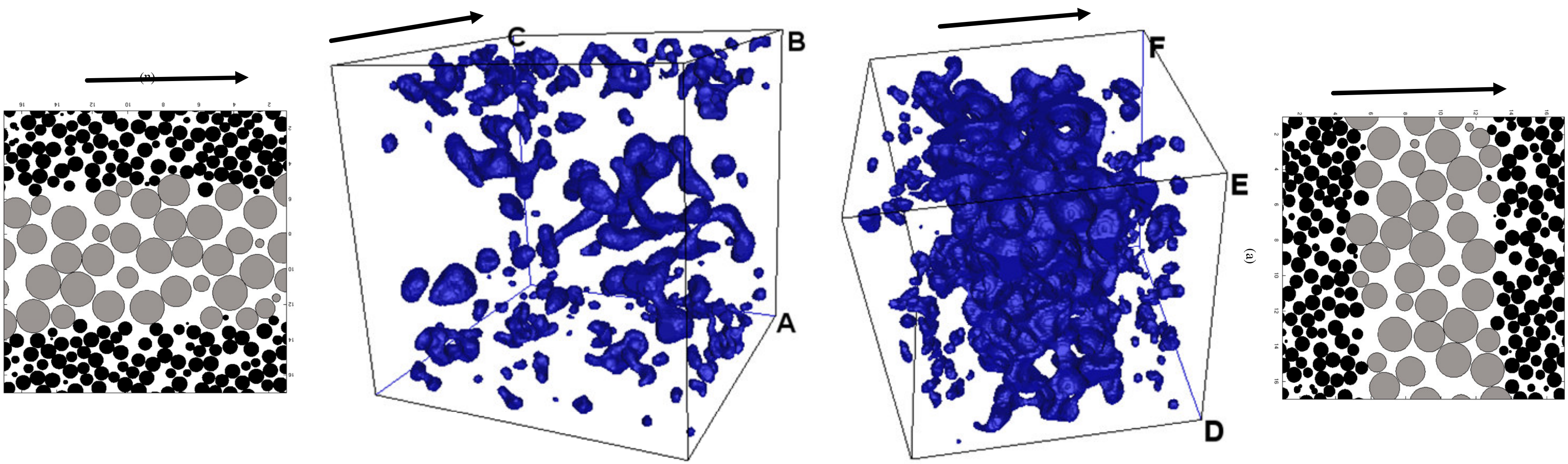
$R_2 = 0.44$

Drainage – matrix begun to drain

C=6.45

C-Sw curve for both drainage and imbibition

Fracture With Proppant: Residual non-wetting phase



(a) (b)

Residual oil at the imbibition endpoint for two directions of invasion

Conclusions

- **Drainage/imbibition modeling is**
  - **Geometrically correct; Haines jumps, Melrose criterion**
  - **Robust with respect to geometry**
- **We can easily obtain Pc-Sw curves, fluid configuration details (volumes, areas)**
- **Modeling (fractional & mixed) wettability possible**
- **Capillarity has an important effect on flow in rough wall fractures with contact points – we find W phase blobs around contacts and hysteresis of C-Sw curves**
- **The extent to which nonwetting phase is trapped in fracture/enclosed gaps is very sensitive to the direction of the displacement**
- **In a reservoir simulation the Pc-Sw curves in matrix+fracture system might serve as an upscaled drainage curve input for a fractured medium.**

Thank you!

More Info:

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